TW-TOA BASED COOPERATIVE SENSOR NETWORK LOCALIZATION WITH UNKNOWN TURN-AROUND TIME

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ABSTRACT

This work aims to estimate multiple node positions in the presence of unknown turn-around times within the context of cooperative sensor network localization. In the adopted scheme, each target can communicate with a set of anchors (probably not in sufficient numbers) and a set of other targets. Two-Way Times-of-Arrival between them are measured, which includes unknown processing delays at both channel endpoints. Since finding the Maximum Likelihood Estimates (MLE) of the positions and turn-around times given those measurements poses a difficult nonconvex optimization problem, it is approximated by a Nonlinear Least Squares problem. Then, the positions and turn-around times of multiple targets are estimated jointly by solving an Euclidean Distance Matrix completion problem. Simulations show that the localization accuracy of the proposed method is good, providing an initial point that subsequently enables MLE to attain the Cramér-Rao Lower Bound for all considered scenarios.

Index Terms— Cooperative sensor network localization, turn-around time, two-way time-of-arrival, Euclidean distance matrix, semidefinite programming.

1. INTRODUCTION

Many wireless sensor network (WSN) applications require the involved sensors to be accurately localized [1]. Among the noisy measurements upon which localization could be based, Times-of-Arrival (TOA) provide a good tradeoff between the accuracy and implementation cost [1, 2]. In this work, the Two-Way Time-of-Arrival (TW-TOA) protocol is adopted, where the anchor node sends the ranging request and the target node responds back. The time of flight of the signal is proportional to the distance between target-anchor if there is no delay in the response time of the target, or if that delay, called turn-around time, is correctly included in the reply packet. However, in practical scenarios, calibration of nodes to determine the turn-around time is undesirable. In addition, the responding target might deceive the anchor by reporting a wrong turn-around time. Therefore, localization algorithms should tackle this issue.

In noncooperative sensor networks, target nodes can communicate only with anchor nodes [3]. The lack of accessible anchor nodes and also limited connectivity among anchor nodes and target nodes lead to the emergence of the cooperative localization paradigm, in which target nodes are able to communicate with both anchor nodes and other target nodes. Therefore, not only are TOAs between target nodes and anchor nodes measured, but also the target nodes themselves are involved and collect TOA measurements from each other.

A closed-form Least Squares (LS) estimator is derived to localize a single target in asynchronous networks, in which clock offset and skew are unknown [4]. The authors propose an asymmetric trip ranging (ATR) protocol, where anchors are not only able to communicate with the target, but also listen to the other anchor-target communications. An asynchronous position measurement system is proposed in [5] and an LS based method is solved for indoor localization of a single target by using the differential TOA. A generalized total LS algorithm is developed for the joint synchronization and localization of an unknown node in [6]. The authors consider hierarchical hop-by-hop time synchronization and localization where only one node needs to be localized and synchronized to the anchors at a time. Recently, an LS based approach using hybrid TW-TOA and TDOA in Cooperative Networks is proposed for the joint estimation of unknown turn-around times and node locations [7]. The authors did not consider TW-TOA measurements between targets to improve the accuracy. All mentioned methods either estimate a single target position or estimate each target position at a time by a linearization-based LS. Therefore, the main contribution of the current work is an accurate Semidefinite Programming (SDP) method which localizes multiple targets simultaneously in the presence of unknown turn-around time in a cooperative network.

To find the Maximum Likelihood Estimator (MLE) for

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the sensor network localization problem with unknown turn-around times, it is necessary to solve a nonlinear and non-convex optimization problem. To avoid this difficulty, the original MLE is transformed into an approximate Nonlinear Least Squares (NLS) problem using squared range measurements. Then, relaxation techniques are applied to convert the NLS problem into a convex optimization problem by resorting to Euclidean Distance Matrix (EDM) completion (a type of SDP). Through this, the target turn-around times are considered as nuisance parameters and estimated jointly with the target locations. The advantage of an SDP is that its cost function does not have local minima and thus convergence to the global minimum is guaranteed [8]. The drawback is that the SDP technique is sub-optimal and cannot achieve the best possible performance under all conditions.

The remainder of the paper is organized as follows. Section 2 formalizes the problem and shows the EDM completion as an SDP method. Simulations and computational complexity analysis are given in Section 3. Conclusions are drawn in Section 4.

2. PROBLEM FORMULATION

This section formulates the cooperative localization problem using TW-TOA measurements, where the target locations and turn-around times are unknown. Two sets of TW-TOA measurements are available to the estimator: target-anchor and target-target measurements. Let $\mathbf{s}_j \in \mathbb{R}^l$, $j \in \mathcal{S} = \{1, \ldots, N\}$ and $\mathbf{a}_i \in \mathbb{R}^l$, $i \in \mathcal{A} = \{N+1, \ldots, N+M\}$ denote N target and M anchor locations, respectively. The following two sets are defined as

$$\mathcal{B}_j = \{\ i \mid \text{anchor } i \text{ can communicate with target } j\ \},$$
$$\mathcal{C}_j = \{\ i \mid \text{target } i \text{ can communicate with target } j\ \}.$$

The cooperative TW-TOA measurement (converted to distance) [1], when the i-th node interrogates the j-th node, is expressed as

$$\hat{d}_{ij} = T_j + 2d_{ij} + n_{ij},\ \ j \in \mathcal{S},\ i \in \mathcal{B}_j \cup \mathcal{C}_j \qquad (1)$$

where T_j is the turn-around time of the j-th target (converted to distance), $d_{ij} = \|\mathbf{s}_i - \mathbf{s}_j\|$, $i \in \mathcal{C}_j$ and $d_{ij} = \|\mathbf{a}_i - \mathbf{s}_j\|$, $i \in \mathcal{B}_j$. In addition, n_{ij} are modeled as independent and identically distributed (i.i.d.) zero mean Gaussian random variables with standard deviation σ_{ij} [1]. Consequently, there are in total $l \times N + N$ unknown elements that should be estimated, including the target locations and the turn-around times defined as $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_N] \in \mathbb{R}^{l \times N}$ and $\mathbf{T} = [T_1, \ldots, T_N]^T \in \mathbb{R}^N$, respectively.

In practical scenarios, we can assume that a target sends a ranging request to another target, which responds back. When the communication initiator gets the response, it then sends a final packet as if it had been interrogated. In this way, with three communications, two TW-TOA measurements are obtained. Whenever an anchor sends a ranging request to a target, that target not only responds back to the ranging message but also sends its TW-TOA measurements obtained with respect to the other targets with which it has communicated. As a result, all TW-TOA measurements are conveyed to the central node via anchors.

2.1. EDM Formulation

By moving T_j to the left hand side (LHS) of the equation and squaring both sides, (1) can be reformulated as

$$\hat{d}_{ij}^2 - 2\hat{d}_{ij}T_j + T_j^2 = 4d_{ij}^2 + 4d_{ij}n_{ij} + n_{ij}^2. \qquad (2)$$

For a sufficiently small noise, we can neglect n_{ij}^2 in the right-hand side of (2) and write

$$\hat{d}_{ij}^2 - 2\hat{d}_{ij}T_j + T_j^2 = 4d_{ij}^2 + \epsilon_{ij}, \qquad (3)$$

where $\epsilon_{ij} = 4d_{ij}n_{ij}$ is a zero-mean Gaussian noise with standard deviation $4d_{ij}\sigma_{ij}$. The NLS formulation that matches predicted (d_{ij}, T_j) vs. observed $\hat{d}_{ij}$ ranges is

$$\underset{\mathbf{S},\mathbf{T}}{\text{minimize}} \quad \sum_{j\in\mathcal{S}} \sum_{i\in\mathcal{B}_j\cup\mathcal{C}_j} (\hat{d}_{ij}^2 - 2\hat{d}_{ij}T_j + T_j^2 - 4d_{ij}^2)^2. \qquad (4)$$

The unknown squared distances can be arranged into a single symmetric EDM matrix of size $(N+M) \times (N+M)$, with elements $E_{ij} = d_{ij}^2$, and satisfying the properties of the EDM cone $\mathcal{E}$ [8, 9]

$$E_{ii} = 0, \qquad E_{ij} \geq 0, \qquad -\mathbf{JEJ} \succeq 0, \qquad (5)$$

where $\mathbf{J} = \left(\mathbf{I}_\rho - \frac{1}{\rho}\mathbf{1}_\rho\mathbf{1}_\rho{}^T\right)$, $\rho = N + M$, is a centering operator which subtracts the mean of a vector from each of its components and $\mathbf{I}_\rho$ is $\rho \times \rho$ identity matrix.

Introducing a vector epigraph variable $\mathbf{K} = [K_1, \ldots, K_N]^T$ yields the following relaxed EDM problem:

$$\begin{aligned} \underset{\mathbf{E},\mathbf{T},\mathbf{K}}{\text{minimize}} \quad & \sum_{j\in\mathcal{S}} \sum_{i\in\mathcal{B}_j\cup\mathcal{C}_j} (\hat{d}_{ij}^2 - 2\hat{d}_{ij}T_j + K_j - 4E_{ij})^2 \\ \text{subject to} \quad & \mathbf{E} \in \mathcal{E},\ \mathbf{E}(\mathcal{A}) = \mathbf{A} \\ & K_j \geq T_j^2\ ,\ T_j \geq 0,\ \text{ and } \beta_j \geq K_j. \end{aligned} \qquad (6)$$

The constraint $\mathbf{E}(\mathcal{A}) = \mathbf{A}$ enforces the known *a priori* spatial information related with anchors in the appropriate EDM submatrix. In (6) the desired nonlinear equality constraint $K_j = T_j^2$ is relaxed to an inequality to obtain a convex optimization problem. However, the relaxation can cause K_j to become arbitrarily large, which is undesirable because estimated locations become arbitrarily far apart. To mitigate this difficulty, large K_j values might be penalized by adding a regularization term to the objective [8, 10]. However, this is sensitive to the penalization term [11]. Another way is to upper bound K_j by a constant β_j, which might be chosen with respect to prior knowledge of system specifications, i.e., knowledge of the maximum possible value of turn-around time or according to the method proposed in Section 2.2.

Note that the solution of (6) is a distance matrix $\mathbf{E}$. Detailed explanations of how to estimate the spatial coordinates of the targets from EDM and the usage of anchors are given in [12]. The basic idea is to use a linear transformation to obtain the Gram matrix $(\mathbf{ZJ})^T\mathbf{ZJ} = -\frac{1}{2}\mathbf{JEJ}$, from which spatial coordinates $\mathbf{Z} = [\mathbf{s}_1, \ldots, \mathbf{s}_N, \mathbf{a}_{N+1}, \ldots, \mathbf{a}_{M+N}]$ are extracted by the singular value decomposition up to a unitary matrix. The anchors are then used to estimate the residual unitary matrix by solving a Procrustes problem [12].

2.2. Estimate of β_j

To provide a reasonably good upper bound, β_j, for the variable K_j, (3) can be approximated by dropping the noise term when $i \in \mathcal{B}_j$ as

$$\hat{d}_{ij}^2 - 2\hat{d}_{ij}T_j + T_j^2 = 4\|\mathbf{a}_i\|^2 - 8\mathbf{a}_i^T\mathbf{s}_j + 4\|\mathbf{s}_j\|^2, \qquad (7)$$

and rearranging terms as

$$\hat{d}_{ij}^2 - 4\|\mathbf{a}_i\|^2 = -8\mathbf{a}_i^T\mathbf{s}_j + 4\|\mathbf{s}_j\|^2 + 2\hat{d}_{ij}T_j - T_j^2, \qquad (8)$$

which can be written as $b_{ij} = \mathbf{H}_{ij}\mathbf{y}_j$, where $b_{ij} = \hat{d}_{ij}^2 - 4\|\mathbf{a}_i\|^2$, $\mathbf{H}_{ij} = [1, \; -8\mathbf{a}_i^T, \; 2\hat{d}_{ij}]$ and $\mathbf{y}_j = [4\|\mathbf{s}_j\|^2 - T_j^2, \; \mathbf{s}_j, \; T_j]^T$. We form the vector $\mathbf{b}_j$ and matrix $\mathbf{H}_j$ from b_{ij} and $\mathbf{H}_{ij}$, $i \in \mathcal{B}_j$, such that $\mathbf{b}_j = \mathbf{H}_j\mathbf{y}_j$. If $\mathbf{H}_j$ has full column rank, coarse estimates of $\mathbf{s}_j$ and T_j are obtained from the LS solution

$$\hat{\mathbf{y}}_j = (\mathbf{H}_j^T\mathbf{H}_j)^{-1}\mathbf{H}_j^T\mathbf{b}_j. \qquad (9)$$

Through simulations it was observed that the accuracy of position estimation is better than the turn-around time estimation with this method. Therefore, estimated turn-around times are calculated as $\tilde{T}_j = (\sum_{i\in\mathcal{B}_j}(\hat{d}_{ij} - 2\tilde{d}_{ij}))/|\mathcal{B}_j|$, where $\tilde{d}_{ij}$ is the estimate of d_{ij} from the estimated target position $\hat{\mathbf{s}}_j$, and $|\mathcal{B}_j|$ is the cardinality of $\mathcal{B}_j$. The upper bound in (6) is set as $\beta_j = \tilde{T}_j^2$. Note that to solve (9), at least 4 (2D) or 5 (3D) anchors are needed. The estimator first estimates the locations and turn-around times of targets that are connected to a sufficient number of anchors, and it uses the estimated position of the neighboring targets as virtual anchors for the remaining ones. When these conditions are not satisfied, the estimator simply assigns a constant to β_j based on prior knowledge of maximum turn-around times.

Note that, ideally, we would like β_j to be large enough (but no larger) so that the true values of $K_j = T_j^2$ for a given network setup are included in the feasible set of (6). Even though the method above for setting β_j does not really guarantee that, simulation results show that it is a good heuristic.

3. SIMULATIONS

In this section, computer simulations are performed to evaluate the performance of the proposed algorithm which will be called "EDM" in the figures. The comparison metric is the total root mean-square error (RMSE) defined as

$$\text{RMSE} = \sqrt{\frac{1}{L}\frac{1}{N}\sum_{k=1}^{L}\sum_{i=1}^{N}\|\mathbf{s}_i - \hat{\mathbf{s}}_i^k\|^2}, \qquad (10)$$

where $\hat{\mathbf{s}}_i^k$ denotes the i-th estimated target position in the k-th Monte Carlo run ($L = 1000$) for the specific noise realization. To assess the fundamental hardness of the position estimation, error plots also show the average Cramér-Rao Lower Bound (CRLB) with known ("CRLB-Known-T") and unknown turn-around times ("CRLB") for each noise variance. The derivation of the CRLB is not given due to the space constraints, but it follows the same reasoning as in [2].

To compare the proposed algorithm with MLE, Matlab's function *lsqnonlin* is initialized with the output of the proposed method and with random initialization, denoted below as EDM-MLE and RAND-MLE, respectively. Additionally, results for EDM localization with true turn-around time ("EDM-Known-T") will be provided. In every realization of the network, the turn-around time is randomly drawn from $[1, 100]$ ns and the measurement noise assumed i.i.d. Gaussian, with $\sigma_{ij} = \sigma \in [0.01, 18]$ m.

Experiment 1: A fully connected (all anchors and targets are within communication range) randomly distributed network in $[-80 \; 80]$ m $\times$ $[-80 \; 80]$ m consisting of 6 targets and 8 anchors is generated at each Monte Carlo run for each noise level. Fig. 1 shows the RMSE of different approaches. The accuracy of the proposed method is good and the degradation in performance due to unknown turn-around time is small when compared to EDM-Known-T. Additionally, EDM-MLE attains the CRLB. The RAND-MLE is the worst one.

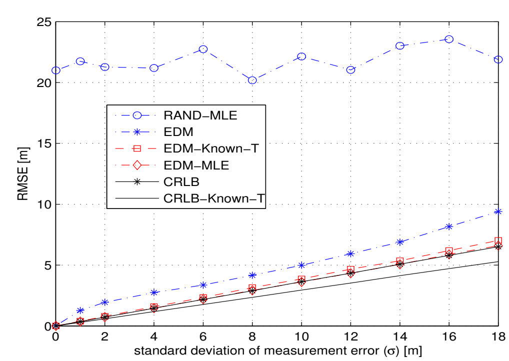


Fig. 1. RMSE comparisons in a fully connected randomly distributed network.

Experiment 2: The behavior of the algorithms is examined for a *structured network*, in which $\mathbf{a}_i \in \{[\pm 50, \pm 50]^T,$

$[0, \pm 70]^T$, $[\pm 70, 0]^T\}$ m and $\mathbf{s}_i \in \{[\pm 20, 40]^T$, $[0, \pm 40]^T$, $[0, 0]^T, [20, 40]^T\}$ m, i.e., when all 6 targets are in the convex hull of 8 anchors and they are fully connected. As shown in Fig. 2, the accuracy of EDM is good and EDM-MLE attains the CRLB. However, for this scenario RAND-MLE also achieves the CRLB because the cost function appears to have a unique minimum. The derivation of an alternative MLE and the effect of the initialization techniques on the number of the MLE iterations will be the subject of future work.

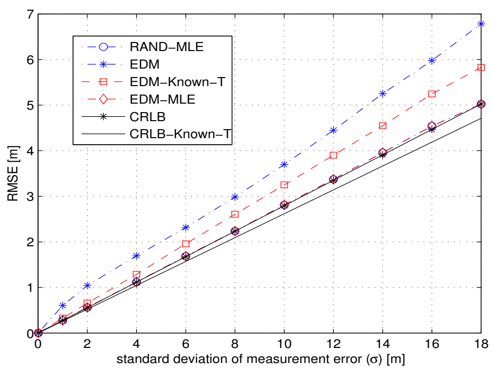
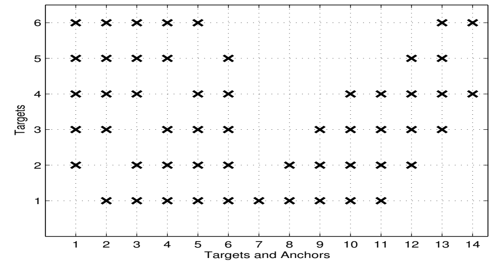


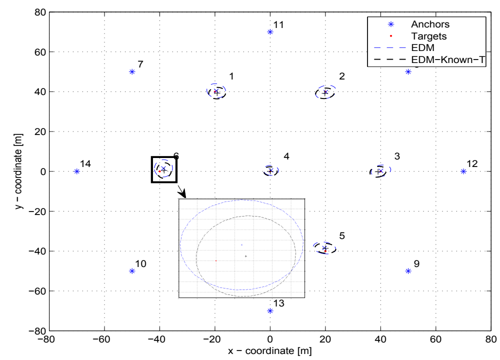
Fig. 2. RMSE comparisons in a fully connected *structured network*.

Experiment 3: The sample mean and the uncertainty ellipsoids of EDM and EDM-Known-T are given in Fig. 3(b) when $\sigma = 10$ m for the *structured network*. The connectivity matrix for the network is shown in Fig. 3(a), where the 5th and 6th targets are only connected to two anchors and all others communicate with five anchors. Although two anchors are not enough for the 5th and 6th targets to be localized in 2D, all positions are eventually determined with good accuracy through cooperation, as the remaining targets are within range of a sufficient number of anchors. Fig. 3(c) shows the RMSE comparisons for this network. With the limited connectivity to anchors the localization problem becomes harder, similarly to what is known to occur even with full connectivity when some of the targets lie outside the convex hull of the set of anchors. This is seen, e.g., in the significant degradation of RAND-MLE for strong observation noise levels ($\sigma > 10$ m). However, EDM still provides good accuracy. Moreover, EDM-MLE attains the CRLB.

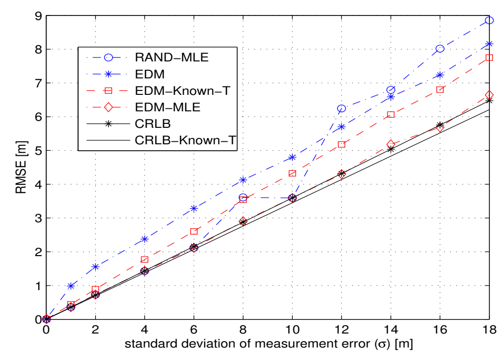
A Note on Practical Computational Complexity: The worst case computational complexity of SDP based algorithms for sensor network localization is bounded by $O((N+M)^6)$ [13]. For the proposed algorithm, CPU time empirically increases with $(N+M)^{4.5}$. The experiments were conducted on a laptop with Intel Core i5-2430M 2.4 GHz CPU and 4 GB of RAM, using MATLAB 7.11, CVX 1.22 and SeDuMi as a general purpose SDP solver. The CPU time to solve the proposed method is about 0.5 seconds for this network.



(a) The connectivity matrix.



(b) The sample mean and uncertainty ellipsoids.



(c) The RMSE comparisons.

Fig. 3. RMSE comparisons, sample mean and uncertainty ellipsoids of localization when the 5th and 6th targets of a *structured network* are connected to only two anchors.

4. CONCLUSION

EDM completion, a type of SDP technique with reasonable computational cost, is proposed to localize multiple targets when target turn-around times are not known under the TW-TOA protocol. It is shown that cooperation among targets provides accurate localization even if some targets are connected to few anchors. Additionally, when the proposed method is used as an initialization of MLE, the latter attains the CRLB.

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