POISSON DENOISING WITH MULTIPLE DIRECTIONAL LOTS

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ABSTRACT

This paper proposes a Poisson denoising with a union of directional lapped orthogonal transforms (DirLOTs). DirLOTs are 2-D non-separable lapped orthogonal transforms with directional characteristics. Its bases overcome a disadvantage of the separable wavelet image denoising for the diagonal textures and edges. Based on this feature, multiple DirLOTs are used to improve the performance by introducing redundant representation with multiple directions. Experimental results show the combination of the variance stabilizing transformation (VST), Stein's unbiased risk estimator-linear expansion of thresholds (SURE-LET) approach and multiple DirLOTs is able to significantly improve the denoising performance, and verify the feasibility of the proposed method.

***Index Terms*—** Multiple DirLOTs, Wavelet shrinkage, Anscombe transform, SURE-LET

1. INTRODUCTION

Image denoising is one of basic problems of image processing, the purpose is to make the quality of the noise image better. Measurement noise are possible to jointly appear in an image obtained by digital image acquisition whose predominant sources are the stochastic nature of the photon-counting process at the detectors and the intrinsic thermal and electronic fluctuations of the acquisition devices. Under many conditions such as low-power light source, short exposure time, and phototoxicity appeared in the photon acquisition systems (e.g., fluorescence microscopy, astronomy and medical devices), only a few photons are collected by the photosensors, appearing the noise that approximately obeys Poisson distribution. This noise is strongly signal-dependent, which leads to the difficulties in denoising process. During the image denoising, the image noise corrupted mentioned above can be modeled as Poisson noise.

The denoising problem for Poisson noise can be modeled by a modular fashion through variance stabilization. The denoising process can be described as follows: First, to modify the noisy data by applying a nonlinear variance stabilizing transformation (VST); Second, to treat the modified noisy data with algorithms designed for the removing of Gaussian noise; Third, to obtain the desired estimation of the unknown noise-free image by applying an inverse VST to the denoised data.

In the first step, since the image is corrupted by the signal-dependent noise whose variance varies with the expectation of the pixel value, the VST should be applied so that the transformed data of Poisson noise can be approximately modeled by the Gaussian noise distribution with a know constant variance [1], [2]. The denoising problem for Gaussian noise becomes possible to apply to Poisson denoising algorithms.

Among Gaussian denoising methods, the SURE-LET is relatively efficient[3]. However, its performance becomes worse for the regions where the interscale correlation is weak, which can generally be overcome by separable transforms (e.g., Haar and Symlets). Nevertheless, the representation of edges, diagonal textures, and gradual changing are inadequate with these separable transforms. To address this problem, the non-separable orthonormal transform was proposed to be applied to the SURE-LET image denoising so that its performance for diagonal textures and edges can be improved [4]. However, this approach is only applicable to a fixed single geometric direction. In order to overcome the problem, a redundant transform with multiple DirLOTs was proposed so that the performance of image denoising can be improved [5].

For Poisson noise, the denoising process is different from that of Gaussian noise. The variance of Gaussian noise is stationary, whereas the variance of Poisson noise is non-stationary. A denoising algorithm designed for Poisson noise based on a Haar-Fize transform has been proposed in [6]. The Haar-Fize transform cannot satisfy directional characteristics. Zhang et al. proposed a hybrid approach that combines VSTs, hypothesis testing, l_1-penalized reconstruction and advanced redundant multiscale representations [7]. The curvelet can efficiently approximate smooth curve edges. But it has a

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question that how to construct a tight curvelet-like transform in discrete domain. In [8], minimizing MSE estimation for Poisson noise based on an unnormalized Haar wavelet transform, so called PURE-LET, was proposed. PURE-LET is very efficient in terms of denoising performance and computational complexity. The proposed PURE-LET exploits a linear denoising function to search the optimal solution. However, the wavelet transform has a disadvantage of representing diagonal geometric structures. Tight frames are preferable for many applications since they reduce mathematical handling of algorithms significantly.

The transforms were adopted in the above methods donot satisfy both of tightness and symmetry simultaneously. The orthogonality, symmetry, repeatability and directivity cannot be satisfied at the same time for traditional wavelet transform. Therefore, the diagonal textures, edges, and gradually changing content cannot be sufficiently represented. In order to solve these problems and to improve the denoising performance, a combination of the VST, SURE-LET approach and multiple DirLOTs will be introduced in this paper.

The remainder of this paper is organized follows: principle of the Poisson noise, and the Anscombe transform will be briefly introduced in Section II. In Section III, we will describe the SURE-LET approach. The proposed multiple DirLOTs will be given in Section IV. We will show the experimental results in Section V, and finally, conclude this paper in Section VI.

2. PRELIMINARIES

In this section, let us review the Poisson noise and Anscombe Transform as well as its inverse transform.

2.1. Poisson noise

Suppose $\mathbf{x} = \{x_i\}$, $i \in \mathbf{R}^2$ is a noiseless image. We use boldface $\mathbf{b}$ to denote the image observed through an image acquisition system. The $\mathbf{b}$ consists of N independent Poisson random variables b_i depending on the underlying intensities x_i, with $b_i \sim P(x_i)$. Each pixel intensity b_i can be considered as a Poisson random variable with the following probability density function

$$P(b_i|x_i) = e^{-x_i}\frac{x_i^{b_i}}{b_i!} \tag{1}$$

where x_i denotes the mean of b_i, which equals to its variance σ_i^2 for Poission distribution. A realization of $\mathbf{b}$ can be thought of as a noisy measurement of the intensity signal $\mathbf{x}$. Poisson noise $\mathbf{p}$ can be modeled by

$$p_i = b_i - x_i, \tag{2}$$

thus, $E(p_i|x_i) = 0$ and $D(p_i|x_i) = x_i$.

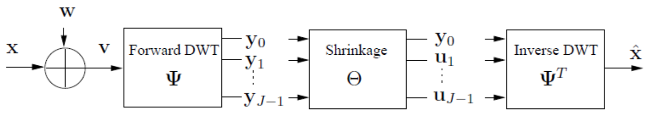

Fig. 1: Principle of orthonormal wavelet denoising

2.2. Anscombe Transform

Many existing Gaussian denoising algorithms cannot be directly applied to Poisson denoising model due to the non-stability and the dependence on the underlying intensity of its variance. Aiming at solving this problem, several VST methods have been adopted [7], [6]. Among them, the Anscombe transform was chosen due to its extensive application, efficiency, and simplicity [1]. The expression of the Anscombe transform can be given by

$$f(\mathbf{b}) = 2\sqrt{\mathbf{b} + \frac{3}{8}} \tag{3}$$

where $\mathbf{b}$ and $f(\mathbf{b})$ denote the observed image contaminated by Poisson noise, the transformed data, respectively.

After the Anscombe transform is performed, the noise throughout the whole image can be approximately modeled by Gaussian distributed. As a result, it is possible to apply Gaussian denoising algorithms for Poisson denoising. Inverse transform of the Anscombe transform is needed in order to return the variance-stabilized and denoised data to the original range. In this paper, a closed-form approximation of this exact unbiased inverse was adopted [9].

3. SURE-LET APPROACH

The pixel values that were observed by an image acquisition device can be defined as $\mathbf{v} = (v_0, v_1, ...v_{N-1})^T$, where N is the number of pixel values, and $(\cdot)^T$ is the transpose operator. The denoising problem of the image corrupted by Poisson noise is then equivalent to solve the noise-free image $\mathbf{x}$ based on the observations $\mathbf{v}$.

The observed picture $\mathbf{v}$ is usually corrupted with noise $\mathbf{w}$ which is generally modeled as an AWGN with zero mean. Let $\mathbf{x}$ be the original clean noiseless picture. Then, the observed image $\mathbf{v}$ can be expressed by

$$\mathbf{v} = \mathbf{x} + \mathbf{w}. \tag{4}$$

Image denoising is to find a good candidate $\hat{\mathbf{x}}$ of unknown noiseless picture $\mathbf{x}$ only from the observed picture $\mathbf{v}$. Figure 1 shows the principle of orthonormal wavelet denoising, where Ψ, Θ and Ψ^T are the forward discrete wavelet transform (DWT), the shrinkage function and the inverse DWT, respectively. The quality of image denoising is determined by the transform Ψ and the shrinkage function Θ.

The SURE-LET approach is a technique to realize the shrinkage function Θ. During the implementation of SURE-LET, all of the priori hypotheses are able to be avoided on

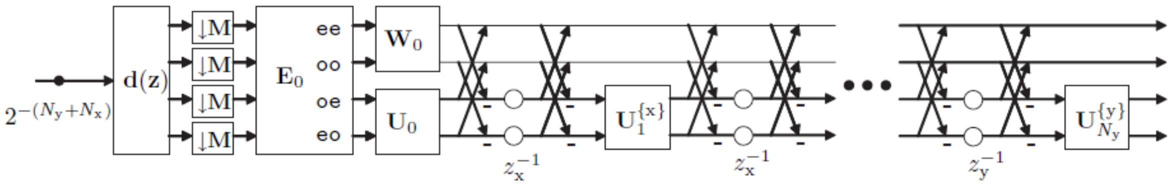

Fig. 2: Lattice structure of DirLOT (forward transform). $\mathbf{d}(\mathbf{z})$ is defined as a 2-D delay chain of size 4×1. Symbols $\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$ are orthonormal matrices with of size $M/2 \times M/2$.

Table 1: Characteristics of transforms

Property	DirLOTs	DWT (5/3,9/7)	Haar DWT	DCT
orthonormal	Yes	No	Yes	Yes
symmetric	Yes	Yes	Yes	Yes
overlapping	Yes	Yes	No	No

the noiseless picture $\mathbf{x}$ with the AWGN assumption. Then, the denoising problem can be reformulated as the search for the denoising process that minimizes the Stein's unbiased risk estimate (SURE) [10], [11].

4. IMAGE DENOISING WITH MULTIPLE DIRLOTS

Compared with other transforms shown in Table 1, we propose to use the DirLOTs as a critically sampled orthonormal wavelet basis. DirLOTs are able to completely satisfy the following three properties: orthogonality, symmetry, and overlapping with a non-separable basis. This transform can be constructed with a lattice structure as shown in Figure 2 [13], [14], [15]. In addition, it can also satisfy the fixed-critically-subsampling, real-valued, and compact-support property. Furthermore, DirLOTs can hold the trend vanishing moments (TVMs) for any direction. The directional property works well for diagonal textures and edges.

The corresponding polyphase matrix of order $[Ny, Nx]$ can be represented by

$$\mathbf{E}(\mathbf{z}) = \prod_{n_y=1}^{N_y} \{\mathbf{R}_{n_y}^{\{y\}} \mathbf{Q}(z_y)\} \cdot \prod_{n_x=1}^{N_x} \{\mathbf{R}_{n_x}^{\{x\}} \mathbf{Q}(z_x)\} \cdot \mathbf{R}_0 \mathbf{E}_0,$$

where

$$\mathbf{Q}(z_d) = \frac{1}{2}\begin{pmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}} \\ \mathbf{I}_{\frac{M}{2}} & -\mathbf{I}_{\frac{M}{2}} \end{pmatrix}\begin{pmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & z_d^{-1} I_{\frac{M}{2}} \end{pmatrix}\begin{pmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}} \\ \mathbf{I}_{\frac{M}{2}} & -\mathbf{I}_{\frac{M}{2}} \end{pmatrix}$$

$$\mathbf{R}_0 = \begin{pmatrix} \mathbf{W}_0 & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & \mathbf{U}_0 \end{pmatrix}, \mathbf{R}_n^{\{d\}} = \begin{pmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{0}_{\frac{M}{2}} \\ \mathbf{0}_{\frac{M}{2}} & \mathbf{U}_n^{\{d\}} \end{pmatrix},$$

where $\mathbf{W}_0$, $\mathbf{U}_0$ and $\mathbf{U}_{n_d}^{\{d\}}$ are orthonormal matrices with of size $M/2 \times M/2$. All of them can be freely controlled during the design process.

As a single DirLOT is not suitable for images with oblique texture and edges in various directions. In order to express the oblique texture and edges better, a dictionary is defined by as the multiple DirLOTs as follows

$$\mathbf{D} = [\Psi_{0\cup\frac{\pi}{2}}^T \ \Psi_{\phi_1}^T \ \Psi_{\phi_2}^T \ \Psi_{\phi_3}^T \ ... \ \Psi_{\phi_{R-1}}^T]^T,$$

where $\Psi_{0\cup\frac{\pi}{2}}^T$ is a nondirectional symmetric orthonormal DWT with the classical two-order vanishing moments (VMs) [12], [13], and Ψ_{ϕ_i} is a directional symmetric orthonormal wavelet transforms (DirSOWTs) constructed by a DirLOTs with the two-order TVMs for the direction ϕ_i. R corresponds to the redundancy. Since the column vectors in $\mathbf{D}$ constructs a normalized tight frame and satisfy

$$\mathbf{D}^T\mathbf{D} = \sum_{k=0}^{R-1} \Psi_k^T \Psi_k = R\mathbf{I},$$

which makes the process simple. In this paper, a heuristic shrinkage was adopted. The heuristic shrinkage takes the average of the denoising results obtained by independent SURE-LET denoising with Ψ_k for k = 1, 2, $\cdots$, R-1 [16], [5]. The heuristic shrinkage is available and simply realized by

$$\hat{\mathbf{x}} = \frac{1}{R}\mathbf{D}^T\Theta(\mathbf{D}\mathbf{v}) = \frac{1}{R}\sum_{k=0}^{R-1} \Psi_k^T \Theta(\Psi_k \mathbf{v}).$$

In this paper, the number of hierarchical levels is set to five.

5. EXPERIMENTAL RESULTS

In order to confirm effectiveness of the Poisson noise removal method based on multiple DirLOTs, experiments were conducted. In these experiments, the interscale shrinkage function was adopted as in [3], where the parameter K was fixed to 2 and $T = \sqrt{6}\sigma$. Several DirLOTs of polyphase order four were adopted. The TVM angles ϕ_1, ϕ_2, ϕ_3 and ϕ_4 were set to $-\frac{\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{6}$ and $\frac{4\pi}{6}$, respectively.

In these experiments, "Galaxy", "Cells", "Lena" and "Cameraman", were used, where the sizes are all 256 × 256 pixels. Since the basis size of DirLOTs is $L_y \times L_x$ = 10 × 10, Symlet of index 5 from the separable orthogonal DWT was used as a reference. The support size of the Symlet

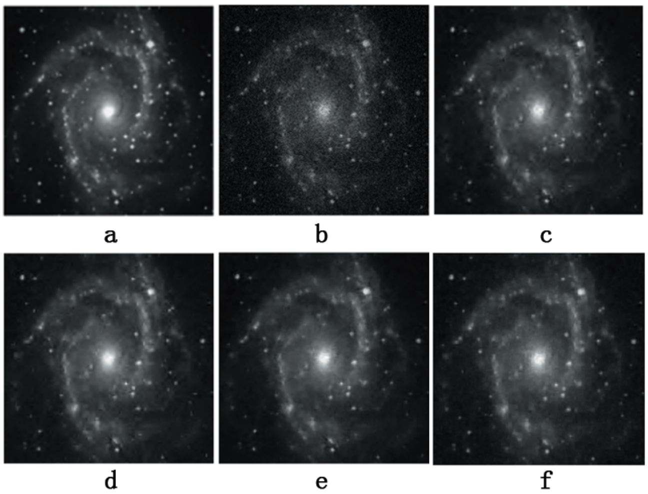

Fig. 3: Denoising results for “Galaxy”. (a) Original image, (b) Noisy image (peak intensity = 30). (c), (d), (e) and (f) are denoised results, where Sym5, SON4, PURE-LET and Multiple DirLOTs denote Symlets of index 5, symmetric orthonormal WT with the classical two-order VMs, Haar WT and Multiple DirLOTs with the two-order TVMs, respectively.

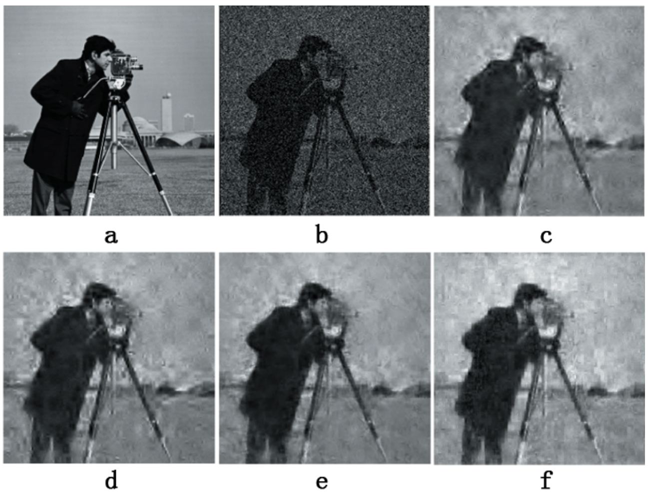

Fig. 4: Denoising results for “Cameraman”. (a) Original image, (b) Noisy image (peak intensity = 5). (c), (d), (e) and (f) are denoised results, where Sym5, SON4, PURE-LET and Multiple DirLOTs, respectively.

of index 5 is identical to the adopted DirLOTs. The number of levels for constructing DWTs is fixed as four. The variance was estimated by applying the robust median estimator to the finest wavelet coefficients [17]. Peak intensity is the maximum intensity of the noise-free signal.

Figure 3 and Figure 4 show parts of the experimental results. It can be seen that the quality (e.g., the diagonal textures and the edges) of denoising image with the multiple DirLOTs is better than the results of Sym5, SON4 and PURE-LET [12]. The multiple DirLOTs shows better quality for diagonal edges. Table 2 compares the denoising performances among four methods for various peak intensities.

Table 2: Comparison of PSNRs among four methods for various peak intensities

Image	Peak	Noise	Sym5	SON4	PURE-LET	M-DirLOTs
Galaxy	5	12.27	27.09	26.86	**27.91**	27.70
	10	16.52	28.73	28.71	28.93	**28.94**
	20	20.35	30.31	30.25	30.54	**30.55**
	30	22.47	31.60	31.54	31.44	**31.83**
	60	25.79	33.59	33.54	33.43	**33.81**
	120	28.97	35.54	35.48	35.21	**35.80**
Cells	5	11.96	26.72	26.58	26.68	**26.78**
	10	16.08	27.97	27.89	28.02	**28.11**
	20	19.85	29.50	29.49	29.46	**29.50**
	30	21.84	**30.28**	30.19	29.96	30.23
	60	25.10	31.86	31.79	31.03	**31.87**
	120	28.24	33.40	33.37	33.07	**33.56**
Lena	5	9.89	23.33	23.28	23.35	**23.46**
	10	13.41	24.54	24.46	24.22	**24.71**
	20	16.63	25.95	25.98	25.88	**26.30**
	30	18.44	26.91	26.93	26.48	**27.28**
	60	21.61	28.58	28.67	28.09	**29.09**
	120	24.64	30.42	30.49	30.16	**30.90**
Camera-man	5	9.55	22.83	22.72	22.78	**23.08**
	10	13.03	24.12	24.04	24.07	**24.46**
	20	16.18	25.58	25.50	25.27	**25.88**
	30	18.04	26.54	26.48	26.48	**26.81**
	60	21.12	28.10	28.08	28.53	**28.54**
	120	24.16	29.70	29.67	**30.39**	29.87

For PSNRs in Table 2, we can see that PURE-LET can achieve highest PSNR when the peak intensity is 5 and 120 for “Galaxy” and “Cameraman”, respectively. Sym5 can achieve highest PSNR when the peak intensity is 30 for “Cells”. For other peak intensities, the proposed multiple DirLOTs can achieve the highest PSNR for all the four images. The experimental results imply that the multiple DirLOTs are not only able to reproduce the diagonal structure appropriately, but also generate fairly good results. However, there is a potential property that the multiple DirLOTs have a tendency to overly smooth fine textures out. This is because multiple scaling filters are applied in the multiple DirLOTs, and two important coefficients of the average of the approximation and the responsible are also diluted out.

6. CONCLUSIONS

The SURE-LET approach for Gaussian noise was reviewed as an orthonormal wavelet-based denoising technique. And it can overcome the disadvantage that the representation of diagonal geometric structures is relatively insufficient by using traditional separable transform. Therefore, the multiple DirLOTs that can improve the SURE-LET approach was proposed to remove Poisson noise. Experimental results show that the combination of the VST, SURE-LET, and multiple DirLOTs significantly improved the denoising performance, and their effectiveness has been verified.

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