REPLICATION AND OPTIMIZATION OF HEDGE FUND RISK FACTOR EXPOSURES

Douglas E. Johnston[†], *Iñigo Urteaga*[*] *and Petar M. Djurić*[*]

[†] Quantalysis, LLC
[*] Department of Electrical & Computer Engineering
Stony Brook University, USA

e-mails: quantalysis.llc@gmail.com, {inigo.urteaga, petar.djuric}@stonybrook.edu

ABSTRACT

In this paper, we propose a novel approach for decomposing hedge fund returns onto observable risk factors. We utilize a vector stochastic-volatility model to extract the time-varying exposure of low frequency hedge fund returns on high frequency market data. We implement the estimation by using particle filtering and the concept of Rao-Blackwellization. With the latter, we remove all the static parameters of the model and thereby reduce the dimension of the parameter space for particle generation. Thus, we are able to obtain accurate estimates of the posterior distributions of the model states. For our model, this reduction is significant because the number of static parameters is large. We use the proposed model to analyze hedge fund performance and to optimally replicate hedge fund strategies economically. We demonstrate the validity and effectiveness of the method by computer simulations.

Index Terms— stochastic volatility, particle filtering, hedge fund, risk-management, VaR, CAPM, beta

1. INTRODUCTION

The hedge fund industry has grown into a large sector of the financial markets with over $2.5 trillion of assets under management. As such, analyzing hedge fund returns has enormous importance for investors [10]. In particular, understanding whether or not a hedge fund is simply taking so-called market exposure (e.g., owning the stock market) or adding incremental value is a critical task. This is especially important in light of the large fees that hedge funds usually charge (2% of assets under management and 20% of profits) [16]. To complicate matters, hedge fund returns are typically only available on a monthly basis and there is a limited history. Investors, such as fund-of-funds, have the arduous task of building portfolios of hedge fund strategies based on these limited and low frequency observations. As such, it is desired to estimate risk factors based on high frequency market observables and reverse engineer hedge fund returns onto those factors [15]. By doing so, portfolios of hedge fund strategies can be optimized while constraining market risk exposures [1, 2].

The model that we propose to investigate will decompose the risk of various hedge fund strategies based on monthly return observations and on common factors that are observable at a higher frequency (e.g., daily, or intra-day). Specifically, we will jointly model the risk-factors (e.g., stock and bond market returns) and the asset returns (e.g., a hedge fund strategy) in a fat-tailed, stochastic volatility environment implemented with a Bayesian approach via a Rao-Blackwellized particle filtering (PF) [4]. The hedge fund estimation problem, for a one-factor setting, was tackled in [12] and in this paper we extend that analysis to multiple factors along with its application.

One of the fundamental tenets in finance is that an asset's returns are proportional to risk and that risk can be decomposed into risk-factors (e.g., CAPM or APT) [14]. For example, let the return of the overall stock market be $r_{m,t}$ and the return of one particular hedge fund strategy be $r_{h,t}$. Then we write

$$r_{h,t} = \alpha_h + \beta_h r_{m,t} + v_t. \tag{1}$$

The first term α_h is the so-called idiosyncratic, or hedge fund specific return ("alpha"), which might be, for example, representative of a fund manager's skill. This is a sought-after commodity which one seeks to maximize. The second term β_h measures the sensitivity of the hedge fund return to the market (e.g., stock) and is not due to any specific skill or algorithm on the part of the fund. This "beta" term does not add value since it can be easily and cheaply replicated using liquid market instruments. Finally, v_t is the residual hedge fund specific risk. This formulation highlights that a hedge fund's performance, and fees they charge, should not be purely based on observed returns since "mimicking" the market does not represent added value and that performance statistics need to be adjusted accordingly [13].

The model introduced in this paper will be based on this basic tenet, but with multiple factors and with the time-varying states of the model being hidden. This implies the need to estimate those hidden random variables from observed data. In particular, we will assume that (1) the unknown states, including the log-variance of the noise, form a vector autoregressive process and (2) the observed returns are functions of the state. We utilize a particle filter and, since the model contains many unknown static parameters, we employ Rao-Blackwellization (RB). In this vector setting, RB reduces the unknown parameter space dimension significantly, leading to improved performance. This is crucial as we increase the number of factors. We show that we can implement PF by generating particles of the dynamic states from multivariate or univariate Student t distributions whose parameters are continuously updated during the tracking. Furthermore, with PF we can readily obtain samples from predictive distributions of the returns of the model. Our model can be used to assess hedge fund performance, divorced from market exposures, as well as to replicate hedge fund returns and to optimally design portfolios with predesigned risk factor exposures.

The problem is formally defined in the next section. In Section 3, we provide our solution based on RB PF. Section 4 contains

The work of the third author was supported by NSF under Award CCF-1018323 and ONR under Award N00014-09-1-1154.

simulation results that demonstrate the performance of the proposed method, and Section 5 concludes the paper.

2. PROBLEM STATEMENT

The vector state process $x_t = [x_{1,t} \dots x_{7,t}]^\top \in \mathbb{R}^{7\times 1}$ is defined as

$$x_t = \Theta z_{t-1} + \Phi u_t \tag{2}$$

where $z_t = [1 \ x_{1,t} \ x_{2,t} \ \cdots \ x_{7,t}]^\top$ and $\Theta \in \mathbb{R}^{7\times 8}$ corresponds to:

$$\Theta = \begin{bmatrix} \theta_{1,0} & \theta_{1,1} & \theta_{1,2} & \theta_{1,3} & 0 & 0 & 0 & 0 \\ \theta_{2,0} & \theta_{2,1} & \theta_{2,2} & \theta_{2,3} & 0 & 0 & 0 & 0 \\ \theta_{3,0} & \theta_{3,1} & \theta_{3,2} & \theta_{3,3} & 0 & 0 & 0 & 0 \\ \theta_{4,0} & 0 & 0 & 0 & \theta_{4,4} & 0 & 0 & 0 \\ \theta_{5,0} & 0 & 0 & 0 & 0 & \theta_{5,5} & 0 & 0 \\ \theta_{6,0} & 0 & 0 & 0 & 0 & 0 & \theta_{6,6} & 0 \\ \theta_{7,0} & 0 & 0 & 0 & 0 & 0 & 0 & \theta_{7,7} \end{bmatrix}, \tag{3}$$

with the noise vector $u_t = [u_{1,t}, \dots u_{7,t}]^\top \sim \mathcal{N}(0, C_u)$, $C_u \in \mathbb{R}^{7\times 7}$ defined by

$$C_u = \begin{bmatrix} 1 & \rho_1 & \rho_2 & 0 & 0 & 0 & 0 \\ \rho_1 & 1 & \rho_3 & 0 & 0 & 0 & 0 \\ \rho_2 & \rho_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

and $\Phi = \mathrm{diag}[\sigma_1, \sigma_2, \cdots, \sigma_7]$. All the parameters contained in Θ, Φ, and C_u are static unknowns. The states $x_{i,t}$, $i = 1, 2, 3$ are the log-variances of the stock market, bond market, and hedge fund returns, respectively. The state $x_{4,t}$ is the beta between the stock market and the bond market return. The state $x_{5,t}$ is the beta between the stock market and the hedge fund return, the state $x_{6,t}$ is the beta between the hedge fund return and the bond market return, that is uncorrelated to the stock market. Finally, the state $x_{7,t}$ is the idiosyncratic return of the hedge fund, commonly referred to as α.

We observe a vector of returns $r_{1,t}, r_{2,t}, r_{3,t} \in \mathbb{R}$ defined by

$$\begin{aligned} r_{1,t} &= \mu_1 + e^{x_{1,t}/2} v_{1,t}, \\ r_{2,t} &= \mu_2 + x_{4,t} r_{1,t} + e^{x_{2,t}/2} v_{2,t}, \\ r_{3,t} &= x_{7,t} + x_{5,t} r_{1,t} + x_{6,t}(r_{2,t} - x_{4,t} r_{1,t}) + e^{x_{3,t}/2} v_{3,t}, \end{aligned} \tag{5}$$

where $r_{1,t}$ is the stock market return, $r_{2,t}$ is the bond market return, $r_{3,t}$ is the hedge fund return, and $v_{i,t} \sim \mathcal{N}(0, 1)$ with $\mathbb{E}(v_{i,t} v_{j,t}) = 0$ for $i, j = 1, 2, 3$, $i \neq j$. The unknown constants μ_1 and μ_2 are the expected stock market and bond market returns. Descriptively, the hedge fund return, $r_{3,t}$, consists of the time varying return $x_{7,t}$ plus two factor returns. The first is associated with the stock market $x_{5,t} r_{1,t}$ and the second is related to the part of the bond market return orthogonal to the stock market $x_{6,t}(r_{2,t} - x_{4,t} r_{1,t})$. Finally, there is the idiosyncratic risk for the hedge fund $e^{x_{3,t}/2} v_{3,t}$.

Our objective is to estimate the state vector, x_t, as well as predict future returns $r_{1,t+1}, r_{2,t+1}, r_{3,t+1}$, given past returns $r_{1,1:t}, r_{2,1:t}, r_{3,1:t}$ and the model defined above. To that end, we want to be able to generate samples from the posterior density $p(x_t | r_{1,1:t}, r_{2,1:t}, r_{3,1:t})$ and the predictive density $p(r_{1,t+1}, r_{2,t+1}, r_{3,t+1} | r_{1,1:t}, r_{2,1:t}, r_{3,1:t})$.

Before we proceed with the proposed method, we comment on the considered state model. The process equation (2) is a vector autoregressive model with lag 1, VAR(1), where only the log-variances ($x_{i,t}, i = 1, 2, 3$) are coupled. This coupling allows for potential strong correlations between the variances of the individual asset return series. For example, it allows for shocks in the stock market to "bleed" into bond market or hedge fund returns with a one-step lag. A specific example would be the occurrences in the fall of 1998, when bond market volatility, caused by the demise of hedge fund Long-Term Management, fed into increased volatility in the stock market as well as the overall hedge fund universe. A similar idea for a univariate SV model was analyzed in [7] where the authors modeled the leverage effect when volatility in the return observation fed back into the variance process itself allowing for shocks in market returns to increase future volatility. While we allow for coupling between the log-variances, we assume that the betas, and the idiosyncratic hedge fund returns, are independent from each other and from the log-variances. While this assumption can certainly be relaxed, we felt the additional flexibility did not warrant the increased parameter set size. An area of future research will be to allow full coupling with, for example, increased volatility potentially leading to higher betas between asset classes.

3. PROPOSED METHOD

The proposed model is characterized by a 7-dimensional dynamic state-space with a large number of static parameters captured by the matrices Θ, Φ, and C_u. The proposed methodology for estimating all the unknowns of the model is PF, which is based on generating streams of particles of the unknowns and evaluating them based on the observations. There are actually three steps that are implemented in such methods at every time instant, (1) particle generation, (2) computation of the particle weights, and (3) resampling. The third step is necessary because it provides the means to remove particles that are non-promising and avoids situations where practically all the weight of the particles is concentrated at one particle.

PF needs to take special care for static parameters because it is designed to track the *changes* of the unknown parameters. We also point out that it is always beneficial to reduce the space of the unknowns where we generate the particles. One way of achieving this is by analytically integrating out some of the unknowns. In our model, this can be done with all the static parameters. Thus, by integrating out the static parameters, we not only resolve the problem of handling them, but also generate vectors of particles of much lower dimension, thereby making the exploration of the space of unknowns much more efficient.

The details of the integration of the static parameters in the state equation are shown in [8] and [12], so we will not repeat them here. In these references we provide the equations that are used for generating particles x_t from $p(x_t | x_{0:t-1})$, where $x_{0,1,\cdots,t-1} \equiv \{x_0, x_1, \cdots, x_{t-1}\}$. First we assume that at time instant $t-1$, we have the random measure $\chi_{t-1} = \{w_{t-1}^{(m)}, x_{0:t-1}^{(m)}\}_{m=1}^M$, where $w_{t-1}^{(m)}$ is the weight of the mth particle stream $x_{0:t-1}^{(m)}$, and M is the number of particles.

We can show that the first three states $p(x_{1:3,t} | x_{1:3,0:t-1}^{(m)})$ have a multivariate Student t distribution and the remaining states, univariate Student t distributions, i.e.,

$$\begin{cases} x_{1:3,t}^{(m)} | x_{1:3,0:t-1} \sim MSt\left(x_{1:3,t} | \vartheta_{1:3,t-1}^{(m)}, 1, \Sigma_{1:3,t-1}^{-1(m)}, \nu_{1:3,t-1}\right) \\ x_{i,t}^{(m)} | x_{i,0:t-1} \sim St\left(x_{i,t} | \vartheta_{i,t-1}^{(m)}, \sigma_{i,t-1}^{2(m)}, \nu_{i,t-1}\right), i = 4, 5, 6, 7, \end{cases} \tag{6}$$

where the parameters of these distributions are obtained from previous data as shown in [8] and [12]. Therefore, once they are known, we can readily generate from them particles for the next time instant.

Once the particles for the time instant t are generated, we proceed with computing their weights. Since we do resampling in every time step, we have

$$\begin{aligned} w_t^{(m)} &\propto p(r_{1,t}, r_{2,t}, r_{3,t}|x_{0:t}^{(m)}, r_{1,1:t-1}, r_{2,1:t-1}, r_{3,1:t-1}) \\ &= p(r_{3,t}|x_{0:t}^{(m)}, r_{1,1:t}, r_{2,1:t}, r_{3,1:t-1}) \\ &\times p(r_{2,t}|x_{0:t}^{(m)}, r_{1,1:t}, r_{2,1:t-1}, r_{3,1:t-1}) \\ &\times p(r_{1,t}|x_{0:t}^{(m)}, r_{1,1:t-1}, r_{2,1:t-1}, r_{3,1:t-1}). \end{aligned} \tag{7}$$

For the first factor in (7) we have

$$p(r_{3,t}|x_{0:t}^{(m)}, r_{1,1:t}, r_{2,1:t}, r_{3,1:t-1}) = \mathcal{N}(r_{3,t}|\,\widetilde{\mu}_{r_{3,t}}^{(m)}, \widetilde{\sigma}_{r_{3,t}}^{2(m)}), \tag{8}$$

where

$$\widetilde{\mu}_{r_{3,t}}^{(m)} = x_{7,t}^{(m)} + x_{5,t}^{(m)} r_{1,t} + x_{6,t}^{(m)}(r_{2,t} - x_{4,t}^{(m)} r_{1,t}), \tag{9}$$

$$\widetilde{\sigma}_{r_{3,t}}^{2(m)} = e^{x_{3,t}^{(m)}}. \tag{10}$$

The second factor is obtained from

$$\begin{aligned} p(r_{2,t}|x_{0:t}^{(m)}, r_{1,1:t}, r_{2,1:t-1}, r_{3,1:t-1}) = \int p(r_{2,t}|\mu_2, x_{4,t}^{(m)}, x_{2,t}^{(m)}) \\ \times p(\mu_2|x_{0:t-1}^{(m)}, r_{1,1:t-1}, r_{2,1:t-1}) \mathrm{d}\mu_2. \end{aligned} \tag{11}$$

Finally, for the third factor we write

$$\begin{aligned} p(r_{1,t}|x_{0:t}^{(m)}, r_{1,1:t-1}, r_{2,1:t-1}, r_{3,1:t-1}) = \int p(r_{1,t}|\mu_1, x_{1,t}^{(m)}) \\ \times p(\mu_1|r_{1,1:t-1}, x_{1,t-1}^{(m)}) \mathrm{d}\mu_1. \end{aligned} \tag{12}$$

The integrations of the unknown parameters μ_1 and μ_2 in (11) and (12) can be carried out analytically. As a result, for the weight of the particle stream m, we can write

$$\begin{aligned} \log(w_t^{(m)}) &= \log c - \frac{1}{2}\left(\log \widetilde{\sigma}_{r_{1,t}}^{(m)^2} + \log \widetilde{\sigma}_{r_{2,t}}^{(m)^2} + \log \widetilde{\sigma}_{r_{3,t}}^{(m)^2}\right) \\ &- \frac{\left(r_1 - \widetilde{\mu}_{r_{1,t}}^{(m)}\right)^2}{2\widetilde{\sigma}_{r_{1,t}}^{(m)^2}} - \frac{\left(r_{2,t} - x_{4,t}^{(m)} r_{1,t} - \widetilde{\mu}_{r_{2,t}}^{(m)}\right)^2}{2\widetilde{\sigma}_{r_{2,t}}^{(m)^2}} \\ &- \frac{\left(r_{3,t} - \widetilde{\mu}_{r_{3,t}}^{(m)}\right)^2}{2\widetilde{\sigma}_{r_{3,t}}^{(m)^2}}. \end{aligned} \tag{13}$$

where c is a proportionality constant, and

$$\widetilde{\mu}_{r_{1,t}}^{(m)} = \left(h_{t-1}^\top C_{1,t-1}^{(m)^{-1}} h_{t-1}\right)^{-1} h_{t-1}^\top C_{1,t-1}^{(m)^{-1}} \rho_{1,t-1}, \tag{14}$$

$$\widetilde{\mu}_{r_{2,t}}^{(m)} = \left(h_{t-1}^\top C_{2,t-1}^{(m)^{-1}} h_{t-1}\right)^{-1} h_{t-1}^\top C_{2,t-1}^{(m)^{-1}} \rho_{2,t-1}^{(m)}, \tag{15}$$

$$\widetilde{\mu}_{r_{3,t}}^{(m)} = x_{7,t}^{(m)} + x_{5,t}^{(m)} r_{1,t} + x_{6,t}^{(m)}(r_{2,t} - x_{4,t}^{(m)} r_{1,t}), \tag{16}$$

$$\widetilde{\sigma}_{r_{1,t}}^{(m)^2} = e^{x_{1,t}^{(m)}} + \left(h_{t-1}^\top C_{1,t-1}^{(m)^{-1}} h_{t-1}\right)^{-1}, \tag{17}$$

$$\widetilde{\sigma}_{r_{2,t}}^{(m)^2} = e^{x_{2,t}^{(m)}} + \left(h_{t-1}^\top C_{2,t-1}^{(m)^{-1}} h_{t-1}\right)^{-1} \tag{18}$$

$$\widetilde{\sigma}_{r_{3,t}}^{2(m)} = e^{x_{3,t}^{(m)}}, \tag{19}$$

where

$$h_{t-1} = [1,\ 1,\ \cdots, 1]^\top \tag{20}$$

$$\rho_{1,t-1} = [r_{1,1}, r_{1,2}, \cdots, r_{1,t-1}]^\top \tag{21}$$

$$\rho_{2,t-1}^{(m)} = [r_{2,1} - x_{4,1}^{(m)} r_{1,1},\ \cdots,\ r_{2,t-1} - x_{4,t-1}^{(m)} r_{1,t-1}]^\top \tag{22}$$

$$C_{1,t-1}^{(m)} = \mathrm{diag}\{e^{x_{1,1}^{(m)}}, e^{x_{1,2}^{(m)}}, \cdots, e^{x_{1,t-1}^{(m)}}\} \tag{23}$$

$$C_{2,t-1}^{(m)} = \mathrm{diag}\{e^{x_{2,1}^{(m)}}, e^{x_{2,1}^{(m)}}, \cdots, e^{x_{2,t-1}^{(m)}}\}. \tag{24}$$

An important feature of the proposed methodology is its ability to generate particles from the predictive distributions of the hidden states, as well as the returns. Consider for example the need to obtain samples of r_{t+1}. In theory, we have to be able to draw from

$$p(r_{t+1}|r_{1,:t}) = \int p(r_{t+1}|x_{t+1}, r_{1,:t}) p(x_{0:t+1}|r_{1,:t}) \mathrm{d}x_{0:t+1}, \tag{25}$$

which first requires that we solve (25) and second, that we are able to generate r_{t+1} from that distribution. Due to the complexity of the integral, this is not feasible, and therefore we resort to an approximation. At time instant $t+1$ we propagate the particles and have the streams $x_{0:t+1}^{(m)}$. We draw randomly the stream from the multinomial distribution with uniform probabilities, and say, we obtain $x_{0:t+1}^{(l_m)}$. Then using the particle $x_{t+1}^{(l_m)}$ and the estimates $\widetilde{\mu}_1^{(l_m)}$ and $\widetilde{\mu}_2^{(l_m)}$, we readily generate first $r_{1,t+1}^{(m)}$, then $r_{2,t+1}^{(m)}$, and finally, $r_{3,t+1}^{(m)}$.

In the next section, we demonstrate the performance of the method with Monte Carlo simulations.

4. SIMULATION RESULTS

In order to evaluate the introduced vector stochastic-volatility model and the proposed PF method, we ran several simulations to produce time-series that mimic real data. In simulating the method, we generated the states and the observations using the following parameters of the model:

$$\Theta = \begin{bmatrix} 0.7 & -0.4 & 0.5 & -0.3 & 0 & 0 & 0 & 0 \\ 1.2 & -0.3 & 0.3 & 0.1 & 0 & 0 & 0 & 0 \\ -0.4 & 0.2 & 0.4 & -0.1 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}, \tag{26}$$

with the noise parameters $\rho_1 = 0$, $\rho_2 = 0$, $\rho_3 = 0$, and $\sigma_i = 0.1$, $i = 1, 2\cdots, 7$. Finally, the expected stock and bond market returns were set to $\mu_1 = 0.03$ and $\mu_2 = 0.02$.

In Fig. 1, we observe one set of simulated stock market, bond market and hedge fund returns from the model. They showed variabilities that mimic well real returns.

In Figs. 2 - 4, we display the mean-square errors (MSEs) of the estimated states $x_{1,t}$, $x_{2,t}$, and $x_{3,t}$ obtained by two particle filters. One of them knew the exact values of all the static parameters of the model and the other had no such knowledge. The purpose of running the first filter was to have a benchmark for the performance ofthe second filter.

The MSEs were obtained from 50 independent realizations, and in each PF implementation, we used $M = 1000$ particles. As

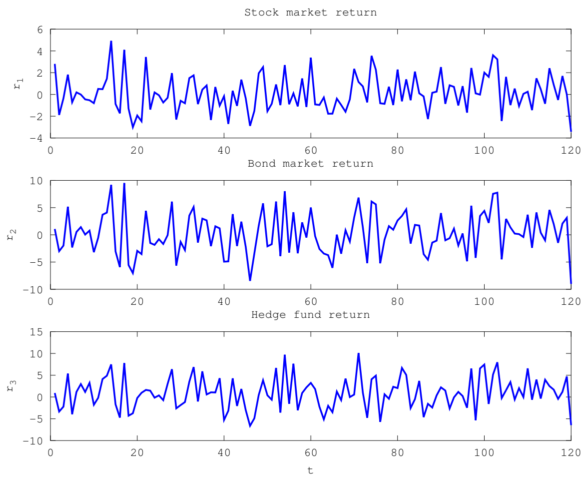

Fig. 1: Simulated return time-series.

expected, the performance of the particle filter that knew all the static parameters performed better, but that of the proposed filter was not far behind. We would like to point out that in initializing the filters, we used particles of the states that were randomly distributed near their true values. Even with this type of initialization, the filters needed about 20 samples to lock on good estimates of the state. The problem of initialization deserves a separate consideration and will be part of our future work.

We should note that in many examples, and certainly in our case, the data are available at different frequencies. For example, stock and bond market returns are available daily, although not necessarily synchronously, while the hedge fund returns are only available monthly. The particle filter is ideally suited to such asynchronous data and therefore, this, too, will be part of our future work.

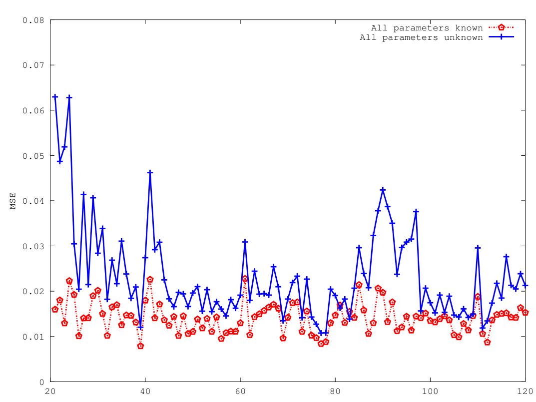

Fig. 2: MSE of $x_{1,t}$ with PF when all the static parameters are known and unknown, respectively.

It is important to emphasize that the proposed method can also provide estimates of the static parameters, even though they were integrated out. Due to lack of space, we do not present results related to the predictive distributions of the returns. They show that the method was also capable to provide accurate predictions.

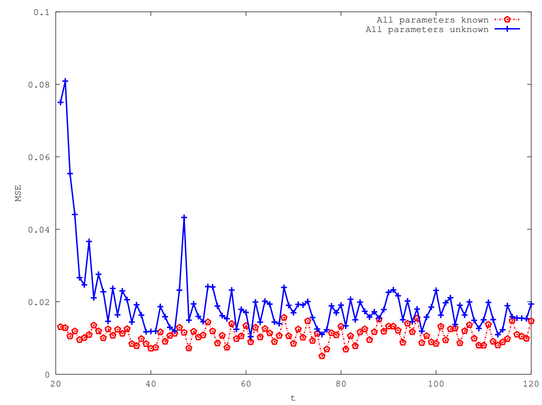

Fig. 3: MSE of $x_{2,t}$ with PF when all the static parameters are known and unknown, respectively.

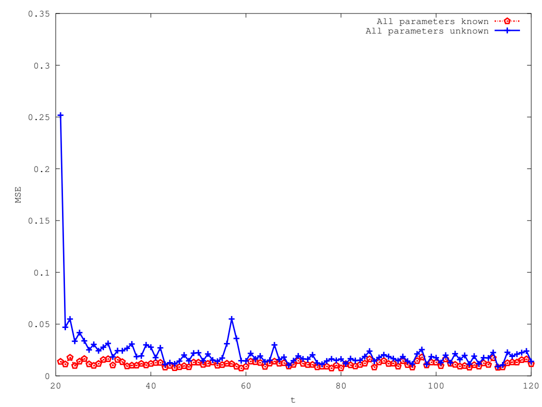

Fig. 4: MSE of $x_{3,t}$ with PF when all the static parameters are known and unknown, respectively.

5. CONCLUSIONS

In this paper, we implemented a Rao-Blackwellized particle filtering method with a vector stochastic volatility model. The intent is that our model captures not only the stochastic volatility in financial markets but also the correlation and beta between asset classes. All this has been demonstrated both from a theoretical and a practical perspective, as shown by the results provided in the previous section.

We showed how we can obtain the predictive distributions of the various hidden processes as well as the returns under rather general conditions. We were able to avoid generating particles of all the static parameters of the model, which allows for enhanced performance of the proposed approach. When applied to analyzing hedge fund returns, we showed how they can be decomposed into idiosyncratic returns and returns related to market factors.

There are many applications, including performance attribution, risk management, portfolio optimization, and replication strategies, that can benefit from this method. In particular, a large percentage of hedge fund returns can be easily replicated using available market instruments, and portfolios of hedge fund strategies can be optimized to constrain, or even eliminate, market risk in order to capture pure alpha related to non-correlated performance. In that way, hedge funds with large market exposures can be avoided and those with large idiosyncratic returns, relative to risk, emphasized.

6. REFERENCES

[1] V. Agarwal and N. Y. Naik, "Risks and Portfolio Decisions Involving Hedge Funds," Review of Financial Studies, vol. 17(1), pp. 63-98, 2004.

[2] O. Aguilar and M. West, "Bayesian dynamic factor models and portfolio allocation," *Journal of Business & Economic Statistics*, vol. 18(3), pp. 338-357, 2000.

[3] M. S. Arulampalam and S. Maskell and N. Gordon and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50(2), pp. 174-188, 2002.

[4] G. Casella and C. P. Robert, "Rao-Blackwellization of sampling schemes," *Biometrika*, vol. 84, pp. 81-94, 1996.

[5] P. M. Djurić and J. H. Kotecha and J. Zhang and Y. Huang and T. Ghirmai and M. F. Bugallo and J. Míguez, "Particle filtering," *IEEE Signal Processing Magazine*, vol. 20(5), pp. 19-38, 2003.

[6] P. M. Djurić and M. F. Bugallo, "Particle filtering," in *Advances in Adaptive Filtering*, Eds. S. Haykin and T. Adali, John Wiley & Sons, Inc.: New York, 2010.

[7] P. M. Djurić, M. Khan, and D. E. Johnston, "Particle filtering of stochastic volatility modeled with leverage," *Journal of Selected Topics in Signal Processing*, vol. 6(4), pp. 327-336, 2012.

[8] P. M. Djurić and M. F. Bugallo, "Particle filtering for multivariate state-space models," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, Asilomar, CA 2012.

[9] A. Doucet, N. de Freitas, and N. Gordon, Eds., *Sequential Monte Carlo Methods in Practice*, Springer, New York, 2001.

[10] W. Fung and D. A. Hsieh, "Hedge fund replication strategies: Implications for investors and regulators," *Financial Stability Review* vol. 10, pp. 55-66, 2007.

[11] S. Geisser, "Bayesian estimation in multivariate analysis," *The Annals of American Statistics*, vol. 36(1) pp. 150–159, 1965.

[12] D. E. Johnston and P. M. Djurić, "Estimating hedge fund risk factor exposures," *Proceedings of Workshop on Signal Processing Advances in Wireless Communications*, Çeşme, Turkey, 2012.

[13] A. Lo, *Hedge Funds: An Analytic Perspective*, Princeton University Press, Princeton, New Jersey, 2012.

[14] D. G. Luenberger, *Investment Science*, Oxford University Press, Oxford, UK, 1998.

[15] T. Roncalli and G. Weisang,"Tracking problems, hedge fund replication and alternative beta," Available at SSRN: http://ssrn.com/abstract=1325190, 2009.

[16] C. W. Wang, "Cloning hedge funds: A factor-based approach," JASSA, vol. 2010(1), pp. 22-29, 2010.