BLOCK-SPARSE SIGNAL RECOVERY WITH SYNTHESIZED MULTITASK COMPRESSIVE SENSING

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ABSTRACT

The paper considers the problem of reconstructing blocks-sparse signals. A new algorithm, called synthesized multi-task compressive sensing (SMCS), is proposed. In contrast to existing methods that rely on the availability of the sparsity structure information, the SMCS algorithm resorts to the multitask compressive sensing (MCS) technique for signal recovery. The SMCS algorithm synthesizes new compressive sensing (CS) tasks via circular-shifting operations and utilizes the minimum description length (MDL) principle to determine the proper set of the synthesized CS tasks for signal reconstruction. An outstanding advantage of SMCS is that it can achieve good signal reconstruction performance without using prior information on the block-sparsity structure. Simulations corroborate the theoretical developments.

***Index Terms*—** Block-sparsity, Bayesian learning, synthesized multitask compressive sensing, minimum description length

1. INTRODUCTION

Compressive sensing (CS) is an innovative signal processing technique that allows recovering a signal from its compressive measurements, as long as the signal representation is sparse in certain domains [1]. In this paper, we consider the efficient reconstruction of block-sparse signals. In addition to sparsity, these signals exhibit additional structure in the form of clustered nonzero coefficients. Block-sparsity arises when we deal with multi-band signals [2] or the measurements of gene expression levels [3]. Utilizing the block-sparsity property has been shown to be able to enable robust signal recovery from fewer compressive measurements [4].

Block-sparse signal reconstruction was examined extensively in recent literatures. Specifically, [4] proposed the block compressive sampling matching pursuit (BCoSaMP) for block-sparse signal recovery. However, BCoSaMP utilizes *a priori* information of original signals, namely, the number of nonzero blocks and the number of nonzero elements in each block. In [5], the block orthogonal matching pursuit (BOMP) was proposed. Through the application of relaxations, Zou *et. al.* developed the block fixed-point continuation algorithm in [6] for block-sparse signal recovery, while Elhamifar and Vidal approached the reconstruction problem via convex optimization [7]. The proposed methods from [6] and [7], nevertheless, require that the information regarding the sizes of different blocks is available. [8] investigated the dictionary optimization problem for block-sparse signal representation but the study also assumed the availability of the knowledge on the maximum block length. Based on the framework of Bayesian sparse learning for temporally correlated signals [9], [10] proposed two algorithms for block-sparse signals recovery. They were referred to as the block-sparse Bayesian learning (BSBL) algorithm and its extended version, the EBSBL algorithm. The BSBL algorithm deals with the case where the block partition is known while the EBSBL algorithm works with unknown block partitions. [10] points out that BSBL as well as EBSBL can have good performance even when the block partition is unknown, but they are sensitive to the choice of the block length parameter.

We shall develop in this paper a novel algorithm for reconstructing block-sparse signals. In contrast to most existing approaches, the new method eliminates the need for the sparsity structure information, such as the number of nonzero blocks required in the BCoSaMP algorithm. The newly proposed method is based on the multi-task compressive sensing (MCS) concept [11]. MCS is an elegant extension of Bayesian compressive sensing (BCS) [12] and it handles multiple CS tasks by jointly recovering multiple signals from their compressive measurements. By exploiting the statistical correlation among the original signals, MCS enables robust reconstruction of signals whose compressive measurements are insufficient when the signal recovery is conducted individually.

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We shall refer to our algorithm as the synthesized multitask compressive sensing (SMCS). It addresses the challenge of reconstructing a single block-sparse signal by transforming it into an MCS problem. The multiple CS tasks are synthesized by simply circular-shifting the columns of the measurement matrix of the original CS model. This corresponds to circular-shifting the elements in the original signal vector and creates signal vectors that have overlapping nonzero clusters, or equivalently, correlated signals, which enables the use of MCS. The number of synthesized tasks is determined by the minimum description length (MDL) principle. The newly proposed SMCS technique outperforms the previously developed block-sparse signal recovery methods in terms of significantly reduced reconstruction errors and the removal of the needs for detailed information on the sparsity structure. Computer simulations are provided to demonstrate the good performance of the proposed SMCS method.

2. MULTITASK COMPRESSIVE SENSING

Assume without the loss of generality that there are L CS tasks in MCS. They are modeled as $\boldsymbol{v}_i = \boldsymbol{\Phi}_i \boldsymbol{\theta}_i + \boldsymbol{\epsilon}_i$, where $i = 1, 2, ..., L$ and $\boldsymbol{v}_i$ is the compressive measurement vector of the ith task and $\boldsymbol{\Phi}_i$ is the $N_i \times M$ measurement matrix ($N_i << M$). $\boldsymbol{\theta}_i$ is the $M \times 1$ signal vector in the ith task, and $\boldsymbol{\epsilon}_i$ represents the measurement noise that follows an i.i.d. Gaussian distribution with zero mean and covariance matrix $\alpha_0^{-1}\mathbf{I}_i$.

For conciseness, here we do not illustrate MCS in detail and interested readers are directed to [11] for more details. MCS exploits the statistical correlation among the original signals of multiple CS tasks, evaluates jointly the sharing parameters $\boldsymbol{\alpha} = \{\alpha_j\}_{j=1,M}$ (the definition of $\boldsymbol{\alpha}$ refers to the equation (4) in [11]), and then recovers the original signals. MCS estimates the sharing parameters $\boldsymbol{\alpha}$ by maximizing the logarithm of resultant marginal likelihood (i.e., the equation (30) in [11])

$$\begin{aligned}\mathcal{L}(\boldsymbol{\alpha}) &= \sum_{i=1}^{L} \log p(\boldsymbol{v}_i|\boldsymbol{\alpha}) \\ &= -\frac{1}{2}\sum_{i=1}^{K}\left[(N_i + 2a)\log\left(\boldsymbol{v}_i^T \mathbf{B}_i^{-1}\boldsymbol{v}_i + 2b\right) + \log|\mathbf{B}_i|\right] \\ &\quad + \text{const}\end{aligned} \tag{1}$$

where $a = 10^2/\mathrm{std}\,(\boldsymbol{v})^2$, $\boldsymbol{v} = \{\boldsymbol{v}_i\}_{i=1,L}$, $b = 1$, $\mathbf{B}_i = \mathbf{I}_i + \boldsymbol{\Phi}_i \mathbf{A}^{-1}\boldsymbol{\Phi}_i^T$, $\mathbf{I}_i$ is an $N_i \times N_i$ identity matrix, $\mathbf{A} = \mathrm{diag}(\alpha_1, \ldots, \alpha_M)$, $|\cdot|$ denotes the determinant operator; and const represents a constant term. Next, the recovery of the signals $\boldsymbol{\theta}_i$ utilizes the equation (28) in [11].

3. SYNTHESIS OF MULTIPLE CS TASKS

We shall present in this section the idea of synthesizing multiple CS tasks from a single CS task. The underlying motivation is to generate CS tasks whose signal vectors are correlated. This would enable the application of the MCS technique described in Section II to the problem of effective block-sparse signal reconstruction. The CS task synthesis is achieved via simple circular-shifting operations.

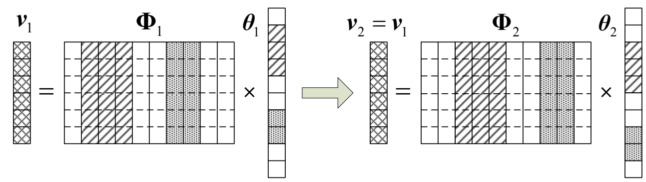


Fig. 1. Synthesis of a new CS task via circular-shifting.

We illustrate the synthesis method using the example depicted in Fig. 1, where for the sake of clarity, the absence of measurement noise is assumed. The original CS task is $\boldsymbol{v}_1 = \boldsymbol{\Phi}_1\boldsymbol{\theta}_1$, where $\boldsymbol{\theta}_1$ is the block-sparse signal to be recovered and it has two nonzero clusters that are highlighted with shadows. For illustration purpose, the columns of the measurement matrix $\boldsymbol{\Phi}_1$ that correspond to the nonzero elements in the signal $\boldsymbol{\theta}_1$ are also shadowed. As shown in Fig. 1, a new CS task can be synthesized from the original one by circularly shifting the columns of $\boldsymbol{\Phi}_1$ to the right by one column to generate a new measurement matrix $\boldsymbol{\Phi}_2$. Correspondingly, the elements in $\boldsymbol{\theta}_1$ are circularly shifted downward by one sample to yield the signal $\boldsymbol{\theta}_2$. Based on the above operation, we can obtain $\boldsymbol{\Phi}_1\boldsymbol{\theta}_1 = \boldsymbol{\Phi}_2\boldsymbol{\theta}_2$, so both CS tasks have the same compressive measurements, i.e., $\boldsymbol{v}_1 = \boldsymbol{v}_2$. We assume that this observation holds also for the case where measurement noise is present. Comparing $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ reveals that the locations of their nonzero elements have overlaps, which indicates that the signals of the original CS task and the synthesized one are correlated. This forms the basis for utilizing MCS in blocks-sparse signal recovery. More CS tasks can be synthesized by following an approach similar to the one shown in Fig.1 but varying the direction and the shift amount of the circular-shifting operations. For instance, we can circularly shift the columns of the measurement matrix $\boldsymbol{\Phi}_1$ to the left by one column to produce another new CS task.

4. SYNTHESIZED MULTITASK COMPRESSIVE SENSING

This section presents the proposed SMCS algorithm. Before providing the details of the computations, we shall first address an essential part of the new SMCS algorithm: the evaluation of the MCS-based signal recovery quality for a given set of synthesized CS tasks. This is crucial for selecting the optimal set of synthesized CS tasks for block-sparse signal recovery and as a result, it largely determines SMCS's performance.

The quality of the signal recovery is evaluated using the

MDL principle. Essentially, the MDL principle states that among a set of competing statistical models, the best model is the one having the minimum code length for the given data [13]-[14]. Mathematically, this is equivalent to solving $\hat{S} = \arg \min_{S \in \mathfrak{M}} \mathcal{CL}(\boldsymbol{v}, S)$, where $\mathfrak{M}$ denotes the set of possible models and $\mathcal{CL}(\boldsymbol{v}, S)$ is the code length function. In this work, we set $\mathcal{CL}(\boldsymbol{v}, S)$ to be the Shannon code length [15], i.e., $\mathcal{CL}(\boldsymbol{v}, S) = -\log_2 p(\boldsymbol{v}, S)$, where $p(\boldsymbol{v}, S)$ is the probability density function of $\boldsymbol{v}$ under the model S.

For a better illustration, denote the model of the original CS task as $\boldsymbol{v}_1 = \boldsymbol{\Phi}_1 \boldsymbol{\theta}_1 + \boldsymbol{\epsilon}_1$. Within the SMCS framework, the synthesized CS tasks have the same compressive measurement vector as the original CS task (see Section III). Define the information vector shared by the synthesized and the original CS tasks as $\boldsymbol{\alpha}$. Suppose under a particular set of synthesized CS tasks, the MCS-based signal recovery algorithm presented in Section II outputs an estimate of $\boldsymbol{\alpha}$, denoted as $\widehat{\boldsymbol{\alpha}}$. Then, the code length for $\boldsymbol{v}_1$ can be evaluated using (1) as

$$\begin{aligned} &\mathcal{CL}(\boldsymbol{v}_1) \\ &= \mathcal{CL}(\boldsymbol{v}_1 | \widehat{\boldsymbol{\alpha}}) + \mathcal{CL}(\widehat{\boldsymbol{\alpha}}) \\ &= -\log_2 \int p(\boldsymbol{v}_1 | \boldsymbol{\theta}_1, \alpha_0) \, p(\boldsymbol{\theta}_1 | \widehat{\boldsymbol{\alpha}}, \alpha_0) \, p(\alpha_0 | a, b) \, d\boldsymbol{\theta} d\alpha_0 \\ &\quad - \log_2 p(\widehat{\boldsymbol{\alpha}}) \\ &= \tfrac{1}{2} \left[(N_1 + 2a) \log_2 \left(\boldsymbol{v}_1^T \mathbf{D}^{-1} \boldsymbol{v}_1 + 2b \right) + \log_2 |\mathbf{D}| \right] \\ &\quad + \mathrm{const1} \end{aligned} \tag{2}$$

where $\mathcal{CL}(\boldsymbol{v}_1 | \widehat{\boldsymbol{\alpha}}) = -\log_2 p(\boldsymbol{v}_1 | \widehat{\boldsymbol{\alpha}})$ measures the goodness of fit between the data and the current model, $\mathcal{CL}(\widehat{\boldsymbol{\alpha}}) = -\log_2 p(\widehat{\boldsymbol{\alpha}})$ represents the model complexity, $\mathrm{const1}$ is a constant term, $\mathbf{D} = \mathbf{I}_1 + \boldsymbol{\Phi}_1 \widehat{\mathbf{A}}^{-1} \boldsymbol{\Phi}_1^T$. This accomplishes the evaluation of the signal recovery quality given a set of synthesized CS tasks.

We are now ready to present the proposed SMCS algorithm for recovering block sparse signals. SMCS improves the signal recovery in an iterative manner. In each iteration, a new CS tasks is synthesized using circular-shifting with different direction and shifting amount and is applied together with the previously synthesized CS tasks and the original CS task to the MCS algorithm. The above process continues until the number of synthesized CS tasks reaches a pre-specified value or including the newly synthesized CS task does not lead to better signal reconstruction quality (or equivalently, the reduced code length for describing the data (see (2))).

The algorithm flow is depicted in Algorithm 1. Here, $k_{\max}$ is the pre-specified maximum number of the synthesized CS tasks. $\mathrm{MCS}^k(\boldsymbol{v}, \boldsymbol{\Phi})$ represents the MCS-based signal reconstruction [11] in the kth iteration and it has k CS tasks. $\boldsymbol{v}$ and $\boldsymbol{\Phi}$ collect the compressive measurement vectors and their associated measurement matrices of the CS tasks utilized in the kth iteration. The outputs of $\mathrm{MCS}^k(\boldsymbol{v}, \boldsymbol{\Phi})$ are denoted by $\widehat{\boldsymbol{\alpha}}^k$ and $\widehat{\boldsymbol{\theta}}_1^k$, i.e., the estimates of the information sharing vector $\boldsymbol{\alpha}$ and the original signal $\boldsymbol{\theta}_1$. The operators $\mathrm{Left}(\boldsymbol{\Phi}_1, k)$ and $\mathrm{Right}(\boldsymbol{\Phi}_1, k)$ represent that the columns of $\boldsymbol{\Phi}_1$ are circularly shifted to the left and to the right by k columns.

Algorithm 1 (SMCS)

1 Inputs: $\boldsymbol{v}_1$, $\boldsymbol{\Phi}_1$, $k_{\max}$.

2 Output: $\widehat{\boldsymbol{\theta}}_1$.

3 Initialize $k \leftarrow 2$; $\boldsymbol{v} \leftarrow \{\boldsymbol{v}_1\}$; $\boldsymbol{\Phi} \leftarrow \{\boldsymbol{\Phi}_1\}$; $\widehat{\boldsymbol{\alpha}}^1, \widehat{\boldsymbol{\theta}}_1^1 \leftarrow \mathrm{MCS}^1(\boldsymbol{v}, \boldsymbol{\Phi})$; calculate $\mathcal{CL}_1(\boldsymbol{v}_1)$ using (2).

4 $\boldsymbol{v} \leftarrow \{\boldsymbol{v}, \boldsymbol{v}_1\}$; $\boldsymbol{\Phi} \leftarrow \{\boldsymbol{\Phi}, \mathrm{Left}(\boldsymbol{\Phi}_1, k-1)\}$ if k is even, or $\boldsymbol{\Phi} \leftarrow \{\boldsymbol{\Phi}, \mathrm{Right}(\boldsymbol{\Phi}_1, k-1)\}$ if k is odd; $\widehat{\boldsymbol{\alpha}}^k, \widehat{\boldsymbol{\theta}}_1^k \leftarrow \mathrm{MCS}^k(\boldsymbol{v}, \boldsymbol{\Phi})$; calculate $\mathcal{CL}_k(\boldsymbol{v}_1)$ using (2).

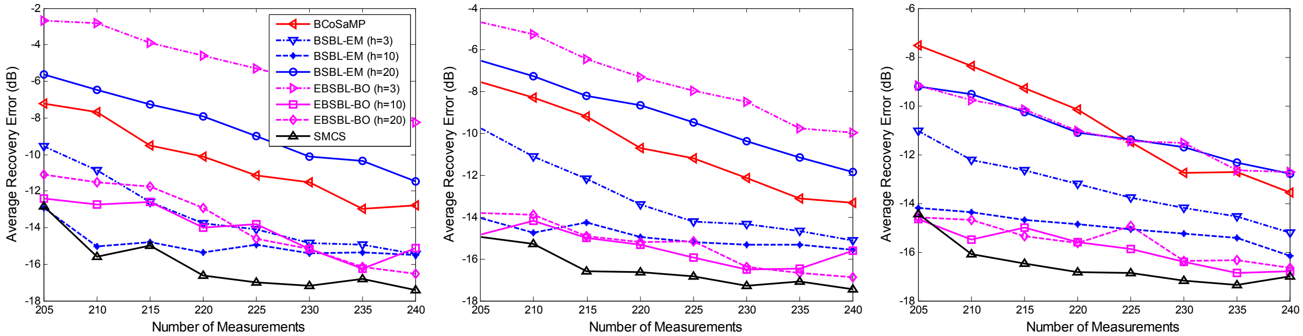
5 If $\mathcal{CL}_k(\boldsymbol{v}_1) < \mathcal{CL}_{k-1}(\boldsymbol{v}_1)$ or $k = k_{\max}$, $\widehat{\boldsymbol{\theta}}_1 \leftarrow \widehat{\boldsymbol{\theta}}_1^k$ and terminate the algorithm; otherwise, $k \leftarrow k+1$ and goto step 4.

5. SIMULATIONS

This section demonstrates via computer simulations the good performance of the newly proposed SMCS algorithm in recovering block-sparse signals. The benchmark algorithms used are BCoSaMP from [4], BSBL-EM and EBSBL-BO developed in [9]. Note that BCoSaMP needs the block partition information of the signal to be reconstructed. The BSBL-EM and EBSBL-BO algorithms both require a tuning parameter, namely, the block length h. They are supplied with those information in the simulations, while the SMCS algorithm operates without exploring any information on the sparsity structure of the original signal. We set the maximum number of CS tasks in the SMCS algorithm to be $k_{\max} = 6$.

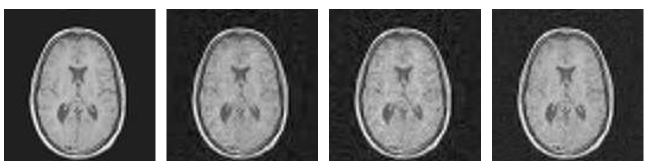
In the first simulation, the Monte Carlo simulations each with a total number of ensemble runs set to be 50 are carried out. In each ensemble run, the elements in the measurement matrix of the original CS task are independently drawn from a Gaussian distribution with mean 0 and a standard deviation $\frac{1}{\sqrt{N_1}}$. Here, N_1 is the number of rows in the measurement matrix, which is also the number of elements in the compressive measurement vector of the original CS task. The signal vector has 512 elements and they are partitioned into 32 blocks with a block size of 16. The locations of the non-zero blocks are chosen randomly at each ensemble run. Zero-mean Gaussian noise with standard deviation 0.01 is added to each of the N_1 measurements that define the data $\boldsymbol{v}_1$. The block length parameter used in the BSBL-EM and EBSBL-BO algorithms can have three possible values $h = 3, 10, \text{or } 20$.

We set the number of non-zero blocks in the original signal to be 8 (i.e, 128 non-zero elements). And we consider three cases, where the intra-block correlation coefficient for each non-zero block is uniformly distributed within [0, 0.1], [0.4, 0.5] and [0.8, 0.9]. The signal reconstruction error is quantified using $\left\| \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_1 \right\|_2 \Big/ \|\boldsymbol{\theta}\|_2$, where $\boldsymbol{\theta}$ and $\hat{\boldsymbol{\theta}}_1$ are the true and the estimated signals. Fig. 2 plots, as a function of N_1, the signal reconstruction mean square error of the sim-



(a) (b) (c)

Fig. 2. Comparison of BCoSaMP, BSBL-EM, EBSBL-BO and SMCS as a function of the number of compressive measurements in the original CS task when the intra-block correlation value for each non-zero block is uniformly randomly varied from 0 to 0.1 (a), from 0.4 to 0.5 (b) , from 0.8 to 0.9 (c).



(a) (b) (c) (d)

Fig. 3. Comparison of BSBL-EM and SMCS in image reconstruction. (a) Linear; (b) BSBL-EM, $h = 10$; (c) BSBL-EM, $h = 20$; (d) SMCS.

ulated BCoSaMP, BSBL-EM , EBSBL-BO and SMCS algorithms in dB scale (i.e., $10\log_{10}(\cdot)$ dB). It can be observed that the proposed SMCS outperforms other algorithms and the performance of BSBL-EM and EBSBL-BO is sensitive to the choice of the tuning parameter h. For example, the reconstruction error of EBSBL-BO decreases greatly as h increases from 3 to a value of 10. It also outperforms BCoSaMP, even though the latter utilizes the block partition information.

In the following simulation, we compare the performance of BSBL-EM with that of SMCS in recovering 2-D images of the MRI image. In this experiment, the elements of the measurement matrices of the two algorithms in consideration are drawn from a uniform spherical distribution. Fig. 3 summarizes the reconstruction results from a particular run. Fig. 3(a) is taken from [11]. The original image has the dimension of 128×128. Here, we utilize the "Daubechies 8" wavelet expansion with a coarsest scale of 3 and a finest scale of 6. We find that the nonzero fine-scale coefficients of the "Daubechies 8" wavelet of the original image also have the block structure. Fig.3(a) gives the result of the inverse wavelet transform with 4096 samples, denoted as Linear in the figure. This is the best performance achievable by all the CS algorithms considered here. The reconstruction results from BSBL, when the tuning parameter $h = 10$ and $h = 20$, are shown in Fig. 3(b) and (c) respectively, where we adopted the hybrid CS scheme that compresses the fine-scale coefficients only as in [11] into N_1=2400 measurements for each task. Figs. 3(d) gives the recovery result of SMCS. The recovery errors of Linear, BSBL-EM ($h = 10$), BSBL-EM ($h = 20$) and SMCS in dB scale are -7.3271dB, -6.8984dB, -6.8387dB and -7.0242 dB, respectively. The results in Fig. 3 shows that SMCS has the best image reconstruction performance, while BSBL-EM ($h = 10$) yields a better performance than that of BSBL-EM ($h = 20$).

The reason why we do not show the result of EBSBL is that Matlab gives the error "Out of memory" when we utilize the EBSBL-EM algorithm with $h = 10$ or $h = 20$ (the parameters of performance about the computer are "Intel(R) Core(TM) i5-3330S CPU @ 2.70GHz, 4.00GB RAM, 64bit Win7 operating system"). From the EBSBL algorithm in [10], we can also find that EBSBL has the heavy computation burden when it deals with the big data.

6. CONCLUSIONS

In this paper, a novel algorithm for recovering a block-sparse signal from its compressive measurements, termed as the SMCS algorithm, was developed. It improves the signal reconstruction performance by synthesizing new CS tasks via simple circular-shifting operations and applying the MCS framework for signal recovery. To determine the proper set of the synthesized CS tasks for reconstructing the block-sparse signal, the MDL principle was adopted. The new method completely eliminates the requirement of *a priori* information on the sparsity structure of the original signal, as usually needed in previously proposed techniques. Computer simulations were carried out and the SMCS algorithm was shown to be able to outperform existing techniques in providing greatly enhanced block-sparse signal reconstruction quality.

7. REFERENCES

[1] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.

[2] M. Mishali and Y. C. Eldar, "Blind multi-band signal reconstruction: Compressed sensing for analog signals," *IEEE Trans. Signal Process.*, vol. 57, no. 3, pp. 993-1009, Mar. 2009.

[3] F. Parvaresh, H. Vikalo, S. Misra, and B. Hassibi, "Recovering sparse signals using sparse measurement matrices in compressed DNA microarrays," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 3, pp. 275-285, Jun. 2008.

[4] R. G. Baraniuk, V. Cevher,M. F. Duarte, and C. Hegde, "Model-based compressive sensing," *IEEE Trans. Inf. Theory*, vol. 56, no. 4, pp. 1982-2001, Apr. 2010.

[5] Y. C. Eldar, P. Kuppinger, and H. Bolcskei, "Block-sparse signals: Uncertainty relations and efficient recovery," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3042-3054, Jun. 2010.

[6] J. Zou, Y. Fu, and S. Xie, "A Block Fixed Point Continuation Algorithm for Block-Sparse Reconstruction," *IEEE Trans. Signal Process. Letters*, vol. 19, no. 6, pp. 364-367, Jun. 2012.

[7] E. Elhamifar, and R. Vidal, "Block-Sparse Recovery via Convex Optimization," *IEEE Trans. Signal Process.*, vol. 60, no. 8, pp. 4094-4107, Aug. 2012.

[8] L. Zelnik-Manor, K. Rosenblum, and Y. C. Eldar, "Dictionary Optimization for Block-Sparse Representations," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2386-2395, May. 2012.

[9] Z. Zhang and B. D. Rao, "Sparse signal recovery with temporally correlated source vectors using sparse Bayesian learning," *IEEE Journal of Selected Topics in Signal Processing*, vol. 5, no. 5, pp. 912-926, 2011.

[10] Z. Zhang and B. D. Rao, "Extension of SBL algorithms for the recovery of block sparse signals with intra-block correlation," to appear in *IEEE Trans. Signal Process.*, downloadable from http://arxiv.org/abs/1201.0862.

[11] S. Ji, D. Dunson, and L. Carin, "Multi-task compressive sensing," *IEEE Trans. Signal Process.*, vol. 57, no. 1, pp. 92-106, Jan. 2009.

[12] S. Ji, Y. Xue, and L. Carin, "Bayesian compressive sensing," *IEEE Trans. Signal Process.*,vol. 56, no. 6, pp. 2346-2356, Jun. 2008.

[13] A. Barron, J. Rissanen, and B. Yu, "The minimum description length principle in coding and modeling," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2743-2760, 1998.

[14] I. Ramirez, and G. Sapiro, "An MDL framework for sparse coding and dictionary learning," *IEEE Trans. Signal Process.*,vol. 60, no. 6, pp. 2913-2927, Jun. 2012.

[15] T. Cover and J. Thomas, *Elements of Information Theory, 2nd ed.* New York: Wiley, 2006.