NON-BAYESIAN QUICKEST DETECTION WITH A STOCHASTIC ENERGY CONSTRAINT

Jun Geng and Lifeng Lai

Department of Electrical and Computer Engineering
Worcester Polytechnic Institute, Worcester, MA, 01605
Email:{jgeng,llai}@wpi.edu

ABSTRACT

Motivated by applications of wireless sensors powered by energy harvested from the environment, we study non-Bayesian quickest change detection problems with a stochastic energy constraint. In particular, a wireless sensor powered by renewable energy is deployed to detect the change of probability density function in a random sequence. The energy in the sensor is consumed by taking observation and is replenished randomly. The sensor cannot take observations if there is no energy left. Our goal is to design power allocation scheme and detection strategy to minimize the delay between the time the change occurs and an alarm is raised. Two types of average run length (ARL) constraint, namely an algorithm level ARL and a system level ARL, are considered. We show that a low complexity scheme, in which the sensor takes observations as long as the battery is not empty coupled with the Cumulative Sum (CUSUM) test for detection, is optimal for the setup with the algorithm level ARL constraint, and is asymptotically optimal for the setup with the system level ARL constraint.

***Index Terms*—** CUSUM test, energy harvested sensor, non-Bayesian quickest detection, sequential detection.

1. INTRODUCTION

Recently, the study of sensor networks powered by renewable energy harvested from the environment has attracted significant attention [1, 2, 3, 4, 5]. Compared with sensor networks powered by batteries, sensor networks powered by renewable energy have several unique features such as unlimited life span and high dependence on the environment, etc. Optimal power management schemes for each individual sensor and scheduling protocols for the whole network have been developed to maximize utility functions of *communication related metrics* such as channel capacity, transmission delay or network throughput. However, the researches on *signal processing related performance metrics* for renewable energy powered sensors have not been investigated. Detection delay, which is one of such performance metrics, is important for sensor networks in many applications. For example, if a sensor network is deployed to monitor the health of a bridge, then the detection delay between the time when a structural problem occurs and the time when an alarm is raised is of interest.

In this paper, we focus on non-Bayesian quickest detection problem, which was first studied by G. Lordon [6] and M. Pollak [7]. Since no prior information about the change point is required, this non-Bayesian setup is very attractive for practical applications. In the classic setups, there is no energy constraint and the sensor can take observations at every time slot. In this paper, we extend Lorden's and Pollak's problems to sensors that are powered by renewable energy. In this case, the energy stored in sensor is replenished by a random process and consumed by taking observations. The sensor cannot take observations if there is no energy left. Hence, the sensor cannot take observation at every time instant anymore. The sensor needs to plan its use of power carefully. Moreover, the stochastic nature of the energy replenishing process will certainly affect the performance of change detection schemes. Since the energy collected by the harvester in each time instant is not a constant but a random variable, this brings new optimization challenges.

There have been some existing works on quickest detection problem taking the sample cost into consideration. [8] considers the design of detection strategy that strikes a balance between the detection delay, false alarm probability and the number of sensors being active. Based on the observations from sensors at each time slot, the fusion center decides how many sensors should be active in the next time slot to save energy. [9] studies the Bayesian quickest detection taking the average number of observations into consideration, and the authors propose the low complexity DE-Shiryeav algorithm which is asymptotically optimal as false alarm probability goes to zero. In [10, 11], the authors propose the DE-CUSUM extending the previous result into non-Bayesian problem formulation. [12] discusses the Bayesian quickest detection problem with a constraint that the sensor could take only a finite number of observations.

The remainder of the paper is organized as follows. The mathematical model is given in Section 2. Section 3 presents the solutions for binary energy arriving model. Section 4, presents asymptotically optimal solutions for general energy arriving model. Numerical examples are given in Section 5. Section 6 offers concluding remarks. Due to space limitations, we present only main ideas of proofs. Details of proofs can be found in [13].

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2. PROBLEM FORMULATION

Let $\{X_k, k = 1, 2, \ldots\}$ be a sequence of random variables whose distribution changes at a fixed but unknown time t. Before t, the $\{X_k\}$'s are independent and identically distributed (i.i.d.) with probability density function (pdf) f_0; after t, they are i.i.d. with pdf f_1. The pre-change pdf f_0 and post-change pdf f_1 are perfectly known by the sensor. We use P_t and $\mathbb{E}_t$ to denote the probability measure and the expectation with the change happening at t, respectively, and use P_∞ and $\mathbb{E}_\infty$ to denote the case $t = \infty$.

For the energy harvested wireless sensor, its energy arrives randomly both in time and in amount. Specifically, we denote $\nu = \{\nu_1, \nu_2, \ldots, \nu_k, \ldots\}$ as the energy arriving process with $\nu_k \in \mathcal{V} = \{0, 1, 2, \ldots\}$, while $\{\nu_k = 0\}$ means that no energy is collected in time slot k, and $\{\nu_k = i\}$ means i units of energy is collected at time k. If we use P^ν to denote its probability measure (correspondingly, $\mathbb{E}^\nu$ is denoted the expectation under P^ν), the pmf of ν_k can be expressed as $p_i = P^\nu(\nu_k = i)$. $\{\nu_k\}$ is i.i.d. over k.

The sensor can decide how to allocate the collected energy. Let $\mu = \{\mu_1, \mu_2, \ldots, \mu_k, \ldots\}$ be the power allocation strategy, where $\mu_k \in \{0, 1\}$. $\mu_k = 1$ means that the wireless sensor spends a unit of energy on taking an observation at time slot k, while $\mu_k = 0$ means that no energy is spent at k and hence no observation is taken.

The energy harvested wireless sensor has a finite battery with capacity C. Denote E_k as the energy left in the battery in time slot k, it would be affected by the energy arriving process and the energy utilizing process:

$$E_k = \min[C, E_{k-1} + \nu_k - \mu_k].$$

The energy allocation policy μ must obey the causality constraint, namely the energy cannot be used before it is harvested. The energy causality constraint can be written as

$$E_k \geq 0 \quad k = 1, 2, \ldots. \tag{1}$$

We use $\mathcal{U}$ to denote the set of μ that satisfy (1).

The sensor spends energy to take observations. The observation sequence is denoted as $\{Z_k, k = 1, 2, \ldots\}$, where

$$Z_k = \begin{cases} X_k & \text{if } \mu_k = 1 \\ \phi & \text{if } \mu_k = 0 \end{cases}. \tag{2}$$

We call an observation Z_k a non-trivial observation if $\mu_k = 1$, i.e, if the observation is taken from the environment. Denote $\left\{\tilde{X}_k\right\}$ as the non-trivial observation sequence, which is the subsequence of $\{Z_k\}$ with all its non-trivial observations.

$\{Z_k\}$ are not necessarily conditionally (conditioned on the change point) i.i.d. due to the existence of μ_k. The distribution of Z_k is related to both μ_k and X_k. Therefore, we use P_t^μ and $\mathbb{E}_t^\mu$ to denote the probability measure and expectation of the observation sequence $\{Z_k\}$ with change happening at t, respectively.

In this paper, we want to find a stopping time T, at which the sensor will declare that a change has occurred, and a power allocation μ that jointly minimize the detection delay. Specifically, we consider three problem setups. The first one is Lorden's quickest detection problem [6] with an *algorithm level* ARL constraint, which is formulated as

$$\text{(P1)} \quad \min_{\mu \in \mathcal{U}, T \in \mathcal{T}} d(\mu, T), \quad \text{s.t. } \mathbb{E}_\infty[N] \geq \eta, \tag{3}$$

where $\mathcal{T}$ is the set of all stopping time with $\mathbb{E}_t^\mu[T] < \infty$, N is the total number of non-trivial observations taken by the sensor before it claims that the change has happened and

$$d(\mu, T) = \sup_{t \geq 1} d_t(\mu, T),$$
$$d_t(\mu, T) = \operatorname{esssup} \mathbb{E}_t^\mu \left[(T - t + 1)^+ | \mathcal{F}_{t-1}\right], \tag{4}$$

where $\mathcal{F}_k = \sigma\{Z_1, \cdots, Z_k\}$. In this formulation, we put a lower bound η on the average number of observations taken before a false alarm is raised. The larger η is, the less frequently a false alarm will be raised. Since this constraint is independent of the power utilizing process μ and energy arriving process ν, this problem setup is more robust against the variation of the ambient environment.

The second problem considered in this paper is Lorden's quickest detection problem with a *system level* ARL constraint, which is formulated as

$$\text{(P2)} \quad \min_{\mu \in \mathcal{U}, T \in \mathcal{T}} d(\mu, T), \quad \text{s.t. } \mathbb{E}_\infty^\mu[T] \geq \gamma. \tag{5}$$

In this formulation, a lower bound γ is set on the expected duration to a false alarm. In contrast to the previous case, this constraint depends on the power allocation μ, which is further related to the energy arriving process ν. Therefore, this setup is more sensitive to the environment.

In some applications, Pollak's formulation [7] is of interest since its delay metric is less conservative than that of Lorden's formulation. In our context, the Pollak's formulation can be written as

$$\text{(P3)} \quad \min_{\mu \in \mathcal{U}, T \in \mathcal{T}} \sup_{t \geq 1} \mathbb{E}_t^\mu [T - t | T \geq t], \quad \text{s.t. } \mathbb{E}_\infty^\mu[T] \geq \gamma. \tag{6}$$

Since the optimal solution for Pollak's formulation is still open [14], we discuss only the asymptotic solution for this formulation in this paper.

3. BINARY ENERGY ARRIVING MODEL

In this section, we consider a relative simple case: the binary energy arriving model. Specifically, we assume $\nu \in \mathcal{V} = \{0, 1\}$, that is, the energy harvester can collect one unit energy at most in one time slot. We denote $p = P^\nu(\nu_k = 1)$. The conclusion in this section will provide valuable insights for the more general model considered in Section 4.

Throughout this paper, we use $L(\cdot)$ to denote the likelihood ratio (LR), and use $l(\cdot) = \log L(\cdot)$ to denote the log likelihood ratio (LLR). For the observation sequence $\{Z_k\}$, LR is defined as

$$L(Z_k) = \begin{cases} \frac{f_1(Z_k)}{f_0(Z_k)}, & \text{if } \mu_k = 1 \\ 1, & \text{if } \mu_k = 0 \end{cases}. \tag{7}$$

The CUSUM statistic and the Page's stopping time can be written as [6]

$$S_k = \max_{1\le q\le k}\left[\prod_{i=q}^{k} L(Z_i)\right] = \max[S_{k-1}, 1]L(Z_k),$$

and

$$T_p = \inf\{k \ge 0 | S_k \ge B\},$$

respectively.

3.1. Optimal solution for (P1)

In this subsection, we show the power allocation $\mu^* = \nu$ and CUSUM strategy is optimal for (P1). An important proposition of this proposed strategy is as follows:

Proposition 3.1. *The power allocation scheme $\mu^* = \nu$ and Page's stopping time T_p together achieve an equalizer rule, i.e., $d_t(\mu^*, T_p) = d_1(\mu^*, T_p), \forall t \ge 1$.*

Lemma 3.2. *The optimal power allocation strategy for (P1) is $\mu^* = \nu$, and the optimal stopping time is T_p, in which the threshold B is a constant such that $\mathbb{E}_\infty[N] = \eta$.*

Proof. Outline: Our proof have two steps. In the first step, we show that T_p is optimal for any given power allocation μ. In the second step, we show $\mu^* = \nu$ is optimal under T_p. □

Remark 3.3. *$\mu^* = \nu$ indicates $\mu_k^* = \nu_k$ for every k, that is, the sensor spends the energy taking observation immediately when it obtains one from the environment. Therefore, we term μ^* as* immediate power allocation scheme *in the following.*

The following proposition shows the performance of (μ^*, T_p) in terms of the detection delay and the algorithm level ARL.

Proposition 3.4. *Suppose $B > 1$, then*

$$\mathbb{E}_\infty[N] = \frac{\mathbb{E}_\infty[\kappa]}{1 - P_\infty(F_0)}, \tag{8}$$

$$d(\mu^*, T_p) = \frac{1}{p}\frac{\mathbb{E}_1[\kappa]}{1 - P_1(F_0)}, \tag{9}$$

where κ is the stopping time

$$\kappa = \min\left\{m \ge 1 \left| \sum_{k=1}^{m} l\left(\tilde{X}_k\right) \notin (0, \log B)\right.\right\},$$

and F_0 denotes the event

$$\left\{\sum_{k=1}^{\kappa} l\left(\tilde{X}_k\right) \le 0\right\}.$$

3.2. Asymptotical optimality for (P2) and (P3)

It is generally difficult to solve (P2) and (P3) since both the detection delay and the system level ARL are related to the power allocation μ. In the following, we show that the proposed scheme (μ^*, T_p) is asymptotically optimal for (P2) and (P3) as $\gamma \to \infty$.

We first provide a lower bound on the the detection delay for any scheme.

Lemma 3.5. *As $\gamma \to \infty$,*

$$\begin{aligned} &\inf\{d(\mu, T) : \mathbb{E}_\infty^\mu[T] \ge \gamma\} \\ &\quad\ge \inf\left\{\sup_{t\ge 1}\mathbb{E}_t^\mu[T - t | T \ge t] : \mathbb{E}_\infty^\mu[T] \ge \gamma\right\} \\ &\quad\ge \frac{1}{p}\frac{|\log\gamma|}{I}(1 + o(1)), \end{aligned} \tag{10}$$

where I is the KL-divergence of f_1 and f_0.

The following lemmas show that this lower bound in Lemma 3.5 can be obtained by (μ^*, T_p) for both (P2) and (P3).

Lemma 3.6. *(μ^*, T_p) is asymptotically optimal for (P2) and (P3) as $\gamma \to \infty$. Specifically,*

$$d(\mu^*, T_p) \sim \frac{1}{p}\frac{|\log\gamma|}{I}. \tag{11}$$

$$\sup_{t\ge 1}\mathbb{E}_t^{\mu^*}[T_p - t | T_p \ge t] \sim \frac{1}{p}\frac{|\log\gamma|}{I}. \tag{12}$$

Proof. Outline: Since (μ^*, T_p) is an equilizer rule for (P2), we have

$$d(\mu^*, T_p) = d_1(\mu^*, T_p) = \mathbb{E}_1^{\mu^*}[T_p] \le \mathbb{E}_1^{\mu^*}[T_{s,1}],$$

where $T_{s,1}$ is the stopping time of one-sided SPRT:

$$T_{s,1} = \inf\left\{k \ge 1 \left| \prod_{i=1}^{k} L(Z_i) \ge B\right.\right\}.$$

For (P3), similar to the discussion in Theorem 6.16 in [15], we have

$$\mathbb{E}_t^{\mu^*}[T_p - t | T_p \ge t] \le \mathbb{E}_1^{\mu^*}[T_{s,1}].$$

Then, the statement follows Proposition 4.11 in [15], by which we have $\mathbb{E}_1^{\mu^*}[T_{s,1}] \sim \frac{1}{p}\frac{|\log\gamma|}{I}$. □

4. GENERAL ENERGY ARRIVING MODEL

For the general case that $\nu_k \in \mathcal{V} = \{0, 1, 2, \ldots\}$ with $p_i = P^\nu(\nu_k = i)$, we propose to use a generalized immediate power allocation strategy:

$$\tilde{\mu}_k^* = \begin{cases} 1 & \text{if } E_{k-1} + \nu_k \ge 1 \\ 0 & \text{if } E_{k-1} + \nu_k = 0 \end{cases},$$

that is, the sensor takes observation as long as the battery is not empty. We notice that $\tilde{\mu}^*$ *degenerates to the immediate power allocation* under binary energy arriving model. We show that the $\tilde{\mu}^*$ coupled with T_p is asymptotically optimal for (P2) and (P3).

Similar to the case in Section 3, we first have the following lower bound on the detection delay for any scheme.

Lemma 4.1. *As* $\gamma \to \infty$*,*

$$\begin{aligned} &\inf\{d(\mu, T) : \mathbb{E}^{\mu}_{\infty}[T] \geq \gamma\} \\ &\quad \geq \quad \inf\left\{\sup_{t\geq 1} \mathbb{E}^{\mu}_{t}[T - t | T \geq t] : \mathbb{E}^{\mu}_{\infty}[T] \geq \gamma\right\} \\ &\quad \geq \quad \frac{1}{\tilde{p}} \frac{|\log\gamma|}{I}(1 + o(1)), \end{aligned} \tag{13}$$

where $\tilde{p} \doteq \mathbb{E}^{\nu}[\tilde{\mu}^*]$

The proposed scheme achieves the above mentioned lower bounds for both (P2) and (P3).

Lemma 4.2. $(\tilde{\mu}^*, T_p)$ *is asymptotically optimal for (P2) and (P3) as* $\gamma \to \infty$*. Specifically,*

$$d(\tilde{\mu}^*, T_p) \sim \frac{1}{\tilde{p}} \frac{|\log\gamma|}{I}, \tag{14}$$

$$\sup_{t\geq 1} \mathbb{E}^{\tilde{\mu}^*}_{t}[T_p - t | T_p \geq t] \sim \frac{1}{\tilde{p}} \frac{|\log\gamma|}{I}. \tag{15}$$

5. NUMERICAL SIMULATION

In this section, we give two numerical examples to illustrate the results obtained in our paper. In these numerical examples, we assume that the pre-change distribution f_0 is $\mathcal{N}(0, \sigma^2)$ and the post-change distribution f_1 is $\mathcal{N}(0, P+\sigma^2)$. The signal-to-noise ratio is defined as $SNR = 10 \log P/\sigma^2$.

In the first scenario, we illustrate the relationship between the detection delay and $\mathbb{E}_{\infty}[N]$ under setup (P1) for binary energy arriving model. The simulation result for $SNR = 0dB$ is shown in Figure 1. In this figure, the blue line with circles is the simulation result for $p = 0.2$, the green line with stars and the red line with squares are the results for $p = 0.5$ and $p = 0.8$, respectively. The black dash line is the performance of the classic Lorden's problem, which is served as a lower bound because the sensor can take observation at every time slot. As we can see, for a given η, the detection delay is in inverse proportion to the energy arriving probability p. The larger p is, the closer is the performance to the lower bound.

In the second scenario, we examine the asymptotic optimality of $(\tilde{\mu}^*, T_p)$ for (P2) and (P3) under general energy arriving model. In the simulation, we assume the battery size $C = 3$, and $\mathcal{V} = \{0, 1, \ldots, 4\}$ with energy arriving pmf $p_0 = 0.8$, $p_1 = 0.1$, $p_2 = 0.05$, $p_3 = 0.025$, $p_2 = 0.025$. In this case, we can find $\tilde{p} = \mathbb{E}^{\nu}[\tilde{\mu}^*] = 0.9964$. We set $SNR = 5dB$. The simulation result is shown in Figure 2. In this figure the blue line with circles is the performance of (P2). The red line with squares is the Performance of (P3),

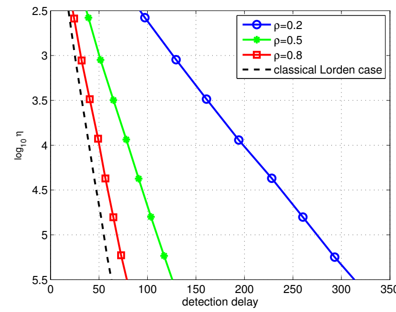

Fig. 1. Detection delay v.s. the algorithm level ARL

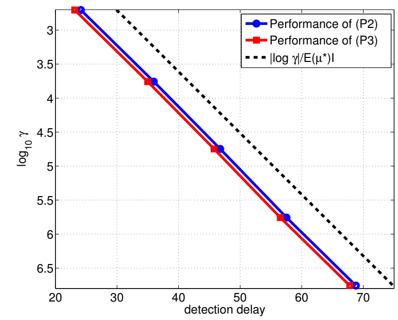

Fig. 2. Detection delay v.s. the system level ARL

and the black dash is calculated by $|\log\gamma|/\tilde{p}I$. Along all the scales, the Pollak's detection delay is smaller than Lorden's detection delay, and these three curves are parallel to each other, which confirms the proposed strategy, $(\tilde{\mu}^*, T_p)$, is asymptotically optimal because the difference between them is negligible as $\gamma \to \infty$.

6. CONCLUSION

In this paper, we have studied the non-Bayesian quickest detection problem with a casual energy constraint. Three non-Bayesian quickest detection problem setups, namely Lorden's problem under the algorithm level ARL, Lorden's problem under the system level ARL and Pollak's problem under the system level ARL, have been considered. For the binary energy arriving model, we have shown that the immediate power allocation coupled with Page's stopping time is optimal for the first problem, and is asymptotically optimal for the second and the third problems. For the multi-energy arriving model, we have shown that the generalized power allocation along with Page's stopping time is asymptotically optimal for the second and the third problems.

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