Opportunistic Scheduling with BIA under Block Fading Broadcast Channels

Weipeng Jiang*, Zhiqiang He, *Member, IEEE,* Kai Niu, *Member, IEEE,* Li Guo, *Member, IEEE,* Weiling Wu, *Member, IEEE*

***Abstract*—We propose an opportunistic scheduling method to achieve DoF (degrees of freedom) gain by BIA (blind interference alignment) in block fading K-user 2×1 MISO broadcast channel. The optimal scheduling method is obtained by solving a general model of linear integer program. All the users are divided into user pairs to form a 2-user 2×1 BIA. Each pair has the same opportunity to be scheduled. When $K \geq 10$, the expectation of the achieved DoF can be very close to $\frac{4}{3}$.**

***Index Terms*—blind interference alignment, DoF, block fading, opportunistic.**

I. Introduction

INTERFERENCE alignment as a new method to improve the system capacity of interference networks has shown remarkable benefits [1]. However, the improvement of DoF often depends on perfect CSIT (channel state information at the transmitters), which is quite impractical for wireless communication. Thus BIA with no CSIT is studied in [2], [3]. They show that with no CSIT, a total DoF of $\frac{M \times K}{M+K-1}$ (the $M \times K$ X channel setting), which is also the DoF outer bound even with perfect and global channel knowledge, can be achieved by exploiting staggered channel correlations. With the help of reconfigurable antenna modes at the receiver, staggered channel correlations required for BIA is easy to implement [4]. On this basis, Wang et al give the beamforming vectors for the $2 \times K$ X channel setting, which reaches the outer bound.

The results above are inspiring but also reveal that the time accuracy of controlling the reconfigurable antenna modes is quite high for wireless communication. The feasibility of BIA in block fading 2×1 MISO broadcast channels for 2 user and 3 user is researched in [5] and [6], respectively. It's proved that there is a high probability to find 2 or 3 out of the K users to realize BIA only taking advantage of the channel correlations. Though a sufficient condition to achieve the DoF bound by using BIA is given, when the condition is not satisfied, the optimal DoF cannot be achieved because not all the channel resources are used to perform BIA. But appropriate scheduling mechanisms to group as many channel resources as possible for BIA will still get DoF benefit.

Inspired by these contributions, we propose an opportunistic scheduling method to group channel resource and users to approximately achieve $\frac{4}{3}$ DoF for the system of block fading 2×1 MISO broadcast channel with K users. A general linear integer program model is given, and the analytical solution is derived. A linear Diophantine equations model is proposed in [7] most recently, which gives the sufficient and necessary condition for the situation where complete BIA is possible. By complete BIA, we mean that all the channel resources are used for BIA. It is just a special case when the inequality constraints in our model satisfy the equal condition, and the solution in [7] is a vertex of the solution space in our model. Conclusions in [5], [6] and [7] show that there is high probability to get 2 or 3 users grouped for complete BIA. Obviously, the DoF bound for 2-user or 3-user 2×1 BIA can be achieved by only allocating resource for complete BIA users at the cost of losing fairness among users, which is not practical. Instead, all the users are classified into pairs to form a 2-user BIA in our scheduling method. The user pairs which don't satisfy the complete BIA condition, that is the inequality situation, can approach the $\frac{4}{3}$ DoF as much as possible. And the achieved DOF expectation of the K-user is very close to $\frac{4}{3}$ as K increases.

Throughout the paper, we will use $\mathbf{A}$ and $\mathbf{a}$ to denote a matrix and a vector, respectively. Let $\mathbf{A}^T$ and rank($\mathbf{A}$) denote the transpose and the rank of $\mathbf{A}$. $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$.

II. System Model and Assumptions

We first consider the setting of 2-user 2×1 MISO broadcast channel, where the transmitter has 2 antennas and each receiver has a single antenna. Block fading channel means the channel remains constant during the coherence time but changes randomly across its coherence interval. The assumptions and conclusions in [4] are quoted here for the convenience to introduce our scheduling model.

BIA is achieved by coding over only a finite number of symbols. If channel resources can be found to construct a supersymbol shown in Fig.1, the outer bound of DoF for BIA can be achieved by using the beamforming vector given in [4]. Fig.1 shows the block fading channel that the two users experience during the transmitting. That the channel remains constant during the coherence time is indicated by the rectangles with the same linetype. The supersymbol which

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The authors are with the Key Laboratory of Universal Wireless Communication, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, China (e-mail: jwpqjty@gmail.com; hezq, niukai, guoli, weilwu @bupt.edu.cn).

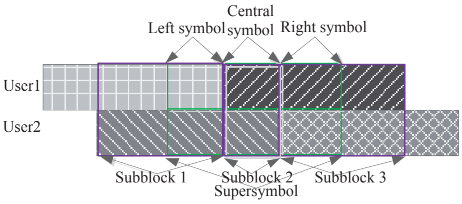

Fig. 1. Supersymbol for BIA of a 2-user 2×1 MISO channel

is outlined with arrows consists of 3 CREs (channel resource elements). Channel remains the same in the first 2 symbols for user 2, but changes for user 1. The situation exchanges when it comes to the last 2 symbols. The transmitted signal for BIA is

$$\mathbf{X} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} \quad (1)$$

where $\mathbf{I}$ is a 2×2 identity matrix. ${u_1}^{[i]}$ and ${u_2}^{[i]}$ are two independently encoded data streams intended to user i. The received signal at user 1 is

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \\ y^{[1]}(3) \end{bmatrix} = \begin{bmatrix} \mathbf{h}^{[1]}(1) \\ \mathbf{h}^{[1]}(2) \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{h}^{[1]}(2) \\ \mathbf{h}^{[1]}(2) \end{bmatrix} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} \quad (2)$$

where $h^{[i]}(n)$ is a 1×2 channel vector standing for the channel between two transmit antennas and user i; $\mathbf{0}$ is a 1×2 zero vector; $y^{[i]}(n)$ is the receiving signal of user i at timeslot n; The interference signal is aligned into one dimension along vector $[0\ 1\ 1]^T$, while the desired signal of user 1 occupies 2 other dimensions. The beamforming vector is also given in (1). So if the CREs constructing the supersymbol are found, BIA can be fulfilled using the method in [4]. The only problem is how to get as many CREs as possible grouped for constructing supersymbols in block fading channel without the help of the reconfigurable antenna at the receiver. This is a question of channel resource scheduling for BIA and it can be implemented in both user layer and CREs layer.

The 2 users are assumed to have the same coherence time N but different initial time offset τ. Without loss of generality, the initial time offsets of user 1 and user 2 are indicated as $\tau^{[0]} = 0$ and $\tau^{[1]} \geq 0$ respectively. Next, we will reanalyse the construction of the supersymbol and find out what kind of CREs are needed.

As shown in Fig.1, the supersymbol for 2-user 2×1 MISO channel includes 3 symbols. The central symbol can be chosen from the intersecting part of a coherence block of either user and then both the left, right symbol can be chosen from the other part of the two coherence blocks. The intersecting part of the slashes coherence block of user 1 and the backslashes coherence block of user 2, which is marked as subblock 2, can be used for central symbol. The left symbol and right symbol can be chosen from subblock 1 and subblock 3 respectively. The channel resource staying constant for both users is defined as a subblock. All the CREs in a single subblock are equivalent in consisting of supersymbol. Fig.2 shows the subblock division of two users with relative time offset $\tau^{[1]}$ and coherence time N. Each subblock is the intersecting part of a coherence

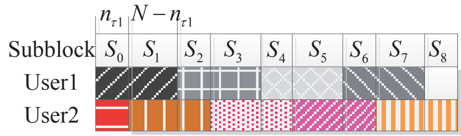

Fig. 2. Subblock segmentation for a 2-user 2×1 MISO channel

block of either user, which means CREs in each subblock can be used as a central symbol. It's also obvious that there is one and only one possible combination for a subblock to construct the supersymbol as the central symbol. Considering about the coherence time and that supersymbol consisting of 3 symbols, $3N$ is a period for analysis. Because of the periodicity, (S_5, S_0, S_1) and (S_4, S_5, S_0) denote combinations where subblock S_0 and S_5 are central symbols. c_i denotes the number of the combination where S_i is the central symbol. All the possible combinations in the $3N$ CREs are shown in (3). The corresponding length of subblock S_i is l_i, and $\mathbb{N}$ is the set of nonnegative integers. Our destination is to find a scheduling method or a combination method to use as many CREs as possible, that is constructing supersymbols to fulfill BIA. So the solution of (3) is the optimal scheduling method for the two users to get the maximum DoF by BIA.

$$\begin{cases} \text{maximize}(c_0 + c_1 + c_2 + c_3 + c_4 + c_5) \\ \text{subject to}: \\ c_0(S_5, S_0, S_1): c_5 + c_0 + c_1 \leq l_0 \\ c_1(S_0, S_1, S_2): c_0 + c_1 + c_2 \leq l_1 \\ c_2(S_1, S_2, S_3): c_1 + c_2 + c_3 \leq l_2 \\ c_3(S_2, S_3, S_4): c_2 + c_3 + c_4 \leq l_3 \\ c_4(S_3, S_4, S_5): c_3 + c_4 + c_5 \leq l_4 \\ c_5(S_4, S_5, S_0): c_4 + c_5 + c_0 \leq l_5 \\ c_0, c_1, c_2, c_3, c_4, c_5 \in \mathbb{N} \end{cases} \quad (3)$$

III. ANALYTICAL SOLUTION

Usually there is no analytical solution for a linear integer program problem. But (3) has rather regular constraint conditions and there is special nature for l_i as well, which is $l_0 = l_{2i} = \tau^{[1]}, l_1 = l_{2i+1} = N - \tau^{[1]}, i = 0, 1, 2....$ So (3) can be rewritten in matrix form as (4).

$$\begin{cases} \text{maximize}(c_0 + c_1 + c_2 + c_3 + c_4 + c_5) \\ \text{subject to}: \mathbf{Ac} \leq \mathbf{b}, c_i \in \mathbb{N}, i = 0, 1, 2...5 \\ \\ \mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} l_0 \\ l_1 \\ l_0 \\ l_1 \\ l_0 \\ l_1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \end{cases} \quad (4)$$

It is well known that the optimal solution of a linear program problem usually lies in the border of the feasible region. For the problem of (4), leaving alone the integer constraints, the maximum value N is achieved when the inequality constraints become equal, which means all the CREs of every subblock are used in a combination. Next, the linear integer program problem is solved by proving 2 theorems. Firstly, a lemma is present.

Lemma 1. *There are always solutions for the objective function to get the maximum value N in integer field.*

Proof: The reduced row echelon form of the augmented matrix as (5a) is obtained with elementary row transformation.

$$\begin{bmatrix} 1\,0\,0\,0 & 1 & 1 & l_1 \\ 0\,1\,0\,0 & -1 & 0 & l_0-l_1 \\ 0\,0\,1\,0 & 0 & -1 & l_1-l_0 \\ 0\,0\,0\,1 & 1 & 1 & l_0 \\ 0\,0\,0\,0 & 0 & 0 & 0 \\ 0\,0\,0\,0 & 0 & 0 & 0 \end{bmatrix} (5a) \quad \begin{cases} c_0 = l_1 - \lambda_0 - \lambda_1 \\ c_1 = l_0 - l_1 + \lambda_0 \\ c_2 = l_1 - l_0 + \lambda_1 \\ c_3 = l_0 - \lambda_0 - \lambda_1 \\ c_4 = \lambda_0 \\ c_4 = \lambda_1 \end{cases} (5b)$$

From (5a), we know $rank(\mathbf{A}) = 4$, $rank(N(\mathbf{A})) = 2$. So the solution space has 2 dimensions and (5b) is the solution set. Because l_0 and l_1 are integers, $(c_0, c_1, c_2, c_3, c_4, c_5)$ is an integer solution if and only if λ_0 and λ_1 are integers. That is to say, the integer solutions are the integer points of a two dimensional plane of (λ_0, λ_1). ■

Theorem 1. *When $l_0 + l_1 \leq 3\min(l_0, l_1)$, there are always solutions for the objective function to get the maximum value N in non-negative integer field.*

$$\begin{cases} \lambda_0 + \lambda_1 \leq l_1 \\ \lambda_0 \geq l_1 - l_0 \\ \lambda_1 \geq l_0 - l_1 \\ \lambda_0 + \lambda_1 \leq l_0 \\ \lambda_0 \geq 0 \\ \lambda_1 \geq 0 \end{cases} (6a) \quad \begin{cases} \lambda_0 + \lambda_1 \leq l_0 \\ \lambda_0 \geq l_1 - l_0 \\ \lambda_1 \geq 0 \end{cases} (6b)$$

Proof: Apparently, the solutions of (5b) which meet the non-negative demand are the optimal solutions. Substituting (5b) in $\mathbf{c} \succeq \mathbf{0}$ we get (6a). Since l_0 and l_1 are integers, there are integer points on the border of the solution space as long as the solution space of (6a) is non-empty. For the case $l_1 \geq l_0$, (6a) equals to (6b), and the sufficient and necessary condition for the existence of the optimal solution is $l_1 - l_0 + 0 \leq \lambda_0 + \lambda_1 \leq l_0$, equivalent to $l_0 + l_1 \leq 3l_0$. Considering about the symmetry of l_0 and l_1 in (6a), we can get the similar result for the case $l_1 < l_0$, which is $l_0 + l_1 \leq 3l_1$. So the sufficient and necessary condition is $l_0 + l_1 \leq 3\min(l_0, l_1)$ for both cases. Fig.3 shows the solutions for the case $l_1 \geq l_0$. Each of the integer points in the triangle shadow, such as the three vertices, is a solution for (4) to reach the maximum value N. When $(\lambda_0, \lambda_1) = (l_1 - l_0, 2l_0 - l_1)$, just the triangle point in Fig.3, $\mathbf{c} = (l_1 - l_0, 0, l_0, 0, l_1 - l_0, 2l_0 - l_1)$ is the corresponding solution given in [5]. The three vertices of the feasible region are special solutions because even while $l_0 + l_1 = 3\min(l_0, l_1)$, all the other integer solutions disappear, these three solutions still exist, although they converge to be one point. With $l_0 = \tau^{[1]}$, $l_1 = N - \tau^{[1]}$, an equivalent condition is $\lceil \frac{N}{3} \rceil \leq \tau^{[1]} \leq \lfloor \frac{2N}{3} \rfloor$. ■

Theorem 2. *When $l_0 + l_1 > 3\min(l_0, l_1)$, there is no solution for the objective function to get the maximum value N in non-negative integer field. The maximum value is $3\min(l_0, l_1)$ and the corresponding solution set can be achieved by letting $l_1' = 2\min(l_0, l_1)$, and $l_0' = min(l_0, l_1)$, then using theorem 1 to (l_0', l_1').*

Proof: Considering about the symmetry of l_0 and l_1 in (4), without loss of generality, suppose $\max(l_0, l_1) = l_1$ and $l_1 > 2l_0$, which is equivalent to $l_0 + l_1 > 3\min(l_0, l_1)$. Let $l_1 = l_1' + \Delta, l_1' = 2l_0$, then $l_0 + l_1' = 3\min(l_0, l_1')$ satisfies the condition in theorem 1. $\mathbf{c}' = (l_0, 0, l_0, 0, l_0, 0)$ is the only solution for the objective function to get the optimal value. From (4), we get $c_0 + c_1 \leq c_5 + c_0 + c_1 \leq l_0$ and $c_1 + c_2 \leq c_1 + c_2 + c_3 \leq l_0$. Adding the two inequalities, we get $c_0 + c_1 + c_2 \leq c_0 + 2c_1 + c_2 \leq 2l_0$, which means at most $2l_0$ elements of the subblock S_1 can be used for BIA. The solution $\mathbf{c}'$ has reached the upper bound and the situation is the same with S_3, S_5. For the subblocks with the length of l_0, all the elements are used in the solution $\mathbf{c}'$. So $\mathbf{c}'$ is the solution for the objective function to reach the maximum value $3\min(l_0, l_1)$. It's referred as patial BIA because not all the CREs are used in BIA. The proof is similar for the case $\max(l_0, l_1) = l_0$. ■

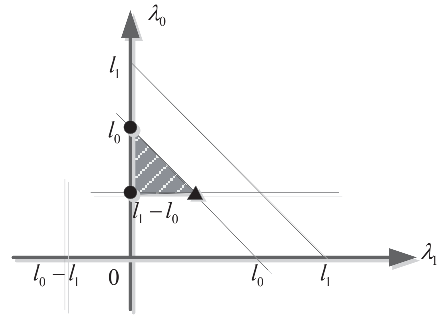

Fig. 3. Solution space of (4)

IV. Scheduling Method and Feasibility Proof

The analytical solution of (4) is given in the two theorems above for different cases. It shows how to use as many CREs as possible to perform BIA for certain two users thus getting DoF gain. The opportunistic scheduling method based on the solution includes two steps. Firstly, user pairs which meet the demand in theorem 1 is found to get complete BIA and achieve the $4/3$ DoF. And for the left users which satisfy the case of theorem 2, patial BIA is done. Secondly, round-robin scheduling among the user pairs can be fulfilled. Next, the feasibility of the opportunistic scheduling method is proved and the achieved DoF expectation is derived with 2 lemmas.

$$\inf(E(P_{(N,K)})) = \frac{2}{K} \sum_{i=0}^{\lfloor K/2 \rfloor - 1} \prod_{j=K-2i}^{K} P_{(N,j)} \quad (7)$$

Lemma 2. *The lower bound of the percentage expectation for the users which satisfy the condition of theorem 1 among K users is (7). When K is even, $K \geq 2$, $j = 0, 2, 4...K$, and when K is odd, $K \geq 3$, $j = 1, 3, 5...K$.*

Proof: [5] has proven that for a K-user 2×1 broadcast channel with coherence time N, the probability that the transmitter can find two users to fulfill complete BIA is $P_{(N,K)} = 1 - f_{(N,K)}$, where $f_{(N,K)} = \frac{3\Theta(\lceil \frac{N}{3} \rceil, K-2) - 2\Theta(\lceil \frac{N}{3} \rceil, K-3)}{N^{K-1}}$ with $\Theta(a,b) = \sum_{i=1}^{a} i^b$, $K \geq 3$. When $K = 2$, $P_{(N,2)} = 1/3$. Suppose that there is a global optimum paring method, which we don't know.

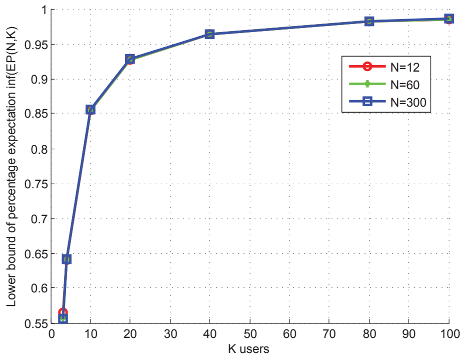

Fig. 4. Percentage expectation of complete BIA users.

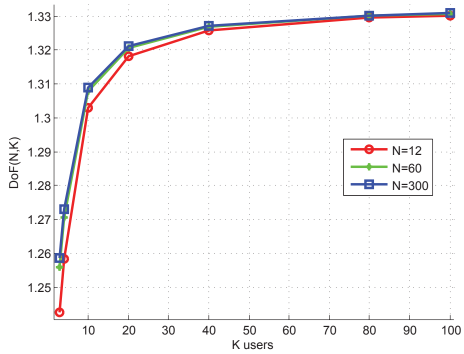

Fig. 5. Expectation of the achieved DoF with respect to total user nember.

It can find maximum user pairs for arbitrary definite user distributions. Let $P_{(N,K,n)}$ denote the probability that there are at most n pairs can be chosen with the optimum paring method. So $P_{(N,K,0)}$ is for the case when no pair of users satisfy the demand and $P_{(N,K)} = 1 - P_{(N,K,0)} = P_{(N,K,1)} + P_{(N,K,2)} + ... + P_{(N,K,\lfloor K/2 \rfloor)}$. Firstly, all the combinations of two users will be tested one by one until a pair can be chosen to form a 2 user $2{\times}1$ BIA, and the probability is $P_{(N,K)}$. Secondly, all the combinations of two users among the left users will be tested until the second pair is chosen, and the probability is $P_{(N,K)}P_{(N,K-2)}$, and so on. The method is not global optimum, because the previous pair is chosen without considering the influence on the following pair. The probability that the second user pair can be chosen with the unknown global optimum paring method is $P_{(N,K,2)} + P_{(N,K,3)} + ... + P_{(N,K,\lfloor K/2 \rfloor)}$. So $P_{(N,K)}P_{(N,K-2)} \leq P_{(N,K,2)} + P_{(N,K,3)} + ... + P_{(N,K,\lfloor K/2 \rfloor)}$. There are $\lfloor \frac{K}{2} \rfloor$ pairs for K users, so $\frac{2}{K}P_{(N,K)} + \frac{2}{K}P_{(N,K)}P_{(N,K-2)} + ... + \frac{2}{K}P_{(N,K)}P_{(N,K-2)}P_{(N,K-4)}...P_{(N,4)}P_{(N,2)} \leq \frac{2}{K}((P_{(N,K,1)}+...+P_{(N,K,\lfloor K/2 \rfloor)}) + (P_{(N,K,2)} + ...P_{(N,K,\lfloor K/2 \rfloor)}) + (P_{(N,K,3)} + ...P_{(N,K,\lfloor K/2 \rfloor)}) + ... + (P_{(N,K,\lfloor K/2 \rfloor)})) = \frac{1}{K}(2P_{(N,K,1)} + 4P_{(N,K,2)} + ... + 2\lfloor K/2 \rfloor (P_{(N,K,\lfloor K/2 \rfloor)})) = E(P_{(N,K)})$, which can be abbreviated to (7). ■

Fig.4 shows that the percentage expectation of the users satisfying theorem 1 increases with the total user number K. When K is larger than 30, almost more than 95% users can form a complete BIA user pair, which means round-robin scheduling among the user pairs to approach the optimal DoF is possible.

Lemma 3. *For the user pairs satisfying theorem 2, the achieved DoF expectation is* $1 + P_N/3$*, where* $P_N = \frac{6}{N\lceil 2N/3 \rceil}\sum_{i=0}^{\lceil N/3 \rceil - 1} i$.

Proof: When $\tau^{[1]}$ doesn't meet the demand $\lceil \frac{N}{3} \rceil \leq \tau^{[1]} \leq \lfloor \frac{2N}{3} \rfloor$, there are $\lceil \frac{2N}{3} \rceil$ possibilities with the same probability. The corresponding $\min(l_0, l_1)$ is $0, 1, 2... \lceil \frac{N}{3} \rceil - 1$, and each $\min(l_0, l_1)$ has two probabilities. So the proportion of the BIA channel resources is $\frac{1}{\lceil 2N/3 \rceil}\frac{2}{N}\sum_{i=0}^{\lceil N/3 \rceil - 1} 3i$, which equals to P_N. Considering that the resource left can achieve DoF 1, the achieved DoF expectation is $\frac{4}{3}P_N + (1 - P_N) = 1 + \frac{1}{3}P_N$. ■

Based on the results above, we get the expectation of the achieved DoF for a K-user $2{\times}1$ broadcast channel with coherence time N using 2 user $2{\times}1$ BIA. The expectation of the achieved DoF is

$$\frac{4}{3}inf(E(P_{(N,K)})) + (1 - inf(E(P_{(N,K)})))(1 + \frac{1}{3}P_N) \quad (8)$$

Fig.5 shows that expectation of the achieved DoF increases with the user number K up to the upper DoF bound of 2-user $2{\times}1$ BIA. When $K \geq 10$, it is almost certain that the achieved DoF can be close to the optimal $4/3$ DoF of 2-user BIA in a setting K-user $2{\times}1$ broadcast channel. The scheduling method is firstly finding user pairs satisfying the condition in theorem 1, and the left user pairs can be dealt with theorem 2.

V. Conclusions

We propose an opportunistic scheduling method to achieve DoF gain by grouping CREs and users to form a 2-user $2{\times}1$ BIA in the setting of K-user $2{\times}1$ broadcast channel. The feasibility of our optimistic scheduling method found according to the analytical solution of a linear integer program problem is proved. All the users can form a 2-user $2{\times}1$ complete or patial BIA pair. With all the user pairs round-robin scheduled, each user has the same opportunity for transmitting. The achieved DoF expectation using our scheduling method is given, which shows that when $K \geq 10$, it can be very close to $4/3$ in a setting K-user $2{\times}1$ broadcast channel.

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