

ONLINE EM FOR INDOOR SIMULTANEOUS LOCALIZATION AND MAPPING

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ABSTRACT

This paper addresses the problem of mobile device localization in wireless sensor networks. The mobile is assumed to receive the signals transmitted by WiFi access points. The localization procedure is performed online (*i.e.* using the observations acquired by the mobile) and relies on the estimation of the propagation maps of the signal associated with each access point. This intermediate estimation step uses a new online Expectation Maximization based algorithm and Sequential Monte Carlo methods.

Index Terms— Simultaneous localization and mapping, Sequential Monte Carlo methods, WiFi signal.

1. INTRODUCTION

The simultaneous localization and mapping (SLAM) problem arises when a mobile (*e.g.* a robot or a human being equipped with sensors) evolves in an unknown environment and seeks to localize itself and to build a map of this environment. In this paper, the environment is assumed to be made of WiFi access points (AP) and the mobile localization is performed using the power of the signals transmitted by the AP. An important step to localize the mobile is to obtain accurate estimations of the propagation maps associated to each AP.

This localization problem in wireless sensor networks has been addressed using several methods to represent the propagation maps. In [1], they are modeled deterministically using some prior information about the environment (*e.g.* the position of walls). In [2, 3], a preliminary calibrating phase is performed: the power of signals transmitted by the AP is measured in previously determined positions in the environment. Using these accurate measurements, the power for each position in the map can be estimated using different techniques (see *e.g.* [4] for a method based on Gaussian processes). All these methods provide a propagation map for each AP based on a prior knowledge on the environment but do not allow any change in the representation of the WiFi signal. If the propagation is modified due to a change in the localization of the

obstacles, the estimated maps are not changed to take this into account. Therefore, the localization procedure is made using a wrong representation of the environment which leads to a degeneration of its performance.

In this paper, the maps are made of an average propagation model, based on a physical representation of wave propagation, and an additive term which corresponds to perturbations not taken into account in the average model. These propagation maps are estimated online, with the observations received by the mobile. Therefore, since any change in the environment impacts the observations, the estimation procedure is adapted accordingly. Simultaneously, the observations received by the mobile are used to estimate its localization. We present here a procedure based on the algorithm introduced in [5, 6], where the estimations are performed using Sequential Monte Carlo methods and a new online Expectation-Maximization (EM) technique.

The organization of the paper is as follows. The statistical model is outlined in Section 2. In Section 3, the online EM based estimation procedure is presented and numerical results are provided in Section 4.

2. MODEL

The mobile is assumed to evolve in a 2-dimensional finite grid $\mathcal{C}$ and its position $\{X_k\}_{k \geq 0}$ is a Markov chain with transition matrix given, for all $(x, x') \in \mathcal{C}^2$, by:

$$q_{x,x'} \propto e^{-\|x-x'\|^2/a}, \quad (1)$$

where a is a known constant and where $\|\cdot\|$ is the usual euclidean norm in $\mathbb{R}^2$. The associated inner product is denoted by $\langle \cdot, \cdot \rangle$. The initial distribution of $\{X_k\}_{k \geq 0}$ is the Dirac mass δ_{x_0} for an unknown $x_0 \in \mathcal{C}$. In the sequel, for any matrix $A \in \mathbb{R}^{B \times |\mathcal{C}|}$, we write A_j for the vector $\{A_{j,x}\}_{x \in \mathcal{C}}$ and A_j^2 for the vector $\{A_{j,x}^2\}_{x \in \mathcal{C}}$.

At each time step $k \geq 1$, the mobile receives the observation $Y_k \in \mathbb{R}^B$, where B is the number of AP in the environment. For any $k \geq 1$ and any $j \in \{1, \dots, B\}$, the observation associated to the j -th AP at time k , $Y_{k,j}$, is given by

$$Y_{k,j} = \mu_{j,X_k}^* + \delta_{j,X_k}^* + \varepsilon_{k,j}, \quad (2)$$

*This work is partially supported by ID Services, 22/24 rue Jean Rostand, 91400 Orsay, France.

where:

- $\mu_{j,x}^*$ is the average term and is modeled using the Friis transmission equation, see [7]: for any $j \in \{1, \dots, B\}$ and any $x \in \mathcal{C}$,

$$\mu_{j,x}^* = c_j^* + d_j^* \log \|x - O_j\|, \quad (3)$$

where O_j is the known position of the j -th AP and where c_j^* and d_j^* are real numbers.

- The additive term $\delta_{j,x}^*$ represents the perturbation not taken into account in the Friis equation. It is assumed that $\{\delta_j^*\}_{j=1}^B$ are embedded with the prior distribution π given, for any $\delta \in \mathbb{R}^{B \times |\mathcal{C}|}$, by

$$\pi(\delta) \propto \exp \left\{ -\frac{1}{2} \sum_{j=1}^B \delta_j^T \Sigma_j^{-1} \delta_j \right\}, \quad (4)$$

where the covariance matrices Σ_j are assumed to be known and where, for any matrix A , A^T denotes the transpose of A .

- $\{\varepsilon_k\}_{k \geq 0}$ is a sequence of i.i.d Gaussian random vectors, independent from $\{X_k\}_{k \geq 0}$, with mean 0 and covariance matrix $\Sigma = \sigma^{*,2} I_B$ (I_B is the identity matrix of size $B \times B$).

The localization of the mobile relies on the estimation of $\theta^* = (c^*, d^*, \delta^*, \sigma^{*,2})$, where $c^* = \{c_j^*\}_{j=1}^B$, $d^* = \{d_j^*\}_{j=1}^B$ and $\delta^* = \{\delta_j^*\}_{j=1}^B$. We denote by $g_\theta(x, Y_k)$ the probability density of the conditional distribution of Y_k given $X_k = x$ when the parameter value is θ .

3. ESTIMATION PROCEDURE

Let n be a positive integer. Define the maximum a posteriori estimator of θ^* based on the observations $Y_{1:n}$ as one maximizer of the function:

$$\theta \mapsto n^{-1} [\log L_\theta(Y_{1:n}) + \log \pi(\delta)], \quad (5)$$

where $L_\theta(Y_{1:n})$ is the likelihood of the observations (*i.e.* the joint probability density of $Y_{1:n}$) when the parameter value is θ . Since the function defined by (5) cannot be maximized explicitly, the estimation is made using an online EM based algorithm.

3.1. Batch EM algorithm

The EM algorithm (see [8]) is an iterative algorithm which produces parameter estimates $\{\theta_p\}_{p \geq 0}$ using a fixed set of observations $Y_{1:n}$. An EM based algorithm can be used to maximize (5). Each iteration of this algorithm is decomposed into two steps:

- 1) The E-step computes the conditional expectation

$$Q_{\theta_p}(Y_{1:n}; \theta) = \mathbb{E}_{\theta_p} \left[\frac{1}{n} \log p_\theta(X_{1:n}, Y_{1:n}) \middle| Y_{1:n} \right], \quad (6)$$

where $p_\theta(X_{1:n}, Y_{1:n})$ is the complete-data likelihood and where $\mathbb{E}_{\theta_p}[\cdot | Y_{1:n}]$ is the conditional expectation given $Y_{1:n}$ when the parameter value is θ_p .

- 2) The M-step defines the new value θ_{p+1} as one maximizer of $\theta \mapsto Q_{\theta_p}(Y_{1:n}; \theta) + n^{-1} \log \pi(\delta)$.

Define, for any $(x, y) \in \mathcal{C} \times \mathbb{R}^B$ and any $j \in \{1, \dots, B\}$,

$$s_1(x) = \{1_{x'}(x)\}_{x' \in \mathcal{C}}, \quad (7)$$

$$s_{2,j}(x, y) = \{1_{x'}(x) y_j\}_{x' \in \mathcal{C}}, \quad (8)$$

$$s_{3,j}(y) = y_j^2. \quad (9)$$

Using the model defined in Section 2, the intermediate quantity computed in the E-step can be written, up to an additive constant (since the parameter a is known), as:

$$\begin{aligned} Q_{\theta_p}(Y_{1:n}; \theta) &= \frac{1}{2\sigma^2} \sum_{j=1}^B \langle S_1^n(\theta_p, Y_{1:n}), F_j^2 \rangle - \frac{B}{2} \log \sigma^2 \\ &\quad - \frac{1}{2\sigma^2} \sum_{j=1}^B \{ S_{3,j}^n(\theta_p, Y_{1:n}) - 2 \langle S_{2,j}^n(\theta_p, Y_{1:n}), F_j \rangle \}, \end{aligned} \quad (10)$$

where, for any $j \in \{1, \dots, B\}$,

$$F_j = \mu_j + \delta_j \quad (11)$$

and

$$S_1^n(\theta_p, Y_{1:n}) = \frac{1}{n} \mathbb{E}_{\theta_p} \left[\sum_{k=1}^n s_1(X_k) \middle| Y_{1:n} \right], \quad (12)$$

$$S_{2,j}^n(\theta_p, Y_{1:n}) = \frac{1}{n} \mathbb{E}_{\theta_p} \left[\sum_{k=1}^n s_{2,j}(X_k, Y_k) \middle| Y_{1:n} \right], \quad (13)$$

$$S_{3,j}^n(\theta_p, Y_{1:n}) = \frac{1}{n} \sum_{k=1}^n s_{3,j}(Y_k). \quad (14)$$

The function $\theta \mapsto Q_{\theta_p}(Y_{1:n}; \theta) + n^{-1} \log \pi(\delta)$ can be maximized explicitly and we write

$$\theta_{p+1} = \bar{\theta}(S^n(\theta_p, Y_{1:n})), \quad (15)$$

where $S^n = (S_1^n, \{S_{2,j}^n\}_{j=1}^B, \{S_{3,j}^n\}_{j=1}^B)$. A detailed expression of $\bar{\theta}(S^n(\theta_p, Y_{1:n}))$ can be found in [9].

This algorithm provides a way to estimate θ^* with a fixed set of observations. Nevertheless, in our localization framework, we are interested in an online estimation procedure which does not store the data and produces new estimates each time new observations are available.

3.2. Block Online EM algorithm

Online EM algorithms have recently been proposed to solve maximum likelihood estimation in hidden Markov models, see [5, 6, 10, 11, 12, 13]. We use the Block Online EM (BOEM) algorithm introduced in [5] and further developed in [6]. The BOEM algorithm allows to update online the estimation of the propagation maps at previously defined time steps. Define, for any $p \geq 0$, $T_p = \sum_{i=1}^p \tau_i$ and $\mathbf{Y}_p = Y_{T_p+1:T_{p+1}}$, where $\{\tau_p\}_{p \geq 1}$ is a sequence of positive integers. For each block of observations $\mathbf{Y}_p$, the BOEM algorithm computes the sufficient statistics $S_1^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$, $S_{2,j}^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$ and $S_{3,j}^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$ using the value θ_p of the parameter and the observations $\mathbf{Y}_p$. The new estimate θ_{p+1} is computed at the end of the block $\mathbf{Y}_p$ by

$$\theta_{p+1} = \bar{\theta}(S^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)). \quad (16)$$

The sufficient statistics $S_1^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$, $S_{2,j}^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$ and $S_{3,j}^{\tau_{p+1}}(\theta_p, \mathbf{Y}_p)$ are all of the form (dropping the dependence on θ_p)

$$S(\mathbf{Y}_p) = \frac{1}{\tau_{p+1}} \mathbb{E} \left[\sum_{k=1}^{\tau_{p+1}} s(X_{T_p+k}, Y_{T_p+k}) \middle| \mathbf{Y}_p \right]. \quad (17)$$

Following [12, 14] this can be written

$$S(\mathbf{Y}_p) = \mathbb{E} \left[\rho_{\tau_{p+1}}^p(X_{T_{p+1}}) \middle| \mathbf{Y}_p \right], \quad (18)$$

where $\rho_0^p(x) = 0$ and for $t \geq 1$,

$$\rho_t^p(X_{T_p+t}) = \frac{1}{t} \mathbb{E} \left[\sum_{k=1}^t s(X_{T_p+k}, Y_{T_p+k}) \middle| \mathbf{Y}_p, X_{T_p+t} \right]. \quad (19)$$

This intermediate quantity ρ_t^p can be computed recursively by

$$\rho_t^p(x) = \frac{1}{t} s(x, Y_{T_p+t}) + \frac{t-1}{t} \int B_t^p(x, dx') \rho_{t-1}^p(x'), \quad (20)$$

where $B_t^p(x, dx')$ is the backward Markov transition kernel given by

$$B_t^p(x, dx') \propto q_{x',x} \phi_{t-1}^p(dx') \quad (21)$$

and where ϕ_t^p is the filtering distribution at time t on the block $\mathbf{Y}_p$ (i.e. the distribution of X_{T_p+t} given $Y_{T_p+1:T_p+t}$). In our context, (18) and (20) cannot be computed explicitly and are approximated using Sequential Monte Carlo methods. These methods produce weighted samples $\{(\xi_t^{p,i}, \omega_t^{p,i})\}_{i=1}^{N_{p+1}}$ for $0 \leq t \leq \tau_{p+1}$ combining sequential importance sampling and resampling steps (see e.g. [15, 16]). At each time step, the filtering distribution ϕ_t^p is approximated by $\hat{\phi}_t^p$ where

$$\hat{\phi}_t^p(dx) = \sum_{i=1}^{N_{p+1}} \omega_t^{p,i} \delta_{\xi_t^{p,i}}(dx). \quad (22)$$

Replacing ϕ_t^p in the recursion (18) and (20) by $\hat{\phi}_t^p$ gives the approximation $\rho_t^{p,i}$ of $\rho_t^p(\xi_t^{p,i})$:

$$\rho_t^{p,i} = \frac{1}{t} s(\xi_t^{p,i}, Y_{T_p+t}) + \frac{t-1}{t} \frac{\sum_{\ell=1}^{N_{p+1}} \omega_{t-1}^{p,\ell} q_{\xi_{t-1}^{p,\ell}, \xi_t^{p,i}} \rho_{t-1}^{p,\ell}}{\sum_{\ell=1}^{N_{p+1}} \omega_{t-1}^{p,\ell} q_{\xi_{t-1}^{p,\ell}, \xi_t^{p,i}}}. \quad (23)$$

Line 16 of Algorithm 1 performs this intermediate step for each statistic. Then, at the end of the block, the approximation $\hat{S}(\mathbf{Y}_p)$ of $S(\mathbf{Y}_p)$ is obtained by using (18):

$$\hat{S}(\mathbf{Y}_p) = \sum_{i=1}^{N_{p+1}} \omega_{\tau_{p+1}}^{p,i} \rho_{\tau_{p+1}}^{p,i}. \quad (24)$$

At each time step, the weighted samples are also used to estimate the mobile localization (line 13 of Algorithm 1).

Algorithm 1 SLAM indoor

Require: $\theta_0, \{\tau_k\}_{k \geq 1}, \{Y_t\}_{t \geq 0}$.

- 1: Sample $\{\xi_0^{0,i}\}_{i=1}^{N_1}$ independently and uniformly in $\mathcal{C}$.
 - 2: Set $\omega_0^{0,i} = N_1^{-1}$ for all $i \in \{1, \dots, N_1\}$.
 - 3: **for** all $p \geq 0$ **do**
 - 4: Set $\rho_0^{p,i} = 0$ for each statistic and $i \in \{1, \dots, N_{p+1}\}$.
 - 5: **for** $t = 1$ to τ_{p+1} **do**
 - 6: *Selection and propagation steps.*
 - 7: **for** $i = 1$ to N_{p+1} **do**
 - 8: Set $I = j$ with probability $\omega_{t-1}^{p,j}$.
 - 9: Set $\xi_t^{p,i} = x$ with probability $q_{\xi_{t-1}^{p,I}, x}$.
 - 10: Set $\omega_t^{p,i} \propto g_{\theta_{p-1}}(\xi_t^{p,i}, Y_{T_p+t})$.
 - 11: **end for**
 - 12: *Localization.*
 - 13: Set $\hat{i} = \underset{i \in \{1, \dots, N_{p+1}\}}{\text{Argmax}} \omega_t^{p,i}$ and $\hat{X}_{T_p+t} = \xi_t^{p,\hat{i}}$.
 - 14: *Forward computation of the intermediate quantity.*
 - 15: **for** $i = 1$ to N_{p+1} **do**
 - 16: Update $\rho_t^{p,i}$ for each statistic using (23).
 - 17: **end for**
 - 18: **end for**
 - 19: *Map estimation.*
 - 20: Update the estimation of each statistic using (24).
 - 21: Compute θ_{p+1} using $\bar{\theta}$.
 - 22: **for** $i = 1$ to N_{p+1} **do**
 - 23: Set $\omega_0^{p+1,i} = \omega_{\tau_{p+1}}^{p,i}$ and $\xi_0^{p+1,i} = \xi_{\tau_{p+1}}^{p,i}$.
 - 24: **end for**
 - 25: **end for**
-

4. NUMERICAL EXPERIMENTS

This section provides numerical experiments to illustrate the performance of the BOEM algorithm. The state-space is given by $\mathcal{C} = \{0, \dots, 25\} \times \{0, \dots, 25\}$ and $B = 12$ AP are displayed in the environment. For all $j \in \{1, \dots, B\}$,

set $c_j^* = -26$, $d_j^* = -17.5$ and $\Sigma_j(x, x') = v_1 \cdot \exp(-|x - x'|^2/v_2)$ with $v_1 = 10$ and $v_2 = 18$. Finally, we set $\sigma^{*,2} = 25$ and $a = 6$.

The initial estimates are $\delta_0 = 0$, $\sigma_0^2 = 30$ and, for all $j \in \{1, \dots, B\}$, $c_{0,j} = -10$ and $d_{0,j} = -30$.

The number of particles on the block p is given by $N_p = 25 + p$ and the block sizes are given by $\tau_p = 25p + 100$.

On each block p , the estimation error is set as the mean normalized L_1 error:

$$\epsilon_p = \frac{1}{B|C|} \sum_{j=1}^B \sum_{x \in C} |F_{j,x}^p - F_{j,x}^*|, \quad (25)$$

where F_j^p is the estimated map of the j -th AP on the current block, see (11). On each block, the localization error is set as the empirical 0.8-quantile of the distance between the true localizations and the estimated positions. Figure 1 displays the error ϵ and Figure 2 the localization error as functions of the number of blocks over 50 independent Monte Carlo runs. Both errors decrease as the number of blocks increases. In order to assess the performance of the algorithm, we also give in Figure 2 the optimal localization error. This error is obtained by applying Algorithm 1 using the true values for each map (*i.e.* without performing the estimation of the maps from line 14 to 21 and using $F_{j,x}^p = F_{j,x}^*$ on each block). After 60 blocks (about 50.000 observations), the localization error almost reaches the optimal performance.

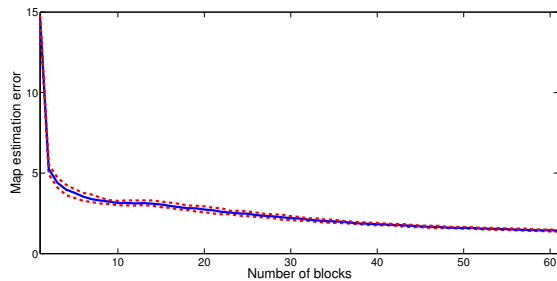


Fig. 1. Map estimation error. Median (bold line) and lower and upper quartiles (dashed lines) over 50 independent Monte Carlo runs for the error ϵ .

Finally, Figure 3 provides the estimation of σ^* as a function of the number of blocks which almost converges to the true value after about 60 blocks.

5. RELATION TO PRIOR WORK

The work presented in this paper provides a new algorithm to perform online simultaneous localization and mapping using WiFi signals. Contrary to previous techniques which use previously calibrated maps (see [2, 3]) or deterministically built maps (see [1]), this algorithm estimates online the propagation maps. Moreover, we provide numerical experiments

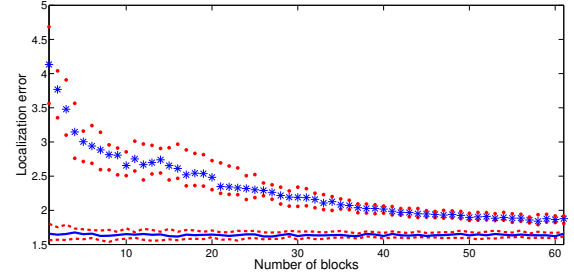


Fig. 2. Localization error. Median (stars) and lower and upper quartiles (balls) over 50 independent Monte Carlo runs. Median (bold line) and lower and upper quartiles (dashed lines) are also displayed for the optimal localization (with known maps).

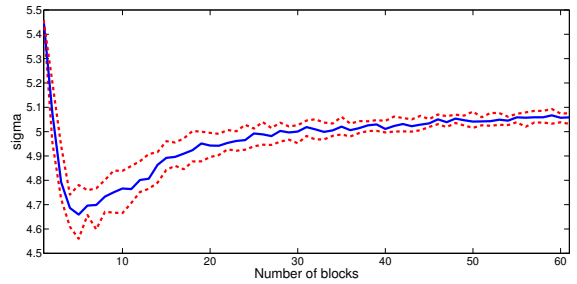


Fig. 3. Estimation of σ^* . Median (bold line) and lower and upper quartiles (dashed line) over 50 independent Monte Carlo runs. The true value is $\sigma^* = 5$.

showing that the stabilization procedure introduced in [9] is not necessary with smaller maps containing few AP.

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