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ON THE DEGREES OF FREEDOM OF PARTIALLY-CONNECTED SYMMETRICALLY-CONFIGURED MIMO INTERFERENCE BROADCAST CHANNELS

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ABSTRACT

In this paper, we study the degrees of freedom (DoF) of partially-connected symmetrically-configured multi-input-multi-output interference broadcast channel (MIMO-IBC) and the impact of partial connectivity on the DoF. We investigate the information theoretic maximal DoF and maximal DoF achieved by linear interference alignment (IA) for the symmetrically-connected symmetrically-configured MIMO-IBC, and prove that the DoF are achievable by asymptotic IA and linear IA for the asymmetrically-connected symmetrically-configured network when the maximal number of interfering cells seen at each BS and each user (denoted as the degrees of connectivity) is bounded. We find that for the symmetrically-connected symmetrically-configured MIMO-IBC, the maximal achievable DoF are independent of the number of total cells but depend on the degrees of connectivity. When the degrees of connectivity decrease, the maximal DoF achieved by linear IA are close to the information theoretic maximal DoF.

***Index Terms*—** Interference alignment (IA), degrees of freedom (DoF), partially-connected, symmetrically-configured

1. INTRODUCTION

The degrees of freedom (DoF) can reflect the potential of interference networks, which are the first-order approximation of sum capacity in high signal-to-noise ratio regime [1, 2]. Recently, significant research efforts have been devoted to find the information theoretic maximal DoF for the multi-input-multi-output (MIMO) interference channel (MIMO-IC) [1–5] and the MIMO interference broadcast channel (MIMO-IBC) [6–8].

For the fully-connected symmetrically-configured G-cell MIMO-IC where each base station (BS) and each mobile station (MS) have M antennas, the study in [3] showed that the information theoretic maximal DoF per user are $M/2$, which can be achieved by asymptotic interference alignment (IA) (i.e., with infinite time/frequency extension). It implies that the sum DoF can increase linearly with the number of cells G, and the interference networks are not interference-limited [3]. Encouraged by such a promising result, many recent works strive to analyze the DoF for the MIMO-IC and the MIMO-IBC with various settings and devise interference management techniques to achieve the maximal DoF.

So far, the DoF for the fully-connected MIMO-IC or MIMO-IBC have been well studied. For the three-cell symmetrically-configured MIMO-IBC, the information theoretic maximal DoF were obtained in [1], which can be achieved by linear IA (i.e., without any symbol extension or only with finite spatial extension) [4]. By constructing a wise genie chain, the information theoretic maximal DoF for the G-cell symmetrically- or asymmetrically-configured MIMO-IBC were investigated in [7, 8]. The results show that only the sum DoF achieved by asymptotic IA can increase linearly with G, while the sum DoF achieved by linear IA are limited by the sum of numbers of transmit and receive antennas.

In fact, such a pessimistic result comes from the full connectivity assumption, which implicitly treats all channel coefficients of all interfering links as equally strong [9]. In practical cellular systems, the natural attenuation effects (e.g., propagation path loss, shadowing, and fading) cause the (at least partial) loss of connectivity of interfering links [11]. The partial connectivity may reduce the aggregated interference and provide throughput gains for interference-limited systems [12]. Therefore, many studies devoted to investigate the DoF and design the IA transceivers for the partially-connected MIMO-IC [10] and the MIMO-IBC [11–13].

Partially-connected MIMO-IBC systems are more complex than fully-connected MIMO-IBC systems, since it is necessary to consider whether the antenna configuration is symmetric or not and whether the connectivity is symmetric or not. For the asymmetrically-connected asymmetrically-configured MIMO-IBC, the authors in [12] only analyzed the DoF achieved by the given IA algorithm. However, what is the optimal IA and what are information theoretic maximal DoF are still unknown. For the symmetrically-connected symmetrically-configured MIMO-IBC, the authors in [11] derived the *proper condition*, which was proved to be a necessary condition for linear IA feasibility. However, when the proper condition is sufficient and what are the maximal DoF achieved by linear IA remain unclear.

In this paper, we consider the partially-connected symmetrically-configured MIMO-IBC. We strive to find the information theoretic maximal DoF and maximal DoF achieved by linear IA for the MIMO-IBC with symmetrical connectivity and the achievable DoF of asymptotic IA and linear IA for the MIMO-IBC with asymmetric connectivity. From the DoF results, we reveal the impact of partial connectivity on the DoF and show when the linear IA can achieve the information theoretic maximal DoF.

2. SYSTEM MODEL

Consider a G-cell downlink multiuser MIMO cellular system, where each BS with M antennas serves K users and each user with N antennas receives d data steams. Assume that there are no data sharing among the BSs and every BS has perfect channel side information (CSI) of all links. This is a scenario of symmetrically-configured MIMO-IBC, denoted as $\left(M \times (N, d)^K\right)^G$.

Consider the MIMO-IBC whose interfering links (i.e., cross links) are partially-connected. We use the connection pattern proposed in [8] to describe the connectivity of interfering links. *Con-*

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nection pattern is a graph that represents which BSs and users are mutually interfering each other. Each connection pattern corresponds to a set of BS-MS pairs, called *connection set* [8], i.e.,

$$\mathcal{J} \triangleq \{(i_k, j) | \forall \mathrm{BS}_j \text{ is connected with } \mathrm{MS}_{i_k}\} \tag{1}$$

where BS_j and MS_{i_k} denote the BS in cell j and the kth user in cell i, respectively.

From $\mathcal{J}$ we can obtain another two sets to describe the connectivity of each BS and user:

$\mathcal{J}_j \triangleq \{i_k | (i_k, j) \in \mathcal{J}\}$ is the set of MSs connected with BS_j

$\mathcal{J}_{i_k} \triangleq \{j | (i_k, j) \in \mathcal{J}\}$ is the set of BSs connected with MS_{i_k}

Therefore, $|\mathcal{J}_j|$ is the number of interfering users for BS_j, while $|\mathcal{J}_{i_k}|$ is the number of interfering BSs for MS_{i_k}, where $|\cdot|$ denotes the cardinality operator.

In this study, we consider a model of the partially-connected MIMO-IBC in [11], the *L-interfering MIMO-IBC*.

Definition 1. *If a partially-connected symmetrically-configured MIMO-IBC whose connection pattern satisfies*

$$\mathcal{J}_{i_1} = \cdots = \mathcal{J}_{i_K} \tag{2a}$$

$$|\mathcal{J}_j|/K \leq L, |\mathcal{J}_{i_k}| \leq L, \forall i, j, k \tag{2b}$$

it is called L-interfering MIMO-IBC*, where L is an integer satisfying* $1 \leq L \leq G-1$.

(2a) means that all users in one cell have the same connectivity. Then, $|\mathcal{J}_j|/K$ and $|\mathcal{J}_{i_k}|$ denote the number of interfering cells seen at BS_j and MS_{i_k}, respectively. Therefore, (2b) indicates that the number of interfering cells seen at each BS and each user is no more than L. Since L denotes the maximal number of interfering cells seen at each BS and each user, it is called *degrees of connectivity* in the sequel.

For MS_{i_k}, the received signal can be expressed as

$$\boldsymbol{y}_{i_k} = \boldsymbol{H}_{i_k,i}\boldsymbol{x}_{i_k} + \sum_{l \neq k} \boldsymbol{H}_{i_k,i}\boldsymbol{x}_{i_l} + \sum_{j \in \mathcal{J}_{i_k}} \boldsymbol{H}_{i_k,j}\boldsymbol{x}_j + \boldsymbol{n}_{i_k} \tag{3}$$

where $\boldsymbol{x}_{i_k} \in \mathbb{C}^{M \times 1}$ is the signal vector transmitted from BS_i to MS_{i_k}, $\boldsymbol{x}_j = \sum_{k=1}^{K} \boldsymbol{x}_{j_k}$ is the signal vector transmitted from BS_j to its desired users, $\boldsymbol{H}_{i_k,j} \in \mathbb{C}^{N \times M}$ is the channel matrix from BS_j to MS_{i_k} whose elements are i.i.d. random variables with a continuous distribution, and $\boldsymbol{n}_{i_k} \in \mathbb{C}^{N \times 1}$ is the noise vector.

The received signal of each user contains the multiuser interference (MUI) from its desired BS and the inter-cell interference (ICI) from its interfering BSs, which are the second and third terms in (3).

Because both the IA with and without symbol extension will be addressed, we define several terminologies to be used. *Linear IA* is the IA without any symbol extension or only with finite spatial extension. *Asymptotic IA* is the IA with infinite time or frequency extension [1]. With the spatial, time or frequency extension, the DoF are not necessary to be an integer.

3. DOF ANALYSIS

In this section, we first derive the DoF achieved by both linear IA and asymptotic IA for the L-interfering MIMO-IBC and then show the impact of the connectivity on the achievable DoF.

3.1. Doubly-symmetric L-interfering MIMO-IBC

From Definition 1, we know that the connectivity of L-interfering MIMO-IBC is not always symmetric. The analysis in [8] indicates that when the connectivity is asymmetric, BSs or users in different cells will see different numbers of ICIs so that the IA feasible conditions are rather involved for analysis. To simplify the analysis, a special class of L-interfering MIMO-IBC was considered in [11], where both the antenna configuration and the connectivity are symmetric. We call it *doubly-symmetric L-interfering MIMO-IBC*.

Definition 2. *If a L-interfering MIMO-IBC whose connection pattern satisfies*

$$|\mathcal{J}_j|/K = |\mathcal{J}_{i_k}| = L, \forall i, j, k \tag{4}$$

it is called doubly-symmetric L-interfering MIMO-IBC.

When $L = G - 1$, the doubly-symmetric L-interfering MIMO-IBC reduces to the fully-connected symmetrically-configured MIMO-IBC.

In Fig. 1, we show an example of connection pattern for the doubly-symmetric L-interfering MIMO-IBC, where the solid lines represent the interfering channels with non-zero coefficients, and the desired links are not shown. Each BS (or user) is connected with the users (or BSs) in the two interfering cells, hence the degrees of connectivity are $L = 2$.

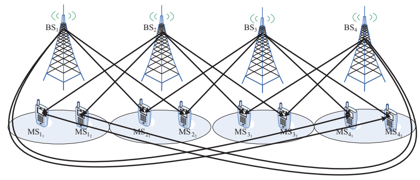

Fig. 1. Example of connection pattern for the doubly-symmetric L-interfering MIMO-IBC.

The study in [11] proved that a L-interfering MIMO-IBC that satisfies the proper condition for the doubly-symmetric L-interfering MIMO-IBC must to be proper. However, the relationship between "proper" and "feasible" is not clear, so that it is still unknown whether the derived proper condition is necessary or sufficient for linear IA feasibility in the L-interfering MIMO-IBC. To answer this question and derive the achievable DoF for the L-interfering MIMO-IBC, we introduce a lemma to show the relationship between the IA feasible conditions of the L-interfering MIMO-IBC and that of the doubly-symmetric L-interfering MIMO-IBC.

Lemma 1. *For two MIMO-IBC systems with the same antenna configuration and different connection patterns (denoted as $\mathcal{J}^\alpha$ and $\mathcal{J}^\beta$), if their antenna configurations are asymmetric and their connection patterns satisfy*

$$\mathcal{J}^\alpha \subseteq \mathcal{J}^\beta \tag{5}$$

or if their antenna configurations are symmetric and their connection patterns satisfy

$$|\mathcal{J}^\alpha_{i_k}| \leq |\mathcal{J}^\beta_{i_k}|, \; |\mathcal{J}^\alpha_j| \leq |\mathcal{J}^\beta_j|, \; \forall i, j, k \tag{6}$$

we have

1. *the necessary condition of IA feasibility for the MIMO-IBC with $\mathcal{J}^\alpha$ is always necessary for that with $\mathcal{J}^\beta$,*
2. *the sufficient condition of IA feasibility for the MIMO-IBC with $\mathcal{J}^\beta$ is always sufficient for that with $\mathcal{J}^\alpha$.*

The proof of this lemma can be obtained from the DoF analysis for the fully-connected asymmetrically-configured MIMO-IBC in [8].

Remark 1. *Since the connection set for the partial connectivity is always a subset of that for the full connectivity, and the symmetric antenna configuration is a special case of the asymmetric antenna configuration, the necessary conditions of IA feasibility for the asymmetrically-connected asymmetrically-configured MIMO-IBC can be obtained from that for the fully-connected asymmetrically-configured MIMO-IBC in [8]. From the necessary conditions, we can obtain the outer-bound of DoF region for the asymmetrically-connected asymmetrically-configured MIMO-IBC.*

For the asymmetrically-connected asymmetrically-configured MIMO-IBC, it is not hard to derive the DoF outer-bound but difficult to prove whether the DoF outer-bound is achievable or not. It is the reason why we consider the doubly-symmetric MIMO-IBC.

Remark 2. *Comparing* (2b) *and* (4)*, we know that for each BS or user, the number of interfering cells for the L-interfering MIMO-IBC is always no more than that for the doubly-symmetric L-interfering MIMO-IBC. As a result, the achievable DoF for the doubly-symmetric L-interfering MIMO-IBC are always achievable for the L-interfering MIMO-IBC. As shown in* (2a)*, we know that all users in one cell are assumed to have the identical connectivity in the L-interfering MIMO-IBC. In fact, from Lemma 1 we know that this assumption is not necessary to draw such a conclusion. In other words, the achievable DoF for the doubly-symmetric L-interfering MIMO-IBC are always achievable for the asymmetrically-connected symmetrically-configured MIMO-IBC when the maximal number of interfering cells seen at each BS and each user is no more than L.*

To derive the achievable DoF for the asymmetrically-connected symmetrically-configured MIMO-IBC (including L-interfering MIMO-IBC), we investigate the maximal achievable DoF for the doubly-symmetric L-interfering MIMO-IBC in the following.

3.2. Maximal Achievable DoF

To understand the potential of the doubly-symmetric L-interfering MIMO-IBC, we investigate the information theoretic maximal DoF.

Theorem 1. *For the doubly-symmetric L-interfering MIMO-IBC with antenna configuration $\left(M \times (N,d)^K\right)^G$, the information theoretic maximal DoF per user are*

$$d^{\text{Info}}(M,N,K,L) = \begin{cases} d^{\text{Decom}}(M,N,K), & \forall M/N \in \mathcal{R}^{\text{I}} \\ d^{\text{Quan}}(M,N,K,L), & \forall M/N \in \mathcal{R}^{\text{II}} \end{cases} \tag{7}$$

where

$$d^{\text{Decom}}(M,N,K) \triangleq \frac{MN}{M+KN} \tag{8}$$

$$d^{\text{Quan}}(M,N,K,L) \triangleq \tag{9}$$

$$\begin{cases} \min\left\{\frac{M}{K+C_n^{\text{A}}(L)}, \frac{N}{1+\frac{K}{C_{n-1}^{\text{A}}(L)}}\right\}, \forall C_n^{\text{A}}(L) \le \frac{M}{N} < C_{n-1}^{\text{A}}(L) \\ \min\left\{\frac{M}{K+C_{n-1}^{\text{B}}(L)}, \frac{N}{1+\frac{K}{C_n^{\text{B}}(L)}}\right\}, \forall C_{n-1}^{\text{B}}(L) < \frac{M}{N} \le C_n^{\text{B}}(L) \end{cases}$$

$$\mathcal{R}^{\text{I}} \triangleq \left(C_\infty^{\text{B}}(L), C_\infty^{\text{A}}(L)\right), \ \forall K \ge 4, L = 1 \text{ or } K \ge 1, L \ge 2$$
$$\mathcal{R}^{\text{II}} \triangleq \begin{cases} (0,\infty), & \forall K \le 3, L = 1 \\ \left(0, C_\infty^{\text{B}}(L)\right] \cup \left[C_\infty^{\text{A}}(L), \infty\right), & \text{otherwise} \end{cases}$$

$C_n^{\text{A}}(L) \triangleq LK - K/C_{n-1}^{\text{A}}(L)$ *and* $C_n^{\text{B}}(L) \triangleq K/(LK - C_{n-1}^{\text{B}}(L))$, $C_0^{\text{A}} = \infty$, $C_0^{\text{B}} = 0$,

$$C_\infty^{\text{A}}(L) \triangleq \lim_{n\to\infty} C_n^{\text{A}}(L) = (LK + \sqrt{L^2K^2 - 4K})/2 \tag{10a}$$

$$C_\infty^{\text{B}}(L) \triangleq \lim_{n\to\infty} C_n^{\text{B}}(L) = (LK - \sqrt{L^2K^2 - 4K})/2 \tag{10b}$$

$\forall K \ge 4, L = 1$ *or* $K \ge 1, L \ge 2$.

In Theorem 1, when $L = G - 1$, (8) and (9) reduce to the *decomposition DoF bound* and the *quantity DoF bound* for the fully-connected symmetrically-configured MIMO-IBC in [7]. Therefore, we also call (8) and (9) *decomposition DoF bound* and *quantity DoF bound* for the doubly-symmetric L-interfering MIMO-IBC.

Due to the lack of space, we only provide the skeleton of proofs for all theorems.

Proof Skeleton. By constructing the similar genie chain with that in [7], we can prove that the decomposition DoF bound and quantity DoF bound are the information theoretic DoF upper-bounds when $M/N \in \mathcal{R}^{\text{I}}$ and $M/N \in \mathcal{R}^{\text{II}}$ (i.e., M/N falls in Region I and Region II, respectively). Following similar derivations in [7], we can show that the decomposition DoF bound can be achieved by asymptotic IA, while the quantity DoF bound can be achieved by linear IA. In addition, the closed-form solution of linear IA exists. Therefore, (7) is information theoretic maximal. □

Considering that asymptotic IA is not feasible for practical systems, this motivates us to find the maximal DoF achieved by linear IA.

Theorem 2. *For the doubly-symmetric L-interfering MIMO-IBC MIMO-IBC with antenna configuration $\left(M \times (N,d)^K\right)^G$, the maximal DoF per user achieved by linear IA are*

$$d^{\text{Linear}}(M,N,K,L) = \begin{cases} d^{\text{Proper}}(M,N,K), & \forall M/N \in \mathcal{R}^{\text{I}} \\ d^{\text{Quan}}(M,N,K,L), & \forall M/N \in \mathcal{R}^{\text{II}} \end{cases} \tag{11}$$

where

$$d^{\text{Prop}}(M,N,K,L) \triangleq \frac{M+N}{(L+1)K+1} \tag{12}$$

Since (12) is one DoF upper-bound obtained from the proper condition for the doubly-symmetric L-interfering MIMO-IBC in [11], it is called *proper DoF bound*.

Proof Skeleton. In Theorem 1, we have proved that the decomposition DoF bound and the quantity DoF bound are the information theoretic DoF upper-bounds in Regions I and II, respectively. The study in [11] has shown that the proper DoF bound is the DoF upper-bound achieved by linear IA in all regions. Following regular but tedious derivations, we find that the proper DoF bound is lower than the decomposition DoF bound in Region I, while the quantity DoF bound is lower than or equal to the proper DoF bound in Region II. As a result, the proper DoF bound and the quantity DoF bound are the DoF upper-bounds of linear IA in Regions I and II, respectively. Moreover, we can prove that there always exists at least one feasible solution for linear IA to achieve the proper DoF bound in Region I following a similar proof in [6]. Theorem 1 shows that the linear IA achieves the quantity DoF bound in Region II. Therefore, (11) is the maximal DoF per user achieved by linear IA. □

Remark 3. *From* (7) *and* (11)*, the DoF gap between the information theoretic maximal DoF and the maximal DoF achieved by linear IA is obtained as*

$$\Delta d \triangleq d^{\text{Info}}(M,N,K,L) - d^{\text{Linear}}(M,N,K,L) \tag{13}$$
$$= \begin{cases} \frac{MN}{M+KN} - \frac{M+N}{(L+1)K+1}, & \forall M/N \in \mathcal{R}^{\text{I}} \\ 0, & \forall M/N \in \mathcal{R}^{\text{II}} \end{cases}$$

In Region I, the information theoretic maximal DoF (i.e., the decomposition DoF bound) are independent of L but the maximal DoF achieved by linear IA (i.e., the proper DoF bound) decrease with L, so that when L decreases Δd decreases. By contrast, in Region II, since the linear IA can achieve the information theoretic maximal DoF, Δd is always equal to zero. Moreover, (10) *shows that $C_\infty^{\text{A}}(L)$ decreases with L but $C_\infty^{\text{B}}(L)$ increases with L, so that when L decreases the range of Region II is expanded. Therefore, as L decreases, the DoF achieved by linear IA approach the information theoretic maximal DoF.*

Remark 4. *No matter asymptotic IA or linear IA, the maximal achievable DoF only depend on the degrees of connectivity L but do not depend on the total number of cells G. From the DoF results for the fully-connected MIMO-IBC in [7], we know that only the sum DoF achieved by asymptotic IA can increase linearly with G but that by linear IA cannot. By contrast, for the partially-connected MIMO-IBC, when L is bounded, the sum DoF achieved by both asymptotic IA and linear IA can linearly increase with G.*

From the above analysis, we know that when L is low, the linear IA can achieve a nearly information theoretic maximal DoF with finite symbol extension. As a result, in the partially-connected MIMO-IBC with the limited degrees of connectivity, the linear IA can achieve a good trade-off between achievable DoF and length of symbol extension.

Remark 5. *From the DoF results in Theorems 1 and 2, we can obtain the necessary and sufficient condition of IA feasibility for the doubly-symmetric L-interfering MIMO-IBC, which is also a sufficient condition of IA feasibility for the L-interfering MIMO-IBC. For example, Theorem 2 indicates that in Region I the proper DoF bound is achievable by linear IA for the doubly-symmetric L-interfering MIMO-IBC, which indicates that the proper condition is the sufficient condition of linear IA feasibility for the L-interfering MIMO-IBC. From the sufficient condition, we can know how many spatial resources are enough to align the interference of partial interfering links.*

To understand the DoF results in Theorems 1 and 2, Figs. 2 and 3 show the feasible and infeasible regions of asymptotic IA and linear IA for the doubly-symmetric L-interfering MIMO-IBC with different degrees of connectivity L, where the feasible region and infeasible region are with legend "FR" and "IR", respectively. From Lemma 1, we know that the feasible region for the MIMO-IBC with the degrees of connectivity $L = l$ must be feasible for that with $L = l - 1$ and the infeasible region for that with $L = l$ must be infeasible for that with degrees of connectivity $L = l + 1$, so that the feasible/infeasible regions for the MIMO-IBC with different L are overlapping. From the boundary of feasible and infeasible regions, the maximal achievable DoF for the doubly-symmetric L-interfering MIMO-IBC are shown, which are also the achievable DoF for the L-interfering MIMO-IBC.

As shown in the figures, when L decreases, the maximal DoF achieved by both asymptotic IA and linear IA increase and their DoF gap decreases. It shows that when L is low, the DoF achieved by linear IA are close to the information theoretic maximal DoF.

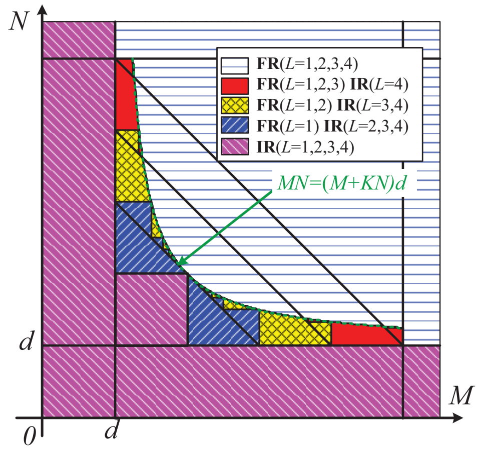

Fig. 2. Feasible and infeasible regions of Asymptotic IA for the doubly-symmetric L-interfering MIMO-IBC.

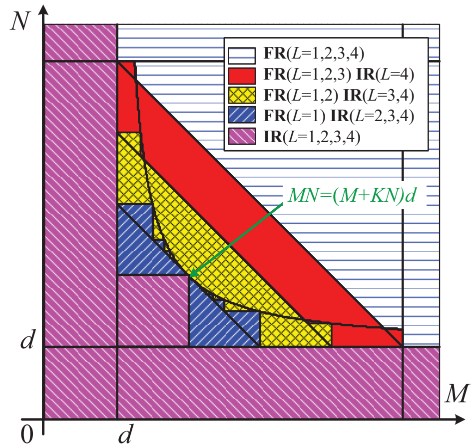

Fig. 3. Feasible and infeasible regions of Linear IA for the doubly-symmetric L-interfering MIMO-IBC.

4. CONCLUSION

In this paper, we investigated the partially-connected symmetrically-configured MIMO-IBC. We analyzed the information theoretic maximal DoF and maximal DoF achieved by linear IA for the doubly-symmetric L-interfering MIMO-IBC and found that the DoF are independent of the total number of cells G but depend on the degree of connectivity L. When L is bounded, the sum DoF achieved by both asymptotic IA and linear IA can linearly increase with G. When L is low, the linear IA can achieve a nearly information theoretic maximal DoF with finite symbol extension. We proved that the maximal achievable DoF for the doubly-symmetric L-interfering MIMO-IBC are also achievable for the L-interfering MIMO-IBC. From the achievable DoF we know what is the sufficient condition of IA feasibility, e.g., when the proper is sufficient for linear IA, which reflects how many spatial resources are enough to align the interference in the partially-connected network.

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