POWER SPECTRUM BLIND SAMPLING USING OPTIMAL MULTICOSET SAMPLING PATTERNS IN THE MSE SENSE

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ABSTRACT

We consider the design of a multicoset sampling pattern to be used in power spectrum blind sampling (PSBS). The criterion for the PSBS pattern design that we propose is based on the minimization of the mean square error of the power spectrum estimate. The design framework appears as a constrained optimization problem, whose complexity increases with the pattern length. We solve such a constrained optimization problem in terms of nonlinear integer programming by using exhaustive search.

Index Terms— Compressed sensing, spectrum sensing, power spectrum blind sampling, minimum mean square error, multicoset sampling, sparse rulers.

1. INTRODUCTION

Cognitive radio (CR) is a radio interface that opportunistically exploits the vacant frequency band (see, e.g., [1, 2]). Thus, a CR system continuously senses the spectrum, dynamically identify the idle spectrum, and operate in the unoccupied spectrum [3, 4, 5]. In this context, spectrum sensing techniques have to deal with multiband signals, which jointly cover a wide spectral band with a significant number of spectral holes.

Some of the previous works that have addressed the problem of compressed spectrum sensing for multiband signals propose a sampling scheme based on compressed sensing theory [6, 7] that guarantees perfect signal reconstruction [8, 9, 10, 11, 12]. However, we will focus here on the approaches that can be classified as power spectrum blind samplers [13, 14, 15, 16], which exploit the fact that for power spectrum estimation only covariance information is of interest, and sampling rate reduction can be achieved even for non-sparse signals.

Thus, in this paper, we consider the design of sampling patterns for a power spectrum blind sampling scheme (or compressive power spectrum estimation), namely herein PSBS pattern (PSBSP). In previous works, minimal sparse rulers (SRs) are proposed as sampling patterns. Our aim is alternatively to consider the final power spectrum reconstruction performance when designing the sampling stage. Therefore, we define sampling patterns that minimize the mean square error (MSE) of power spectrum estimate. To ensure full power spectrum reconstruction in a similar fashion to [14, 12, 16], new constraint of certain quantities is imposed herein by means of an existence condition of the pseudo inverse of a pattern correlation matrix.

2. PROBLEM STATEMENT

Consider a complex-valued wide-sense stationary signal x(t) with bandwith B. Our aim is to sample this signal at a rate lower than the Nyquist frequency 1/T, such that the power spectrum of x(t) can be accurately estimated.

For the acquisition stage, we consider a multicoset sampling strategy [8] implemented with M interleaved analog-to-digital converters working at a rate 1/NT, being 1/T the Nyquist sampling rate and N the block length. This sampling device can be modeled as in [16, 14]: a high rate integrate and dump process followed by a bank of M branches, consisting each of one of a filtering operation followed by a downsampling operation, as illustrated in Figure 1. Taking into account that multicoset sampling consists of selecting M Nyquist-rate samples in each block of length N, the coefficients of the filter $c_i[n], i = 1, ..., M$, can be written as

$$c_i[n] = \begin{cases} 1, & n = -n_i, \\ 0, & n \neq -n_i, \end{cases}$$
(1)

where there is no repetition in n_i , i.e.

$$n_i \neq n_j, \quad \forall i \neq j.$$
 (2)



Fig. 1. Digital model of the sampling device.

The output of the *i*-th branch of this PSBS scheme is given by

$$y_i[k] = z_i[kN], \tag{3}$$

where $z_i[\cdot]$ is given by

$$z_i[n] = c_i[n] * x[n] = \sum_{m=1-N}^{0} c_i[m] x[n-m].$$
(4)

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In Compressive Power Spectrum Estimation, the objective is to estimate the power spectrum of x(t) from the subNyquist samples $\{y_i[k]\}_{i,k}$, that is, to estimate the power spectrum spectrum of x[n], which is equivalent to obtain the autocorrelation function of x[n] given by $r_x[n] = \mathcal{E}\{x[m]x^*[m-n]\}$. In [16], Ariananda and Leus propose a method to recover the autocorrelation function $r_x[n]$ given the cross correlations $r_{y_i,y_j}[k]$, for $i, j = 0, \ldots, M - 1$, given by

$$r_{y_i,y_j}[k] = \mathcal{E}\{y_i[l]y_j^*[l-k]\}.$$
(5)

They show that

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$$r_{y_i,y_j}[k] = \sum_{l=0}^{1} \mathbf{r}_{c_i,c_j}^{\mathsf{T}}[l] \mathbf{r}_x[k-l],$$
(6)

where

$$\boldsymbol{r}_{c_{i},c_{j}}[k] = \begin{bmatrix} r_{c_{i},c_{j}}[kN] & r_{c_{i},c_{j}}[kN-1] \\ \cdots & r_{c_{i},c_{j}}[(k-1)N+1] \end{bmatrix}^{\mathsf{T}},$$
(7a)

$$[k] = \begin{bmatrix} r_x[kN] & r_x[kN+1] \\ \cdots & r_x[(k+1)N-1] \end{bmatrix}^{\mathsf{T}}.$$
 (7b)

By cascading all these cross correlation functions they compose vector $\boldsymbol{r}_y[k] = [\dots, r_{y_i,y_j}[k], \dots]^T$, which can be written as:

$$\boldsymbol{r}_{y}[k] = \sum_{l=0}^{1} \boldsymbol{R}_{c}[l]\boldsymbol{r}_{x}[k-l], \qquad (8)$$

where

$$\mathbf{R}_{c}[k] = \begin{bmatrix} \mathbf{r}_{c_{0},c_{0}}[k] & \cdots & \mathbf{r}_{c_{0},c_{M-1}}[k] \\ \mathbf{r}_{c_{1},c_{1}}[k] & \cdots & \mathbf{r}_{c_{M-1},c_{M-1}}[k] \end{bmatrix}^{\mathsf{T}}, \qquad (9)$$

From this equation, and after some algebraic manipulations, they arrive to the following matrix equation:

$$\boldsymbol{r}_{\mathrm{y}} = \boldsymbol{R}_{\mathrm{c}} \boldsymbol{r}_{\mathrm{x}},\tag{10}$$

where $r_y \in \mathbb{C}^{\frac{1}{2}M(2L+1)(M+1)\times 1}$ and $r_x \in \mathbb{C}^{N(2L+1)\times 1}$ are given by

$$\boldsymbol{r}_{y} = \begin{bmatrix} \boldsymbol{r}_{y}^{\mathsf{T}}[0] & \cdots & \boldsymbol{r}_{y}^{\mathsf{T}}[L] & \boldsymbol{r}_{y}^{\mathsf{T}}[-L] & \cdots & \boldsymbol{r}_{y}^{\mathsf{T}}[-1] \end{bmatrix}^{\mathsf{T}}, \quad (11a)$$

$$\mathbf{r}_{x} = [\mathbf{r}_{x}^{T}[0] \cdots \mathbf{r}_{x}^{T}[L] \mathbf{r}_{x}^{T}[-L] \cdots \mathbf{r}_{x}^{T}[-1]]^{T},$$
 (11b)

with L being a design parameter related to the support of $r_x[k]$ and $R_c \in \mathbb{C}^{\frac{1}{2}M(2L+1)(M+1) \times N(2L+1)}$ is given by

$$\mathbf{R}_{c} = \begin{bmatrix} \mathbf{R}_{c}[0] & \mathbf{O} & \cdots & \mathbf{O} & \mathbf{R}_{c}[1] \\ \mathbf{R}_{c}[1] & \mathbf{R}_{c}[0] & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \mathbf{R}_{c}[1] & \mathbf{R}_{c}[0] & \mathbf{O} & \vdots \\ \vdots & \ddots & \ddots & \ddots & \mathbf{O} \\ \mathbf{O} & \cdots & \mathbf{O} & \mathbf{R}_{c}[1] & \mathbf{R}_{c}[0] \end{bmatrix}, \quad (12)$$

with \boldsymbol{O} being a dimension-coresponding zero matrix.

The power spectrum of x[n] can be written as $s_x = \mathbf{F}_{2L+1}r_x$, being $\mathbf{F}_n \in \mathbb{C}^{n \times n}$ the DFT matrix of size n. From (10), a Time Domain (TD) estimator for the power spectrum is proposed as [16]

$$\hat{\boldsymbol{s}}_{\mathrm{x}} = \boldsymbol{F}_{(2L+1)N} \left(\boldsymbol{R}_{\mathrm{c}}^{\mathsf{H}} \boldsymbol{R}_{\mathrm{c}} \right)^{-1} \boldsymbol{R}_{\mathrm{c}}^{\mathsf{H}} \hat{\boldsymbol{r}}_{\mathrm{y}}, \qquad (13)$$

where $\hat{r}_y \in \mathbb{C}^{M^2(2L+1)\times 1}$ is an estimate of r_y . An unbiased estimator of r_y can be obtained using that

$$\hat{r}_{y_i,y_j}[k] = \frac{1}{K - |k|} \sum_{l=\max(0,k)}^{K-1+\min(0,k)} y_i[l]y_j^*[l-k], \quad (14)$$

where K is the number of measurements.

From (13), it is clear that the selected sampling pattern has to lead to a full column rank matrix \mathbf{R}_c . In [16], a suboptimal solution for the sampling patterns is proposed based on Minimal Sparse Rulers. Our contribution in this paper is the design of multicoset sampling patterns that lead to a full column-rank matrix \mathbf{R}_c and, at the same time, provide a time domain, minimum mean square error estimaton of the power spectrum.

3. DESIGN OF THE MULTICOSET SAMPLING MATRIX

Let $C_{\hat{r}_y} \in \mathbb{C}^{M^2(2L+1) \times M^2(2L+1)}$ be the covariance matrix of the estimate of r_y , defined as

$$\boldsymbol{C}_{\hat{\boldsymbol{r}}_{y}} = \mathcal{E}_{x} \left\{ \left(\hat{\boldsymbol{r}}_{y} - \mathcal{E}_{x} \left\{ \hat{\boldsymbol{r}}_{y} \right\} \right) \left(\hat{\boldsymbol{r}}_{y} - \mathcal{E}_{x} \left\{ \hat{\boldsymbol{r}}_{y} \right\} \right)^{\mathsf{H}} \right\}.$$
(15)

In what follows, we assume that the PSBSP length, i.e. M, is known. Next we consider an assumption that restricts the input signal x[n] to a certain class. However, in real application, e.g., in numerical simulation, the result derived from this assumption can be used for any signal waveform. Let the second-order statistics of x[n] be

$$\mathcal{E}_{\mathbf{x}} \{ x[n] x^*[m] \} = \sigma_{\mathbf{x}}^2 \delta[n-m], \qquad \forall m, n, \qquad (16a)$$

$$\mathcal{E}_{\mathbf{x}} \{ x[n] x[m] \} = 0, \qquad \forall m, n, \qquad (16b)$$

where σ_x^2 is the variance of x[n]. Under the assumption of circularlysymmetric complex-valued zero-mean signal with the second-order statistics in (16), it is shown in [16, eq. (45)] that $C_{\hat{r}_y}$ in (15) holds a block-diagonal structure according to

$$\boldsymbol{C}_{\hat{r}_{y}} = \begin{bmatrix} \ddots & \cdots & \boldsymbol{O} \\ \vdots & \boldsymbol{C}_{\hat{r}_{y}\hat{r}_{y}}[k] & \vdots \\ \boldsymbol{O} & \cdots & \ddots \end{bmatrix}, \quad (17)$$

for $k \in \{0, 1, ..., L, -L, ..., -1\}$, where $C_{\hat{r}_y \hat{r}_y}[k] \in \mathbb{C}^{M^2 \times M^2}$ is the covariance matrix of the estimate of $r_y[k]$ given in (18) shown on top of the next page. By observing

$$\boldsymbol{C}_{\hat{r}_{y}\hat{r}_{y}}[k] = \frac{1}{K - |k|} K \boldsymbol{C}_{\hat{r}_{y}\hat{r}_{y}}[0], \qquad (19)$$

we have

$$\boldsymbol{C}_{\hat{r}_y} = K\left(\boldsymbol{\Lambda}(\boldsymbol{\beta}) \otimes \boldsymbol{C}_{\hat{r}_y \hat{r}_y}[0]\right), \qquad (20)$$

where $\boldsymbol{\beta} \in \mathbb{R}^{(2L+1) \times 1}$ is given by

$$\boldsymbol{\beta} = \begin{bmatrix} \frac{1}{K} & \frac{1}{K-1} & \cdots & \frac{1}{K-L} & \frac{1}{K-L} & \cdots & \frac{1}{K-1} \end{bmatrix}^{\mathsf{T}}.$$
 (21)

Let us introduce a vector of indices $\boldsymbol{n} \in \mathbb{N}^{M imes 1}$

$$\boldsymbol{n} = \begin{bmatrix} n_0 & n_1 & \cdots & n_{M-1} \end{bmatrix}^{\mathsf{T}}.$$
 (22)

These indices correspond to the positions of the Nyquist rate samples to be obtained when acquiring the signal, that is, the indices of $c_i[n]$ which contain a nonzero element. Our goal is to find vector

$$\boldsymbol{C}_{\hat{r}_{y}\hat{r}_{y}}[k] = \frac{1}{K - |k|} \sigma_{x}^{4} \begin{bmatrix} r_{c_{0},c_{0}}[0] & r_{c_{0},c_{0}}[0] & r_{c_{0},c_{0}}[0]r_{c_{0},c_{1}}^{*}[0] & \cdots & r_{c_{0},c_{M-1}}[0]r_{c_{0},c_{M-1}}^{*}[0] \\ r_{c_{0},c_{0}}[0]r_{c_{1},c_{0}}^{*}[0] & r_{c_{0},c_{0}}[0]r_{c_{1},c_{1}}^{*}[0] & \cdots & r_{c_{0},c_{M-1}}[0]r_{c_{1},c_{M-1}}^{*}[0] \\ \vdots & \vdots & \ddots & \vdots \\ r_{c_{M-1},c_{0}}[0]r_{c_{M-1},c_{0}}^{*}[0] & r_{c_{M-1},c_{0}}[0]r_{c_{M-1},c_{1}}^{*}[0] & \cdots & r_{c_{M-1},c_{M-1}}[0]r_{c_{M-1},c_{M-1}}^{*}[0] \end{bmatrix}.$$
(18)

n that leads to a minimum MSE estimation of the power spectrum of the acquired signal. To this aim, we define the cost function to be minimized as $f_{\text{TD}-\text{MSE}}(n)$, a scaling amount of the mean square error (MSE) of the power spectrum estimate defined in (13), given by

$$f_{\text{TD-MSE}}(\boldsymbol{n}) = \frac{\mathcal{E}_{\text{x}}\left\{\|\hat{\boldsymbol{s}}_{\text{x}} - \boldsymbol{s}_{\text{x}}\|_{\text{E}}^{2}\right\}}{N(2L+1)\left(\frac{1}{K} + 2\sum_{l=1}^{L}\frac{1}{K-l}\right)\sigma_{\text{x}}^{4}}$$
(23)

where $\|\cdot\|_{\mathsf{E}}$ is the Euclidean norm.

With this formulation we will find next the sampling patterns that minimize this function, and at the same time, provide a full column-rank R_c matrix. To reach this objective, we need to obtain first an explicit form of the cost function in (23).

Proposition 1 (TD-MSE Function). By assuming the input signal x[n] to be a circularly-symmetric complex-valued random process with the second-order statistics according to (16), the MSE function $f_{\text{TD-MSE}}(n)$ in (23) is given by

$$f_{\text{TD}-\text{MSE}}(\boldsymbol{n}) = \sum_{m_1=0}^{M-1} \sum_{m_2=0}^{M-1} \frac{1}{\alpha_{1-(n_m_2-n_m_1)}^2(\boldsymbol{n})} + \frac{1}{\alpha_{N+1-(n_m_2-n_m_1)}^2(\boldsymbol{n})},$$
(24)

where $\alpha_{n_r}(\boldsymbol{n}) \in \mathbb{Z}^{1 \times 1}$ for $n_r \in \{1, 2, \dots, N\}$ is given by

$$\alpha_{n_r}(\boldsymbol{n}) = \sum_{m_1=0}^{M-1} \sum_{m_2=0}^{M-1} \delta[-n_r + 1 - (n_{m_2} - n_{m_1})] + \delta[N - n_r + 1 - (n_{m_2} - n_{m_1})].$$
(25)

The derivation of these equations is provided in [17].

Since the signal is assumed to be temporally-white Gaussian random, its power spectrum is well known to be flat. One may observe that it is unnecessary to estimate the power spectrum at all, because the power spectrum is known a priori and should be flat according to the Gaussian signal assumption. We, however, will not restrict the result in (24) to only the Gaussian signal or a signal with an extremely low signal-to-noise ratio. The noise-like Gaussian signal assumption paves the way for an alternative multicoset sampling pattern design based on minimum mean square error criterion, which outperforms, as we will see later, the minimal sparse rulers proposed in [16]. Similar to the derivation of (24) in the proof of Proposition 1, the extension of the MSE function derivation to any signal with a specific power spectrum shape is possible, when i) the covariance matrix $C_{\hat{r}_y}$ in (15) is known in advance, or ii) a consistent estimate of $C_{\hat{r}_u}$ is available. However, the computation is intensive. Furthermore, there is no guarantee that the final expression of the MSE function can be written in an explicit form as same as in (24), which may later affect the optimization complexity.

The result of Proposition 1 brings an insight into a sufficient condition such that the power spectrum reconstruction in (13) is perfect. **Lemma 1** (Power Spectrum Reconstruction Condition). Under the assumption of circularly-symmetric complex-valued zero-mean signal with the second-order statistics in (16), the power spectrum estimation in (13) is valid, when

$$\alpha_{n_r}(\boldsymbol{n}) \ge 1, \quad \forall n_r \in \{1, 2, \dots, N\}.$$
(26)

Proof of this lemma is given in [17].

The expression of the MSE function in Proposition 1 is not the final form we will use. We obtain a more concentrated form, which can reduce the computational burden, as follows.

Lemma 2 (TD-MSE Function Refinement). *The TD-MSE function in* (24) *yields*

$$f_{\text{TD}-\text{MSE}}(\boldsymbol{n}) = 2\tilde{f}_{\text{TD}-\text{MSE}}(\boldsymbol{n}),$$
 (27)

where $\tilde{f}_{\text{TD}-\text{MSE}}(\boldsymbol{n})$ is given by

$$\tilde{f}_{\text{TD}-\text{MSE}}(\boldsymbol{n}) = \sum_{m_1=0}^{M-1} \sum_{m_2=0}^{M-1} \frac{1}{\alpha_{1-(n_m_2-n_m_1)}^2(\boldsymbol{n})}.$$
 (28)

A straightforward derivation of (27) given (28) is provided in [17].

For $n_r = 1$ and $n_r = N + 1$, we can see that

$$\alpha_1(\boldsymbol{n}) = \boldsymbol{M},\tag{29a}$$

$$\alpha_{N+1}(\boldsymbol{n}) = M. \tag{29b}$$

Let n_0 be zero, i.e. $n_0 = 0$. From (29a) and (28), the optimization problem for the TD approach can be finally written as:

$$\hat{\boldsymbol{n}}_{\text{TD-MMSE}} = \arg\min_{\boldsymbol{n}} \tilde{f}_{\text{TD-MSE}}(\boldsymbol{n})
\alpha_{n_r}(\boldsymbol{n}) \ge 1,
\forall n_r \in \{2, 3, \dots, \lfloor \frac{1}{2}N \rfloor + 1\},
n_0 = 0,
\text{s.t.} \quad n_{M-1} = \lfloor \frac{1}{2}N \rfloor,
n_m \in \{n_{m-1} + 1, \dots, \lfloor \frac{1}{2}N \rfloor - M + m + 1\},
\forall m \in \{1, 2, \dots, M - 2\}.$$
(30)

4. NUMERICAL EXAMPLES

A circularly-symmetric complex-valued Gaussian signal is passed onto a digital filter as in [16]. The filtered signal has the first band as a rectangular shape that occupies the normalized frequency from -0.9π to -0.65π , the second band from 0.1π to 0.35π , and the third band from 0.55π to 0.8π . We have solved the optimization problem in (30) using the ES. Furthermore, we generate a set of random sampling points, which are redundant and not overlapped

Sampling Patterns for $N = 39$										
$n_{ m SR}$	[0	1	10	11	13	15	17	$[19]^{T}$		
$\hat{n}_{ ext{MMSE}}$ #1	[0	1	2	5 1	$0 \ 1$	31'	7 1	9] ^T		
$\hat{n}_{ ext{MMSE}}$ #3	[0	1	3	7 9	14	18	19	$]^{T}$.		

Table 1. Designed sampling patterns for K = 1,786, L = 2, $M_0 = 8$, N = 39, $N_R = 1,000$.

Sampling Patterns for $N = 78$											
$n_{ m SR}$	[0	1	12	15	26	29	31	33	35	37	39] ^T
$\hat{n}_{ ext{MMSE}}$ #1	[0	1	2	10	15	16	28	32	35	37	$39]^{T}$
$\hat{n}_{ ext{MMSE}}$ #5	[0	1	2	10	15	16	28	32	35	37	$39]^{T}$

Table 2. Designed sampling patterns for K = 1,786, L = 2, $M_0 = 11$, N = 78, $N_R = 1,000$.

with the designed sampling patterns, i.e. from $\lfloor \frac{1}{2}N \rfloor + 1$ to N - 1. The purpose is to see how the additional redundant sampling points affect the designed sampling patterns.

Tables 1, 2 and 3 show the designed sampling patterns for different parameters, in addition to the sparse rulers of the same length.

Sampling Patterns for $N = 128$											
$n_{ m SR}$	[0	1	3	6	13	20	27	34	41	48	55
									59	63	$[64]^{T}$
$\hat{n}_{ ext{MMSE}}$ #1	[0	1	2	6	8	20	29	38	47	50	53
									60	63	$64]^{T}$
$\hat{m{n}}_{ m MMSE}$ #2	[0	1	3	9	13	22	30	39	46	50	57
									61	62	$[64]^{T}$

Table 3. Designed sampling patterns for K = 1,786, L = 2, $M_0 = 14$, N = 128, $N_R = 1,000$.

Fig. 2, Fig. 3, and Fig. 4 show the theoretical and simulation results in terms of the normalized MSE in the estimate of the power spectrum as a function of the compression ratio M/N. It can be clearly observed that MMSE sampling patterns outperform minimal sparse rulers, and that the gain increases when the compression ratio decreases. For $(M_0, N) = (8, 39)$ in Fig. 2, the MMSE patterns slightly outperforms the minimal-SR patterns in low compression ratio. However, when the compression ratio approximately is greater than 0.359, the MMSE patterns starts to provide worse error performance than the minimal-SR patterns. In Fig. 3 and Fig. 4, one can see that in the cases of $(M_0, N) = (11, 78)$ and $(M_0, N) = (14, 128)$ the MMSE patterns provide lower NMSE than the minimal-SR patterns in the entire range of the compression ratio, unlike the case of $(M_0, N) = (8, 39)$. The performance improvement by the MMSE patterns is more significant in the case of $(M_0, N) = (14, 128)$ than in the case of $(M_0, N) = (11, 78)$.

5. CONCLUSIONS

We have derived the cost function to be optimized in order to design a sampling pattern that minimizes the MSE of a power spectrum estimate. The constrained optimization problem has been solved by using ES. The minimal-SR pattern previously proposed is shown to be fairly comparable to the PSBSP design based on the MMSE in terms of the MSE for short PSBSPs, while the MMSE-based PSB-SPs outperform those based on minimal-SR for long PSBSPs.



Fig. 2. NMSE as a function of compression ratio for sampling patterns in Table 1.



Fig. 3. NMSE as a function of compression ratio for sampling patterns in Table 2.



Fig. 4. NMSE as a function of compression ratio for sampling patterns in Table 3.

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