

PRECODING FOR SYSTEMS WITH SOFT COMBINING TO COUNTERACT INSTATIONARY INTERCELL INTERFERENCE

Hans H. Brunner, Jonas Braun, Amine Mezghani, Josef A. Nossek

Institute for Circuit Theory and Signal Processing
Technische Universität München, 80290 Munich, Germany
e-mail: {brunner|amine.mezghani|josef.a.nossek}@tum.de

ABSTRACT

We consider the downlink of a cellular network with multiple antenna base stations and single antenna user terminals. In cellular networks with and without cooperation, the problem of instationary intercell interference arises. Some base stations change their beamforming unpredictably and the signal to interference plus noise ratios of the served user terminals are unknown at the base station. Consequently, the precoding and link rate adaption are outdated and the transmission might fail. Hybrid automatic repeat request can be used to mitigate the risk of such a fail. We propose to optimize the precoders at the base stations based on the expectation of the rate, where we include the effects of soft combining in the optimization.

1. INTRODUCTION

The problem of the instationarity of the *intercell interference* (ICI) is investigated in this paper. Whenever a *base station* (BS) changes its beamforming strategy, it changes the ICI at every *mobile device* (MD) in the whole network. These changes are usually not predictable and a BS cannot keep track of the *signal to interference plus noise ratio* (SINR) of its associated MDs. Therefore, it is not optimal to choose beamforming strategies and link rate adaptations based on measured or assumed SINR values.

This problem was already addressed in [1]. An upper bound to the possible rates in systems with ICI instationarity was defined, where the actual SINR is simply known in each time slot. In [2], we proposed to optimize the beamforming vectors at each BS based on the expected rate of the associated MDs. With this approach, the system for which the precoders are optimized and the system in which the precoders are utilized become the same.

A different method was suggested in [3]. The BSs are forced to transmit with scaled identity matrices, which still leaves room for an optimization of the beamforming vectors. This constraint completely removes the uncertainty in the ICI variance and the SINR values of the served MDs can be known at the BSs. But, the shaping constraint on the

transmitter also reduces the possible rates.

The problem of the instationary ICI can also be mitigated with *hybrid automatic repeat request* (HARQ) with soft combining. The soft combining in HARQ can either be *Chase combining* (CC) [4], where each retransmission contains the same bits, or *incremental redundancy* (IR) [5], where each retransmission adds additional redundancy bits.

The authors of [6] present an algorithm, which optimizes the scheduling decisions based on the expected rates, where the effects of HARQ are already taken into account. This is very similar to our approach. In contrast, we propose an algorithm, which optimizes beamforming vectors at the BSs. In [6, 7] the authors propose to increase the number of retransmissions L for HARQ with IR in LTE. They show that the ergodic upperbound rate with known SINR values can be reached for $L \rightarrow \infty$.

In [8, 9] the performance of HARQ with CC and IR is investigated for wireless standards with link level simulations. Based on such simulations, an abstraction for system level simulations is proposed in [10], where each retransmission is associated with an equivalent SINR gain. The authors of [11] propose several HARQ combining schemes for receivers with multiple antennas in cellular networks.

The used system model based on the Winner channel model is described in Section 2. The instationarity of the ICI is discussed in more detail in Section 3. Section 4 describes HARQ with CC and IR for mitigating the ICI instationarity and general utilities. In Section 5 the required optimization algorithm is presented for the sum rate utility. Simulation results are shown in Section 6.

2. SYSTEM MODEL

We consider a cellular network with 19 three faced sites and, therefore, 57 BSs. Each BS serves the MDs of the hexagonal shaped cell it covers. A MD in the set \mathcal{K} of all MDs is specified by the tuple $(b, k) \in \mathcal{K}$, where $b \in \mathcal{B}$ identifies the BS in the set \mathcal{B} of all BSs and $k \in \mathcal{K}_b$ the MD in the set \mathcal{K}_b of all MDs in the cell of BS b . The wrap-around method is used to treat all cells equally and the channels are found with the

3GPP MIMO urban macro cell model [12].

We assume block fading, where the channel stays constant for T_{block} transmit symbols. In this paper, each BS has N antennas and serves $K = |\mathcal{K}_b|$ single antenna MDs, respectively. The vectors $\mathbf{h}_{\hat{b},b,k} \in \mathbb{C}^N$ contain the channel coefficients between the antennas of BS \hat{b} and MD (b, k) . With $(\bullet)^T$ and $(\bullet)^H$ we denote the transposition and the complex conjugate transposition, respectively. The achievable, normalized rate of MD (b, k) can be expressed as

$$r_{b,k} = \log_2 \left(1 + \frac{|\mathbf{h}_{\hat{b},b,k}^T \mathbf{p}_{b,k}|^2}{\sigma^2 + \sum_{\hat{k} < k} |\mathbf{h}_{\hat{b},b,k}^T \mathbf{p}_{b,\hat{k}}|^2 + \theta_{b,k}} \right), \quad (1)$$

$$\theta_{b,k} = \sum_{\hat{b} \in \mathcal{B} \setminus b} \mathbf{h}_{\hat{b},b,k}^H \mathbf{Q}_{\hat{b}} \mathbf{h}_{\hat{b},b,k}, \quad (2)$$

where $\mathbf{p}_{b,k} \in \mathbb{C}^N$ is the beamforming vector for MD (b, k) and $\sum_k \mathbf{p}_{b,k} \mathbf{p}_{b,k}^H = \mathbf{Q}_b \in \mathbb{C}^{N \times N}$ is the sum transmit covariance matrix of BS b . $\sum_{\hat{k} < k} |\mathbf{h}_{\hat{b},b,k}^T \mathbf{p}_{b,\hat{k}}|^2$ is the variance of the intracell interference with dirty paper coding, $\theta_{b,k}$ is the variance of the received ICI, and $\sigma^2 = \sigma_\eta^2 + \theta_{\text{bg}}$ is the sum variance of the thermal noise σ_η^2 and the background ICI. The Gaussian background ICI θ_{bg} models the BSs further away than the closest 57 BSs for a given signal variance per transmit antenna. All BSs have to satisfy the transmit power constraint $\text{tr}(\mathbf{Q}_b) \leq P$. We assume that the CSI measurements are error free and we ignore the costs of any signaling overhead.

3. INSTATIONARITY OF THE INTERCELL INTERFERENCE

In our scenario, the BSs do not coordinate their beamforming. The interference channels are not measured and the interference has to be regarded as noise. In scenarios with cooperation such an interference exists as well. Cooperation is always limited in realistic systems, because the measurement of all interference channels and a coordination of all beamformers cannot be implemented [13, 14]. This interference over the unmeasured channels scales with the common transmit power at the BSs and such systems are always interference limited.

In addition, the interference variance at the receivers cannot be known before the transmission. The BSs are assumed to calculate their beamforming in a distributed manner and the update process is not synchronized between the BSs. Even if all BSs would update their beamforming at the same time, the ICI could not be known before the BSs have chosen their beamformers. The ICI at each MD will change the moment any BS applies a new beamforming. Therefore, the BSs compute their beamforming based on assumed ICIs $\tilde{\theta}_{b,k}$. The BSs are blind to the ICI changes and take the risk, that the actual ICI $\theta_{b,k}$ increases and the MD cannot decode the transmitted symbols or that $\theta_{b,k}$ decreases and valuable resources are wasted [1]. With a monotonic rising utility $U(r_{b,k})$ of the rate and without the use of ARQ, this problem can be formulated

as

$$U_{b,k}^{\text{no ARQ}} = \begin{cases} U(\tilde{r}_{b,k}) = U\left(r_{b,k} |_{\theta_{b,k} = \tilde{\theta}_{b,k}}\right), & \text{for } \tilde{\theta}_{b,k} \geq \theta_{b,k}, \\ U(0), & \text{for } \tilde{\theta}_{b,k} < \theta_{b,k}. \end{cases} \quad (3)$$

Most optimizations in the literature utilize the expectation of the ICI or an ICI realization from a previous step as the assumed ICI. This results in a mismatch between the cost function of the optimization and the actual performance measure. To counteract this problem, we consider the expectation of the utility with respect to the random ICI variance [2]:

$$E \left[U_{b,k}^{\text{no ARQ}} \right] = U(\tilde{r}_{b,k}) p_{\tilde{\theta}_{b,k}} + U(0) (1 - p_{\tilde{\theta}_{b,k}}), \quad (4)$$

where $p_{\tilde{\theta}_{b,k}} = P(\tilde{\theta}_{b,k} \geq \theta_{b,k})$ is the probability, that the transmission is successful. This probability is equal to the *cumulative distribution function* (CDF) of the random ICI variance evaluated at $\tilde{\theta}_{b,k}$. With these steps, we reach the cost function

$$C^{\text{no ARQ}} = \sum_{(b,k) \in \mathcal{K}} U(\tilde{r}_{b,k}) p_{\tilde{\theta}_{b,k}} + U(0) (1 - p_{\tilde{\theta}_{b,k}}), \quad (5)$$

which corresponds to the performance measure. The beamforming vectors $\mathbf{p}_{b,k}$ and the assumed ICI $\tilde{\theta}_{b,k}$ in (5) can be optimized with an alternating optimization as described in [2]. For a fixed assumed ICI, the probability of a successful transmission is fixed and a weighted utility optimization with respect to the beamforming vectors remains. For fixed precoders, the assumed ICI can be optimized with a root finding algorithm.

To perform the described procedure, the CDFs of the ICI at each associated MD need to be available at the serving BS. The CDFs can be approximated with long term measurements at the MDs. It could also be possible to estimate a rough CDF directly based on the channel measurements. This would not require any additional measurements and feedback for the CDF. If the update process at the BSs is synchronized, it will be possible to measure $\theta_{b,k}$ with a second pilot, which removes the uncertainty in the ICI afterwards but increases the overhead [15].

4. HARQ AND SOFT COMBINING

The transmit data is encoded with *forward error correction* (FEC) and *error detection* (ED). With HARQ, an additional transmission of the same data block will be requested from the transmitter, if an unrecoverable error is detected at the receiver. Typically, there exists a maximum number of transmissions L in order to respect rate or delay requirements. If the receiver cannot decode the data after L transmissions, the data will be discarded and the higher layer will be informed.

We decide that the regarded utility depends on the sum rate over the transmission blocks $T_{b,k}^{\text{HARQ}}$ required for completing the HARQ process:

$$U_{b,k}^{\text{HARQ}} = \begin{cases} U(\tilde{r}_{b,k}), & \text{if decoded within } L \text{ transmissions,} \\ U(0), & \text{else.} \end{cases} \quad (6)$$

Based on the renewal-reward theorem the cost function can be found as [16]

$$C^{\text{HARQ}} = \sum_{(b,k) \in \mathcal{K}} \frac{E_{H_b, \Theta_{b,k}} [U_{b,k}^{\text{HARQ}}]}{E_{H_b, \Theta_{b,k}} [T_{b,k}^{\text{HARQ}}]}, \quad (7)$$

where the expectation is taken over the HARQ processes with respect to the channel realizations H_b and the ICI realizations $\Theta_{b,k}$.

We assume that many HARQ processes can be completed during the coherence time T_{block} . We also assume that the end of the coherence time always coincides with the end of an HARQ process. This cannot be true in general, but, it only introduces a very small error if the maximum length of an HARQ process will be much smaller than the coherence time, $L \ll T_{\text{block}}$. Under this assumptions, the renewal-reward theorem can be rewritten as the expectation over the block fading blocks of the renewal-reward theorems within one block. Therefore, the maximization of the cost function can be done per block:

$$\max C^{\text{HARQ}} = \sum_{(b,k) \in \mathcal{K}} E_{H_b} \left[\max \frac{E_{\Theta_{b,k}} [U_{b,k}^{\text{HARQ}}]}{E_{\Theta_{b,k}} [T_{b,k}^{\text{HARQ}}]} \right]. \quad (8)$$

To simplify the analysis and to concentrate on the ICI blindness, we assume that the MD set associated with the regarded BS. All channels to these MDs stay constant during the coherence time, while the transmit covariance matrices at the interfering BSs vary randomly. We also assume that the HARQ processes for other MDs do not introduce correlation to the interference. To support this assumption, it can be argued that multiple HARQ processes are handled in parallel for an MD and that the time between retransmissions of the same bits varies randomly [6].

In typical HARQ implementations all received data blocks of an HARQ process are stored at the receiver and decoded jointly. This procedure of soft combining allows the receiver to recover data from multiple transmissions blocks, which could not be decoded individually.

4.1. Chase Combining

With CC the FEC and ED is wrapped in a repetition code. Every transmission in an CC-HARQ process contains the same bits. With maximum ratio combining at the receiver and if

all interference and noise is uncorrelated over different transmit blocks, the effective SINR after m transmissions will be equal to the sum of all individual SINRs of these m transmission [17].

The HARQ process can be completed successfully, if the sum of all SINR values $\gamma_{b,k}^{(m)}$ within this process is larger than the assumed SINR $\tilde{\gamma}_{b,k}$:

$$U_{b,k}^{\text{CC}} = \begin{cases} U(\tilde{r}_{b,k}), & \text{for } \sum_{m=1}^M \gamma_{b,k}^{(m)} \geq \tilde{\gamma}_{b,k} \\ U(0), & \text{for } \sum_{m=1}^M \gamma_{b,k}^{(m)} < \tilde{\gamma}_{b,k}. \end{cases} \quad (9)$$

Depending on the assumed SINR and the SINR realizations, the number of required transmit blocks for an CC-HARQ process can be found as

$$T_{b,k}^{\text{CC}} = \begin{cases} 1, & \text{for } \gamma_{b,k}^{(1)} \geq \tilde{\gamma}_{b,k} \\ 2, & \text{for } \gamma_{b,k}^{(1)} + \gamma_{b,k}^{(2)} \geq \tilde{\gamma}_{b,k} \cap \gamma_{b,k}^{(1)} < \tilde{\gamma}_{b,k} \\ \vdots \\ M, & \text{for } \sum_{m=1}^M \gamma_{b,k}^{(m)} \geq \tilde{\gamma}_{b,k} \cap \sum_{m=1}^{M-1} \gamma_{b,k}^{(m)} < \tilde{\gamma}_{b,k} \\ M, & \text{for } \sum_{m=1}^M \gamma_{b,k}^{(m)} < \tilde{\gamma}_{b,k}. \end{cases} \quad (10)$$

We define the probability that the sum over m consecutive SINR values is larger than the assumed SINR as

$$p_{\tilde{\gamma}_{b,k}}^{(m)} = P \left(\sum_{m'=1}^m \gamma_{b,k}^{(m')} \geq \tilde{\gamma}_{b,k} \right), \quad (11)$$

where $p_{\tilde{\gamma}_{b,k}}^{(0)} = 0$. With (11) the expectation of (9) can be found as

$$E[U_{b,k}^{\text{CC}}] = U(\tilde{r}_{b,k}) p_{\tilde{\gamma}_{b,k}}^{(M)} + U(0) (1 - p_{\tilde{\gamma}_{b,k}}^{(M)}). \quad (12)$$

The probability that exactly m transmit blocks are necessary is $p_{\tilde{\gamma}_{b,k}}^{(m)} - p_{\tilde{\gamma}_{b,k}}^{(m-1)}$, therefore, the expectation of (10) reads as

$$\begin{aligned} E[T_{b,k}^{\text{CC}}] &= M \left(1 - p_{\tilde{\gamma}_{b,k}}^{(M)} \right) + \sum_{m=1}^M m \left(p_{\tilde{\gamma}_{b,k}}^{(m)} - p_{\tilde{\gamma}_{b,k}}^{(m-1)} \right) \\ &= M - \sum_{m=1}^{M-1} p_{\tilde{\gamma}_{b,k}}^{(m)}. \end{aligned} \quad (13)$$

The cost function per block fading block with CC-HARQ is

$$C_{b,k}^{\text{CC}} = \sum_{(b,k) \in \mathcal{K}} \frac{U(\tilde{r}_{b,k}) p_{\tilde{\gamma}_{b,k}}^{(M)} + U(0) (1 - p_{\tilde{\gamma}_{b,k}}^{(M)})}{M - \sum_{m=1}^{M-1} p_{\tilde{\gamma}_{b,k}}^{(m)}}. \quad (14)$$

4.2. Incremental Redundancy

The data bits are encoded with ED and a FEC code, which adds many redundancy bits. Most of these bits are then punctured to reach the desired code rate. In every transmission of

an IR-HARQ process different punctured versions are transmitted. Therefore, the code rate changes with every retransmission. The decoding profits from the improved SINR and a coding gain. If the FEC code is infinitely long and infinitely many retransmissions are acceptable, the supported rate of the channel with instationary ICI can be reached [6, 7].

The IR-HARQ process can be completed successfully in m transmissions, if the sum of the individually possible rates within these m transmissions is larger than the assumed rate [16]. Equations (9) - (14) can be written equivalently for IR, where the assumed SINR and the SINR realizations have to be exchanged with assumed rates and rate realizations, respectively.

5. SUM RATE COST FUNCTION OPTIMIZATION

The optimization of the cost function (14) for CC and the corresponding cost function for IR proceeds similarly to the optimization of the cost function (5) as discussed in [2]. For fixed precoders, the optimal $\tilde{\gamma}_{b,k}$ in (14) and the optimal $\tilde{r}_{b,k}$ for IR can be found numerically and they are associated with an optimal ICI. But, the precoders are part of the SINR and the rate. Therefore, the weight cannot be fixed for the optimization of the precoders. The water spilling algorithm from [18] is adapted to solve this problem. The CC cost function for the sum rate utility $U(r) = r$ reads

$$C_{b,k}^{\text{CC}} = \sum_{(b,k) \in \mathcal{K}} \tilde{r}_{b,k} \frac{p_{\tilde{\gamma}_{b,k}}^{(M)}}{M - \sum_{m=1}^{M-1} p_{\tilde{\gamma}_{b,k}}^{(m)}} = \sum_{(b,k) \in \mathcal{K}} w_{b,k} \tilde{r}_{b,k}. \quad (15)$$

With the uplink-downlink duality, the cost function is transformed to the uplink. The weights $w_{b,k}$ do not change in this transformation. But, the weights have to be evaluated in the downlink, because different uplink-downlink transformations follow from different ICI realizations. The gradient of the cost function with respect to the transmit powers in the uplink for the water spilling algorithm is found numerically. The alternating optimization, which finds the optimal precoders and the optimal assumed interference in turns, converges in less than five iterations.

6. SIMULATIONS

The following results are obtained with Monte Carlo simulations. Every BS has $N = 4$ transmit antennas. In every cell, $K = 4$ MDs are placed uniformly distributed and suffer from a thermal noise variance of $\sigma_{\eta}^2 = 8.3 \cdot 10^{-14}$ W, respectively. The background interference is set to $\theta_{\text{bg}} = 9.53 \cdot 10^{-13} \cdot P$, where P is the used transmit power. The HARQ processes stop after $L = 4$ transmissions.

We operate on an histogram of ICI realizations instead of the probability distribution. The first round of ICI realizations is generated with scaled identities as transmit covariances for

the interfering BSs. New transmit covariances are found with these ICI realizations and these new transmit covariances are used for the calculation of new ICI realizations. Only with the second set of ICI realizations the calculated expectation of the rates and the simulated expectation become equal. The scaled identity matrices are always of full rank, while the second set of covariances is not. A further iteration with the ICI realizations and covariances does not change the results.

The normalized average user rate is plotted over the transmit power in Fig. 1. All curves saturate for high power because of the ICI. The saturation starts already at 1 W as we assume that all BSs transmit in the same frequency band. The “ici aware” rate is the upper bound, which can only be achieved, if the ICI is known at the transmitter. The ICI could be made available with a second pilot at the cost of an additional overhead, if the BSs synchronize the update of their beamforming. “no ARQ” has the rates optimized according to the expected rate without ARQ, which is then combined with CC in “HARQ-CC” and IR in “HARQ-IR”. It can be seen that IR performs better than CC and no ARQ. The interference robustness method from [3] is plotted as “identity” and the conservative gambling algorithm from [1] with an optimized backoff factor as “gambling”. The gambling algorithm can also be improved with CC and IR. We omitted these results, which perform worse than the expected rate algorithm without ARQ, as the plot is already overloaded.

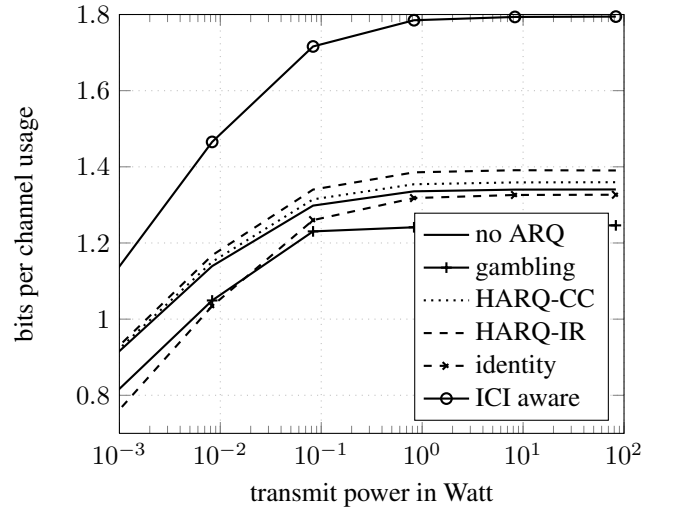


Fig. 1. average user rate over transmit power

7. CONCLUSION

HARQ with soft combining can be used to mitigate the disadvantage of ICI instationarity. Our main contribution is to include this effect of HARQ in the optimization of the beamforming vectors in cellular systems. We could show that the combination of HARQ with the expected rate optimization outperforms any other methods which handle the instationarity of the ICI.

8. REFERENCES

- [1] M.T. Ivrlač and J.A. Nossek, "Inter-cell-Interference in the Gaussian MISO Broadcast Channel," in *GLOBECOM '07, IEEE Global Telecommunications Conference*, Nov. 2007.
- [2] H.H. Brunner and J.A. Nossek, "Handling Unknown Interference in Cellular Networks with Interference Coordination," in *WSA 2012, International ITG Workshop on Smart Antennas*, Mar. 2012.
- [3] A. Dotzler, M. Riemensberger, W. Utschick, and G. Dietl, "Interference Robustness for Cellular MIMO Networks," in *SPAWC 2012, IEEE International Workshop on Signal Processing Advances in Wireless Communications*, 2012.
- [4] D. Chase, "A Combined Coding and Modulation Approach for Communication over Dispersive Channels," *IEEE Transactions on Communications*, vol. 21, no. 3, pp. 159–174, 1973.
- [5] S. Kallel, "Analysis of a Type II Hybrid ARQ Scheme with Code Combining," *IEEE Transactions on Communications*, vol. 38, no. 8, pp. 1133–1137, 1990.
- [6] H. Shirani-Mehr, H. Papadopoulos, S.A. Ramprasad, and G. Caire, "Joint Scheduling and ARQ for MU-MIMO Downlink in the Presence of Inter-Cell Interference," *IEEE Transactions on Communications*, vol. 59, no. 2, pp. 578–589, 2011.
- [7] S. Mukherjee, S.A. Ramprasad, and Yueqian Li, "Performance Bounds and Improvements to HARQ in LTE under Unknown and Highly Variable Interference Due to MIMO," in *VTC 2011, IEEE Vehicular Technology Conference*, 2011.
- [8] P. Frenger, S. Parkvall, and E. Dahlman, "Performance Comparison of HARQ with Chase Combining and Incremental Redundancy for HSDPA," in *VTC 2001, IEEE Vehicular Technology Conference*, 2001.
- [9] A. Das, F. Khan, Ashwin Sampath, and H.-J. Su, "Performance of Hybrid ARQ for High Speed Downlink Packet Access in UMTS," in *VTC 2001, IEEE Vehicular Technology Conference*, 2001.
- [10] J.C. Ikuno, M. Wrulich, and M. Rupp, "Performance and Modeling of LTE H-ARQ," in *WSA 2009, International ITG Workshop on Smart Antennas*, 2009.
- [11] E.W. Jang, Jungwon Lee, Hui-Ling Lou, and J.M. Cioffi, "On the Combining Schemes for MIMO Systems with Hybrid ARQ," *IEEE Transactions on Wireless Communications*, vol. 8, no. 2, pp. 836–842, 2009.
- [12] "Spatial Channel Model for Multiple Input Multiple Output (MIMO) Simulations," Tech. Rep. 25.996 V9.0.0, 3rd Generation Partnership Project, Technical Specification Group Radio Access Network, Dec. 2009.
- [13] A. Lozano, R. Jr. Heath, and J. Andrews, "Fundamental Limits of Cooperation," *IEEE Transactions on Information Theory*, 2013.
- [14] H.H. Brunner, M.T. Ivrlac, and J.A. Nossek, "Upper Bound to Interference Coordination with Channel State Information Outdating," in *EW 2011, European Wireless Conference*, Apr. 2011.
- [15] R. Bendlin, H.H. Brunner, M.T. Ivrlač, J.A. Nossek, and Y. Huang, "Two-Phase Scheduling and Leakage-Based Precoding in Wireless Cellular Networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3148–3157, 2013.
- [16] G. Caire and D. Tuninetti, "The Throughput of Hybrid-ARQ Protocols for the Gaussian Collision Channel," *IEEE Transactions on Information Theory*, vol. 47, no. 5, pp. 1971–1988, 2001.
- [17] J. Proakis, *Digital Communications*, McGraw-Hill Science/Engineering/Math, 4th edition, Aug. 2000.
- [18] R. Hunger, D.A. Schmidt, M. Joham, and W. Utschick, "A General Covariance-Based Optimization Framework Using Orthogonal Projections," in *SPAWC 2008, IEEE Workshop on Signal Processing Advances in Wireless Communications*, July 2008, pp. 76–80.