MODELING INFORMATION DIFFUSION DYNAMICS OVER SOCIAL NETWORKS

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ABSTRACT

Information diffusion over social networks becomes a hot topic recently. Most of the existing works are based on the machine learning method with social network structure analysis and empirical data mining. However, the results learned from some specific dataset may not apply to the future networks, since the social network structure is in a highly dynamic environment. Moreover, the dynamics of information diffusion are also heavily influenced by network users' decisions, actions and their socio-economic interactions, which is generally ignored by existing works. In this paper, we propose an evolutionary game theoretic framework to model the dynamic information diffusion process in social networks, which focuses on the users' behavior analysis from a microeconomics points of view. We also conduct experiments by using real-world Twitter information diffusion dataset, which shows that the proposed evolutionary game theoretic model is effective and practical in modeling the social network users' information diffusion dynamics.

***Index Terms*—** Social networks, information diffusion, information spreading, game theory, evolutionary game.

1. INTRODUCTION

Nowadays, social networks have become ubiquitous in our daily life. Due to its diverse implication, researchers from different disciplines have been working on it from various perspectives [1]. The topic of how information diffuse over social networks draws great attentions from both industry and academia recently. On one hand, the study of information diffusion can help the enterprises/polititians to achieve efficient and effective advertisement/advocation. On the other hand, from the security point of view, the study of information diffusion can also help to prevent the detrimental information spreading, e.g. computer virus, rumors and inauthentic news.

In the literature, the existing works can be classified into two categories: diffusion dynamics analysis and diffusion stability analysis. The first category focuses on analyzing the dynamic diffusion process over different kinds of networks using different mathematical models [2]-[8]. While the second category focuses on the stability and consequence of information diffusion [9]-[13]. In this paper, our analysis falls into the first category, i.e., the dynamic diffusion process. Most of existing works on information diffusion analysis are based on the machine learning method through empirical data mining, which is based on the assumption that the training set is statistically consistent with the testing set. One conspicuous shortcoming is that the results learned from some specified dataset rely on the corresponding social network structure and may not be able to analyze or predict the future networks since the social networks are usually in a highly dynamic environment. Secondly, such machine learning based method totally ignores the actions and decision making of users. While the influence of users' decisions, actions and socio-economic connections on information forwarding also plays an important role in the diffusion process. Considering these problems, we propose a game theoretic framework to analyze the dynamics of information diffusion over social network. Compared with the machine learning based method, the game theoretic one focuses on the users' behavior analysis from a microeconomics point of view, the results of which do not rely on the network structure and can be generally used to analyze and predict the future networks.

Specifically, we find that in essence the information diffusion process on social networks follows similarly the evolution process in natural ecological systems [14]. It is a process that evolves from one state at a particular instance to another when information is being forwarded and diffused around. Therefore, we consider the evolutionary game to model and study the social network users' information forwarding strategies and the dynamic diffusion process. The proposed model reveals the dynamics of information diffusion among users through analyzing their learning, interactions and decision making. Based on the evolutionary game theoretic formulation, we analyze the dynamics of diffusion process over complete networks and uniform degree networks. In the rest of this paper, we first model the dynamics of information diffusion over complete networks using evolutionary game in Section 3. Then, we model the dynamics of information diffusion over uniform degree networks using graphical evolutionary game in Section 3. Experiments results are shown in Section 4 and conclusions are drawn in Section 5.

2. EVOLUTIONARY GAME FORMULATION FOR INFORMATION DIFFUSION DYNAMICS OVER COMPLETE NETWORKS

In this section, we modeling the information diffusion dynamics over complete networks using graphical evolutionary game. We first introduce the basic concepts of evolutionary game, and then elaborate how to formulate the dynamic information diffusion process using evolutionary game theory.

2.1. Basic Concepts of Evolutionary Game Theory

The evolutionary game theory (EGT), originated from ecological biology [14], imagines that a game is played over and over again by biologically or socially conditioned players who are randomly drawn from a large population [15]. It emphasizes more on the dynamics of the whole population's strategies by studying the population shift and evolving process due to the influence of mutants. Recently, EGT has been widely used to model users' behaviors in communication and networking area [16][17], including network selection [18], cooperative sensing [19], cooperative peer-to-peer (P2P) streaming

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[20] and dynamic spectrum access [21], and also image processing area [22]. In these literatures, evolutionary game has been shown to be an effective approach to model the dynamic social interactions among users in a network.

One of the most important concepts in EGT is the "replicator dynamics", which illustrates the dynamic process of the whole population's strategies and can provide the system state information at a particular time instance. Consider an evolutionary game with a population of N players and M pure strategies $\mathcal{X} = \{1, 2, ..., M\}$. Let n_i denote the number of players adopting strategy i and $x_i = \frac{n_i}{N}$ denote the proportion of players adopting strategy i among the whole population. In such a case, the population state can be illustrated by a vector $\mathbf{x} = [x_1, x_2, ..., x_M]$. In EGT, the utility of a player is referred as "fitness" [23], which is defined as follows:

$$\Psi = (1 - \eta) \cdot B + \eta \cdot P, \tag{1}$$

where B is the baseline fitness representing the player's inherent property. For example, in a social network, a user's baseline fitness can be interpreted as his/her own interests on the released news. P is the player's payoff which is determined by the predefined payoff matrix and the player's interactions with others. The parameter η represents the selection intensity, i.e., the relative contribution of the game to fitness. The case $\eta \rightarrow 0$ represents the limit of weak selection [24], while $\eta \rightarrow 1$ denotes strong selection, where fitness equals payoff. Note that the selection intensity can be time variant, e.g., $\eta(t) = \beta e^{-\alpha t}$ which means that the contribution of game interaction decreases along with time.

According to EGT, the replicator dynamics can be defined by a set of discrete differential equations as follows:

$$\dot{x}_i(t) = x_i(t)\left[\overline{\Psi}_i(t) - \overline{\Psi}(t)\right], \forall i = 1, 2, ..., M. \tag{2}$$

where $\dot{x}_i(t)$ represents the variation of x_i at time t, i.e., $x_i(t+1) = x_i(t) + \dot{x}_i(t)$, $\overline{\Psi}_i(t)$ represents the average fitness of players adopting strategy i at time t and $\overline{\Psi}(t)$ represents the average fitness of the whole population at time t. We can see that if adopting strategy i can lead to a higher fitness than the average level, the proportion x_i will increase and the increasing rate $\dot{x}_i/x_i$ is proportional to the difference between $\overline{\Psi}_i$ and $\overline{\Psi}$.

2.2. Evolutionary Game Formulation

In a social network, when a series of new information are originated from one user or a small group of users, the dynamic information diffusion process heavily depends on other users' actions: to forward the information or not. For each user, whether to forward the information is determined by several factors, including the user's own interest on this information, as well as his/her neighbor's actions in the sense that if all his/her neighbors forward the information, the user may also forward the information with relatively high probability. Such a dynamic process of users' information forwarding is quite similar to the players' strategies update in the aforementioned EGT. Therefore, we can model the information diffusion dynamics over complete network using the evolutionary game, where the users can be regarded as players and each user has two possible strategies:

$$\begin{cases} \boldsymbol{S}_f, & \text{forward the information,} \\ \boldsymbol{S}_n, & \text{not forward the information.} \end{cases} \tag{3}$$

In social networks, users' payoff is determined by multiple factors, including the cost of forwarding the information, the reward obtained by forwarding/not forwarding the information (e.g, the popularity of a user in a social network or the hit rate of a website). In this paper, we model the users' payoff matrix as follows:

$$\begin{array}{c} \\ \boldsymbol{S}_f \\ \boldsymbol{S}_n \end{array} \begin{array}{c} \begin{array}{cc} \boldsymbol{S}_f & \boldsymbol{S}_n \end{array} \\ \begin{pmatrix} u_{ff} & u_{fn} \\ u_{fn} & u_{nn} \end{pmatrix} \end{array} \tag{4}$$

where a symmetric payoff structure is considered, i.e., when a user with strategy $\boldsymbol{S}_f$ meets a user with strategy $\boldsymbol{S}_n$, each of them receives the same payoff u_{fn}. Moreover, we assume that the payoff has been normalized within interval $(0, 1)$, i.e., $0 < u_{ff}, u_{fn}, u_{nn} < 1$. In a complete network, each user has possible interactions with all other users. Under such a circumstance, once some information are released by a user, all other users are supposed to receive the information. However, whether to forward the information depends on the strategies of different users. In this scenario, we consider the network as a group of users that continuously release new information. For instance, in practical social networks, such a group of users can be a circle in the Google pluse or a group in the Facebook. Since each user in the group also connects to other users outside the group, the more users in the group forward the information, the wider the information can diffuse. Therefore, through analyzing the dynamic of users' strategies on information forwarding, we can infer how the information propagate to other users outside the group. Let us define the proportion of users adopting strategy $\boldsymbol{S}_f$, i.e., forward the information, as x_f; and the proportion of users adopting strategy $\boldsymbol{S}_n$ as $x_n = 1 - x_f$. In such a case, the network state can be described by $\mathbf{x} = [x_f, x_n]$.

To analyze the dynamic changing of $\mathbf{x}$ along with time, we discretize the dynamic information diffusion process into time slot. In each time slot, the users in the complete network are assumed to be able to observe the strategies and fitness of other users in the population. Based on the observed information, in the next time slot, each user's decision on whether forwarding the information or not is determined by which strategy can give him/her higher fitness. Thus, along with the users' strategies update slot by slot, the network state $\mathbf{x}$ also keeps changing slot by slot. Let us define the changing rate of the network state as the *population dynamics* of information diffusion, $[\dot{x}_f, \dot{x}_n]$. The following theorem shows the population dynamics of information diffusion in complete networks, where we can see that no network scale information, e.g., how many users in the network, were utilized. The information diffusion dynamics in (5) only rely on the initial state $x_f(0)$ and the values of payoff matrix, which also shows the scale-free property.

Theorem 1: The population dynamics of information diffusion over complete networks can be described as follows:

$$\dot{x}_f(t) = \eta x_f(t)(1 - x_f(t))\left(a_1 x_f(t) + b_1\right), \tag{5}$$

$$x_f(t+1) = x_f(t) + \dot{x}_f(t), \tag{6}$$

$$\text{where } \begin{cases} a_1 = u_{ff} - 2u_{fn} + u_{nn}, \\ b_1 = u_{fn} - u_{nn}. \end{cases} \tag{7}$$

Proof: Due to page limitation, we show the proof in the supplementary information [25].

3. GRAPHICAL EVOLUTIONARY GAME FORMULATION FOR INFORMATION DIFFUSION DYNAMICS OVER UNIFORM DEGREE NETWORKS

In this section, we model the information diffusion dynamics over uniform degree networks, where users do not fully connect with each other. We first introduce the basic concepts of graphical evolutionary game, and then elaborate how to formulate the dynamic information diffusion process over uniform degree networks.

3.1. Basic Concepts of Graphical Evolutionary Game Theory

The traditional evolutionary game theory considers a population with full connections, i.e., the population is based on a complete graph. However, in many scenarios, players' spatial locations may lead to an incomplete graph structure. Graphical evolutionary game theory is introduced to study the strategies evolution in such a structured population [26]. In graphical EGT, in addition to the entities of players, strategy and payoff matrix, each game model is associated with a graph structure, where the vertexes represent players and the edges determine which player to interact with. Since the players only has limited connections with others, each player's fitness is locally determined from interactions with all adjacent players. In essence, the traditional evolutionary game can be regarded as a special case of graphical EGT, where the corresponding graph structure is complete. Previously, we have used graphical EGT to model the adaptive networks [27], as well as the stable state of information diffusion over social networks [28]. The major difference is that, we focus on the dynamics analysis of information diffusion in this paper using replicator dynamics, while [28] focused on the final stable state of information diffusion by analyzing the evolutionarily stable state (ESS), which is also an important concept in the EGT.

Similar to that in the traditional EGT, the concept of replicator dynamics is also of importance in the graphical EGT. The difference is that it is usually analyzed under some predefined strategy updating rules, including birth-death (BD), death-birth (DB) and imitation (IM) [29]. These strategy updating rules are from the evolutionary biology field and used to model the resident/mutant evolution process as follows: (a) For BD update rule, a player is chosen for reproduction with the probability being proportional to fitness (Birth process). Then, the chosen player's strategy replaces one neighbor's strategy with uniform probability (Death process). (b) For DB update rule, a random player is chosen to abandon his/her current strategy (Death process). Then, the chosen player adopts one of his/her neighbors' strategies with the probability being proportional to their fitness (Birth process). (c) For IM update rule, each player either adopts the strategy of one neighbor or remains with his/her current strategy, with the probability being proportional to fitness. In this paper, we adopt BD update rule when modeling the information diffusion dynamics. Note that all the analytical method and results can be easily extended to the DB and IM update rules.

3.2. Graphical Evolutionary Game Formulation

Based on the graphical evolutionary game formulation above, we analyze the information diffusion dynamics over uniform degree networks in this subsection. In the uniform scenario, an N-user social network based on a homogenous graph with general degree k is considered. Similar to the complete network scenario, the network state of information diffusion can also be described by $\mathbf{x} = [x_f, 1 - x_f]$, where x_f denotes the proportion of users who forward the information among the whole population. In this uniform degree networks scenario, our target is also to derive the dynamics of x_f along with time, which reflects the diffusion scale of the information. On the other hand, unlike the complete network scenario where the probability that a player meets an a player adopting strategy $\boldsymbol{S}_f$ is equal to the global network state x_f, in a social network based on an incomplete graph, this is not necessarily true since each user only has possible connections with his/her neighbors. In such a case, due to the limited dispersal, those who adopt the same strategy, i.e., either forward the information or not, tend to form clusters. In order to take into account the correlation in strategies of two adjacent players, we define the local network states as $x_{f|f}$ and $x_{f|n}$, which represents the proportion of a user's neighbors adopting strategy $\boldsymbol{S}_f$, given the user is using strategy $\boldsymbol{S}_f$ and $\boldsymbol{S}_n$, respectively. In other words, $x_{f|f}$ or $x_{f|n}$ is the local network state around a user adopting strategy $\boldsymbol{S}_f$ or $\boldsymbol{S}_n$. Note that the local network state and the global network state has the relationship as follows:

$$x_{f|f} = x_{ff}/x_f, \quad (1 - x_{f|f})x_f = x_{f|n}x_n, \tag{8}$$

where x_{ff} represents the global edge state, i.e., the proportion of edges on which both users adopting strategy $\boldsymbol{S}_f$. Similarly, we have x_{fn} and x_{nn}, where $x_{ff}+x_{fn}+x_{nn} = 1$. Thus, with the definitions of global and local network states, as well as the global edge states, we can define three dynamics of information diffusion over graph based networks as follows:

- *Population dynamics*: $\dot{x}_f$, which is similar to that in the complete networks.
- *Relationship dynamics*: $\dot{x}_{ff}$ and $\dot{x}_{nn}$, which are the dynamics of global edge states and illustrate the dynamics of relationship among users. Note that $\dot{x}_{fn} = -\dot{x}_{ff} - \dot{x}_{nn}$.
- *Influence dynamics*: $\dot{x}_{f|f}$ and $\dot{x}_{f|n}$, which are the dynamics of local network state and illustrate the influence of one user on his/her neighbors. For instance, $\dot{x}_{f|f} = 1$ means that all the user's neighbors adopt the same forwarding strategy with him/her, i.e., the user's neighbors are inclined to be influenced by him/her or the user is more influential. On the other hand, $\dot{x}_{f|n} = 1$ represents an opposite characteristic.

In the following, we will analyze those dynamics of information diffusion over uniform degree networks, with the objective of deriving the close-form expression of population dynamics.

Similar to the complete network scenario in Section II, we also discretize the dynamic information diffusion process into time slot and analyze the local and global dynamics under the BD strategy updating rule. According to the BD strategy rule, in each time slot, a user is selected from the whole population with probability proportional to his/her fitness. Then, the selected user's strategy, i.e., either forward the information or not, replaces one of his/her neighbors' strategy randomly. In other words, one of the user's neighbor is influenced by the user and replicates the user's strategy. Since the user selected for reproduction probably adopts a strategy with higher fitness than the average, the physical meaning of such a dynamic strategy updating rule is equivalent to that of the replicator dynamics. Therefore, the dynamics of the network states updated under BD rule is also expected to be derived as a set of differential equations, as in (2). In the following derivation, we only consider the weak selection scenario, i.e., the selection intensity parameter $\eta \to 0$. Under the weak selection, the payoff obtained from the interactions is considered as limited contribution to the overall fitness of each player, as we can see in (1). Note that the results derived from weak selection often remain as valid approximations for larger selection strength [24]. Moreover, the weak selection assumption can help to achieve a close-form analysis of the dynamic information diffusion process and better reveal how the strategy diffuses over the network. The following theorem shows the population dynamics of information diffusion in uniform degree networks.

Theorem 2: The population dynamics of information diffusion over uniform degree networks under Birth-Death strategy update rule can be described as follows:

$$\dot{x}_f(t) = \frac{\eta(k-2)}{(k-1)} x_f(t)(1 - x_f(t)) \left(a_2 x_f(t) + b_2\right), \tag{9}$$

$$x_f(t+1) = x_f(t) + \dot{x}_f(t), \tag{10}$$

$$\text{where} \begin{cases} a_2 = (k-2)(u_{ff} - 2u_{fn} + u_{nn}), \\ b_2 = u_{ff} + (k-2)u_{fn} - (k-1)u_{nn}. \end{cases} \quad (11)$$

Proof: Due to page limitation, we show the proof in the supplementary information [25].

Remarks: From *Theorem 2*, we can see that the form of population dynamics of information diffusion over uniform degree networks is quite similar to that over complete network in (5). The dynamics in (9) only rely on the initial state $x_f(0)$, the values of payoff matrix and the degree of the network, regardless of the network scale information. Therefore, the population dynamics of information diffusion over uniform degree networks also shows the scale-free property. Moreover, in real social networks, the degree of each user usually exhibits that $k \gg 2$. In such a case, (9) can be further approximated by

$$\begin{aligned} \dot{x}_f &= \frac{\eta(k-2)^2}{(k-1)} x_f(1-x_f)\big[(u_{ff} - 2u_{fn} + u_{nn})x_f \\ &\quad + \frac{u_{ff} - u_{nn}}{k-2} + u_{fn} - u_{nn}\big] \\ &= \eta' x_f(1-x_f)\big[(u_{ff} - 2u_{fn} + u_{nn})x_f + u_{fn} - u_{nn}\big], \end{aligned} \quad (12)$$

where $\eta' = \frac{\eta(k-2)^2}{(k-1)}$. We can see that, the population dynamics of information diffusion over uniform degree networks are exactly same with that over complete network as in (5). This is because, in a uniform degree network with sufficiently large degree, i.e., each user is with sufficiently large number of neighbors, the information forwarding strategy of one user is influenced by a large number of other users, which is similar to that in the complete networks. In essence, the complete network is a special case of the uniform degree networks when $k \to N$. Moreover, such a phenomenon also validates that the dynamics derived by the BD strategy update rule is equivalent with the replicator dynamics in complete networks.

4. EXPERIMENTS

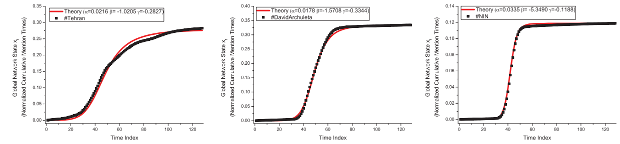
In the experiment, we use the Twitter hashtag dataset to estimate the payoff matrices corresponding to different hashtags. The Twitter hashtag dataset contains the the number of mention times per hour of 1000 Twitter hashtags with corresponding time series, which are the 1000 hashtags with highest total mention times among 6 million hashtags from Jun. to Dec. 2009 [30]. Let us first derive the closed-form expression for the global network state $x_f(t)$. In *Theorem 1* and *Theorem 2*, we can see that the dynamics of information diffusion over two kinds of networks share the same form as follows:

$$\frac{dx_f}{dt} = \beta e^{-\alpha t} x_f(1-x_f)(x_f + \gamma), \quad (13)$$

where $\eta = e^{-\alpha t}$ is considered as time-variance and different kinds of networks have different coefficients β and γ. Using the separation of variables method, we can derive the implicit closed-form expression of x_f as follows:

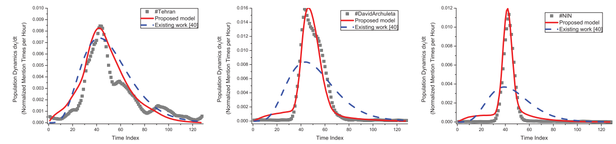
$$\frac{(\gamma+1)\ln x_f - \gamma \ln(1-x_f) + \ln(-x_f-\gamma)}{\gamma(\gamma+1)} = -\frac{\beta}{\alpha} e^{-\alpha t} + c, \quad (14)$$

where c is a constant and can be calculated by the initial condition $x_f(t=0)$. In such a case, we can estimate the parameters α, β and γ using (14) through fitting the Twitter hashtag dataset. Fig. 1 shows the curve fitting results of four hashtags using least squares method, where the vertical axis is global network state $x_f(t)$. The mention times of different hashtags per hour in the Twitter dataset are first normalized within interval $[0,1]$ and then accumulated over time to get the cumulative mention times as shown by solid black square. From the figure, we can see that our model can fit the real-world information diffusion data very well, which means that the global network state of information diffusion can be accurately predicted by the proposed evolutionary game theoretic model.



(a) Tehran. (b) DavidArchuleta. (c) NIN.

Fig. 1. The curve fitting of different hashtags diffusion dynamics.



(a) Tehran. (b) DavidArchuleta. (c) NIN.

Fig. 2. Comparison with the existing work.

According to (5), (9) and (13), we can obtain the relationship of the payoff matrix as follows:

$$u_{ff} - 2u_{fn} + u_{nn} = \beta, \quad (15)$$

$$u_{fn} - u_{nn} = \beta\gamma. \quad (16)$$

If u_{fn} is normalized as 1, then we can calculate u_{ff} and u_{nn} through solving the equation set above. Based on the estimated payoff matrix, we can further simulate the dynamics $\dot{x}_f(t)$ using our proposed model. In this experiment, we compare our results with one of the most related exsiting works [31] using data mining method, in which the dynamics of information diffusion are predicted by

$$\frac{dx_f}{dt} = q_1 t^{q_2} e^{-q_3 t}, \quad (17)$$

where the parameters q_1, q_2 and q_3 can also be estimated through least-squares curve fitting in a similar way. Fig. 2 shows the comparison results, where the vertical axis is the dynamics $\dot{x}_f(t)$ and the mention times of different hastags per hour in the Twitter dataset are normalized within interval $[0,1]$ and denoted by solid black square. From the figure, we can see that our model can fit the real-world information diffusion dynamics better than the data mining method in [31] since the users' interactions and decision making behaviours are taken into account.

5. CONCLUSION

In this paper, we formulate the dynamics of information diffusion over social networks using evolutionary game theory. We defined the players, strategies and payoff matrix in this problem, and highlighted the correspondence between the EGT and information diffusion. To validate our theoretical analysis, we conducted experiments on Twitter hashtags dataset, which corroborated that our proposed EGT model is effective and practical for modeling the dynamics of information diffusion problem. One of our future works is to extend the system model into non-uniform degree networks.

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