A NEW METHODOLOGY FOR OPTIMAL DELAY DETECTION IN MOBILE LOCALIZATION CONTEXT

A. Boudjellal[1] *K. Abed-Meraim*[1] *A. Belouchrani*[2] *Ph. Ravier*[1]

[1] Polytech'Orleans, Prisme Laboratory, France; [2] Ecole Nationale Polytechnique, Algiers, Algeria.

ABSTRACT

In this paper, we address the problem of delay detection in mobile localization context. A new methodology for delay detection is introduced, namely the *Cell-Averaging Minimum Error Rate CA-MER* detector, based on the minimization of the true error probability instead of minimizing only the miss probability for a constant false alarm rate. Simulation results show that the CA-MER detector operates better than the classical ones especially for low SNR values.

1. INTRODUCTION

Mobile Station (MS) positioning is one of the most important issues in mobile communications systems [7].

Existing solutions use directional measurements such as the Angle of Arrival (AoA), or distance-related measurements such as the Time of Arrival (ToA), Time Difference of Arrival (TDoA), and Received Signal Strength (RSS).

For each MS-BS (Base Station) link, under the assumption of *Line of Sight* (LoS) path, the considered ToA can be obtained as the position of the first peak (finger) of the communications channel coefficients. In practice, the latter are classically obtained by carrying out correlations of the received signal with delayed versions of a pilot sequence introduced within the transmitted signal for this purpose [2].

Finding the position of the first finger of the communications channel imposes the use of robust detection techniques, since the accuracy of ToA-based mobile localization methods is directly based on the quality of communications channels peak detection. The latter requires the use of optimal threshold to get rid of the noise peaks and to retain only peaks corresponding to effective channel paths. Usually, this thresholding is done in an ad-hoc way which may affect the performance of this mobile positioning.

In [4] and [5], a constant ad-hoc threshold value equal to a percentage of the main peak value has been considered. This ad-hoc thresholding finds difficulties to distinguish false noise peaks from the true desired ones; a high threshold might hide the first path if the latter is of relatively low power and a low threshold would lead to false peak detection especially when the background noise has an important power.

As a solution to this problem, Bartelmaos et al. [2] propose to use a sub-optimal detector whose inputs are incoherently integrated outputs of a square law detector in order to optimally detect the first arriving path. These authors consider a threshold value that minimizes the sum of false alarm and miss probabilities.

In reference [1], a CA-CFAR (Cell Average - Constant False Alarm Rate) detector is used for detecting the first arriving path by using a threshold value ensuring a constant false alarm probability. In fact, in mobile localization, false alarm and non-detection events are equally harmful and as a result, both introduce a bias on the mobile location estimation. For this reason, giving more importance to a false alarm event than a non-detection event seems to not take advantage from the context in which the detection process is applied.

Herein, we introduce a new methodology for the detection of the first finger of the communications channel. We propose to use a threshold which is optimal in the sense of minimizing the error probability as a cost function. The proposed solution is evaluated over extensive simulations on different scenarios.

2. STOCHASTIC CHANNEL MODELLING

In mobile communications, the *Channel Impulse Response* (*CHIR*) can be modelled as a noisy version of a time-varying linear filter [11]:

$$h(t) = \left\{ \sum_{r=1}^{R} \alpha_r(t)\delta(t-\tau_r(t)) \right\} + w(t) \qquad (1)$$

where $\tau_r(t)$ and $\alpha_r(t)$ are the time-varying delay and the time-varying complex amplitude of the r-th path, respectively, and $w(t)$ is a zero-mean Gaussian complex-valued noise with variance σ_w^2. The equivalent discrete version of the baseband channel model is given by:

$$h(k) = \left\{ \sum_{r=1}^{R} \alpha_r(k)\delta(k-\tau_r(k)) \right\} + w(k) \qquad (2)$$

where k is the sampled version of t.

In the context of fading mobile communications, it is usual to assume that paths' complex amplitudes remain constant over a slot (burst) duration. The latters are modelled as i.i.d Gaussian complex-valued processes. Time delays are usually assumed constant over several slots (L slots)

[14], hence, $\tau_r(k) = \tau_r$. Taking into account these two assumptions, the shaping filter effect, and assuming that *CHIR* vanishes outside an interval $[0, M-1]$, where M represents the channel length, the *CHIR* over the slot l will be [6]:

$$h_l(k) = \left\{ \sum_{r=1}^{R} \alpha_{r,l}\, g(k - \tau_r) \right\} + w_l(k), k = 0 : M - 1 \quad (3)$$

where $g(.)$ denotes the shaping filter. Note that under the above assumptions $h_l(k)$ is a noisy Rayleigh channel.

2.1. Time of Arrival Estimation

In a downlink mobile communications scenario, the ToA of the signal of interest is the corresponding delay τ_1 of the first channel peak $\alpha_{1,l}$. Herein we define a *Power Delay Profile* (PDP)-like function, over L slots, and we estimate the ToA as the first peak location of this function:

$$P(k) = \sum_{l=1}^{L} |h_l(k)|^2, k = 0 : M - 1 \quad (4)$$

Our objective now is to detect the first peak of the PDP-like function $P(k)$ and estimate its corresponding delay τ_1. However, peak detection requires the use of a threshold γ to avoid noise peak detection and to detect only peaks corresponding to effective channel paths.

3. EXISTING THRESHOLDING METHODS

3.1. Constant Ad-Hoc Thresholding

ToA estimation accuracy is based on peak detection results. Using an ad-hoc thresholding may affect the performance of the detection methods. Indeed, a high threshold might hide the first path if the latter is of relatively low power and a low threshold would lead to false peak detection.

In [4] and [5], a constant ad-hoc threshold value equals to a percentage α of the main peak value has been considered. An ad-hoc thresholding finds difficulties to distinguish false noise peaks from the true desired ones, especially when the background noise has a large variance.

Assuming that the main peak corresponds to the first path, the ad-hoc threshold is then given as $T_{adhoc} = \alpha\ \sigma_1^2$. In this case, the false alarm and the detection probabilities [1] for incoherently integrated signals, are both function of the signal-to-noise ratio ($S = \sigma_1^2/\sigma_w^2$):

$$P_{fa} = e^{\left(\frac{-\alpha L S}{2}\right)} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{\alpha L S}{2}\right)^l \quad (5)$$

$$P_d = e^{\left(\frac{-\alpha L S}{2(1+S)}\right)} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{\alpha L S}{2(1+S)}\right)^l \quad (6)$$

[1]We have calculated these probabilities expression by using the technique shown in subsection 4.1. Due to space limitation, all proofs are omitted.

3.2. Bartelmaos Thresholding

Bartelmaos et al. [2] propose a detector which minimizes the sum of false alarm and miss probabilities. Its inputs are incoherently integrated outputs of a square law detector over L slots. For simplicity, this detector replaces the true expression of the probabilities by the ones corresponding to the non-integrated case (i.e. $L = 1$) given by:

$$P_{fa} = \left(1 + \frac{T}{2m}\right)^{-2m} \quad (7)$$

$$P_d = \left(1 + \frac{T}{2m(1+S)}\right)^{-2m} \quad (8)$$

where $2m$ represents the number of reference cells used to estimate the background noise variance and T is the threshold value. The optimal threshold given in [2] is (we set $\beta = 1/(1+S)$):

$$T_{bar} = \underbrace{argmin}_{T}(P_{fa} + 1 - P_d) = 2m \left(\frac{1 - \beta^{\frac{1}{2m+1}}}{\beta^{\frac{1}{2m+1}} - \beta}\right) \quad (9)$$

3.3. CA-CFAR Thresholding

In reference [1] a CA-CFAR detector is considered. A threshold value is used ensuring a constant false alarm probability (i.e. $P_{fa}(T_{CFAR}) = 10^{-6}$). Since P_{fa} is SNR independent (see eq.(7)), the threshold selection of this method is quite simple. However , in mobile localization, false alarm and non-detection events are equally harmful and as a result, both introduce a bias on the mobile location estimation. For this reason, giving more importance to a false alarm than a non-detection event is inappropriate in this case.

Next, we propose an alternative approach where both error types are equally taken into account in such a way the detection error probability is minimized.

4. CELL-AVERAGING MINIMUM ERROR RATE CA-MER DETECTOR

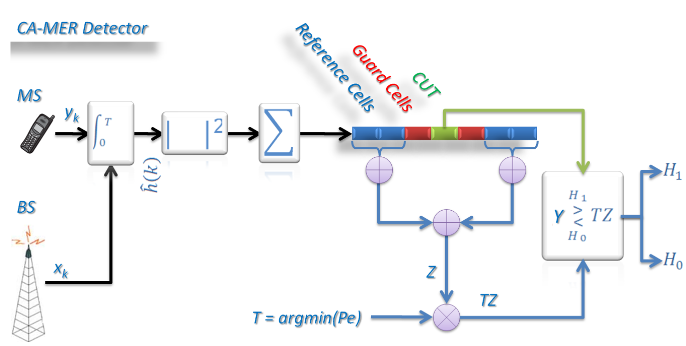


Fig. 1. CA-MER Optimal Thresholding.

4.1. Stochastic Modelling and Error Probability

Channel fading coefficients $\alpha_{r,l}$ are slot-varying Gaussian variables of variance σ_r^2 and their corresponding delays τ_r are constant over an observation period of L slots. Under these assumptions, the PDP-like samples $P(k)$ are χ_2 distributed random variables with $2L$ degrees of freedom.

A total of $2m$ noise samples are used to estimate the background environment noise variance level Z. The output of $2m$ *Reference Cells* (sum of m leading cells and m lagging cells) is multiplied by a constant T to obtain the threshold value.

In our case, each reference cell corresponds to one sample of the PDP-like function, thus, $Z = \sum_{k=1}^{2m} P(k)$ is χ_2 distributed random variables with $4mL$ degrees of freedom. In view of this, the PDF function describing the random variable $\gamma = TZ$, in the noise alone case, is given by [12]:

$$f_\gamma(\gamma) = \frac{\gamma^{2mL-1}\, e^{-\gamma/2T\sigma_w^2}}{(2T\sigma_w^2)^{2mL}\,\Gamma(2mL)}; \gamma > 0 \qquad (10)$$

Under the channel model assumptions of section 2, the peak model at the *Cell Under Test* (CUT) is a slowly fluctuating signal of Swerling 1 type. Since both the noise and Rayleigh fading coefficients have Gaussian quadrature components, the output of the square-law detector has an exponential probability density function and the CUT output Y, after the non-coherent integration stage, is χ_2 distributed random variable with $2L$ degrees of freedom. The conditional density function of the output of the CUT is given by:

$$\begin{cases} f_{Y|H_0}(y|H_0) = \frac{y^{L-1}\, e^{-y/2\sigma_w^2}}{(2\sigma_w^2)^L\,\Gamma(L)}; y > 0 \\ f_{Y|H_1}(y|H_1) = \frac{y^{L-1}\, e^{-y/2\sigma_w^2(1+S)}}{(2\sigma_w^2(1+S))^L\,\Gamma(L)}; y > 0 \end{cases} \qquad (11)$$

Hypothesis H_0 represents the case of noise alone, while hypothesis H_1 represents the noise plus fading signal case. Under these assumptions, the probability of false alarm is independent of the noise variance and is given by [3] and [12]:

$$P_{fa} = \frac{1}{(1+T)^{2mL}} \sum_{l=0}^{L-1} \frac{1}{l!} \frac{\Gamma(2mL+l)}{\Gamma(2mL)} \left(\frac{T}{1+T}\right)^l \qquad (12)$$

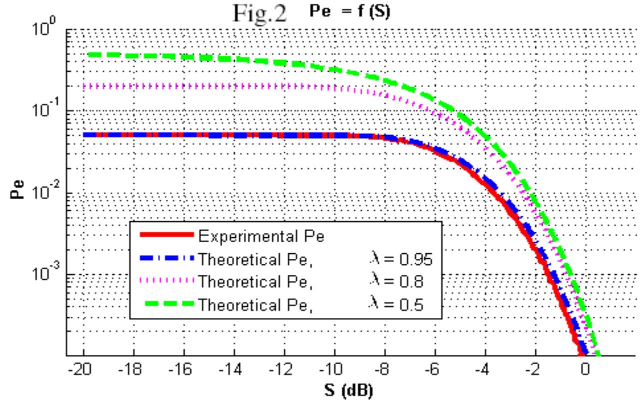
The probability of detection is:

$$P_d = \frac{1}{(1+V)^{2mL}} \sum_{l=0}^{L-1} \frac{1}{l!} \frac{\Gamma(2mL+l)}{\Gamma(2mL)} \left(\frac{V}{1+V}\right)^l \qquad (13)$$

with $V = \frac{T}{1+S}$, and $\Gamma(.)$ is the gamma function. Note that equations (7) and (8) are particular cases of equations (12) and (13) for non-integrated signals (i.e. $L = 1$).

4.2. CA-MER Optimal Thresholding

For finding the optimal threshold, we develop a new methodology that takes advantage from the context of mobile localization in which a false alarm and a non-detection events are equally harmful. Also, it takes into account the hypothesis of presence or absence of a peak value, and the fact that reference cell's input are resulting from a non-coherent integration over L slots. In the view of the previous discussion, the proposed cost function is the error probability, given by:



$$P_e = P(H_0)P_{fa} + P(H_1)\overline{P_d} = \lambda P_{fa} + (1-\lambda)(1-P_d) \qquad (14)$$

where $P(H_0) = \lambda$ and $P(H_1) = 1-\lambda$ represent prior probabilities of absence and presence of a peak value, respectively.

The optimal scaling parameter in this case is given by:

$$T_{CA-MER} = \underbrace{argmin}_{T}(P_e) \qquad (15)$$

The solution of this equation $T_{CA-MER} = f_{L,m,\lambda}(S)$ is a parametric function of the SNR S which depends of three parameters; the integration factor L, the reference cell number m, and the peak probability λ. In order to automate the processing of the proposed method, the SNR should be estimated based on the observed data[2].

5. NUMERICAL RESULTS

In order to validate the proposed CA-MER thresholding method and compare it to the classical methods, a set of numerical experiments is considered. A Rayleigh fading channel modelling a fading suburban outdoor macrocell environment is generated. According to [8], it is a 'B channel' corresponding to a Pedestrian mobile with speed of 3 km/h. Channel B contains six fingers and is the median spread delay case that occurs frequently. For simplicity, we consider only the first path with complex amplitude and time-delay characteristics as given in [8]. This generated fading channel is then corrupted by background Gaussian noise of variance σ_w^2.

The first simulation aims at validating the experimental results w.r.t. the theoretical ones and getting better clarification of their behaviour versus the signal-to-noise ratio S. In Fig.2, we made a comparison between an experimental error

[2]This is the focus of a future work on this topic.

Fig.3 Pe = f (S), L= 1

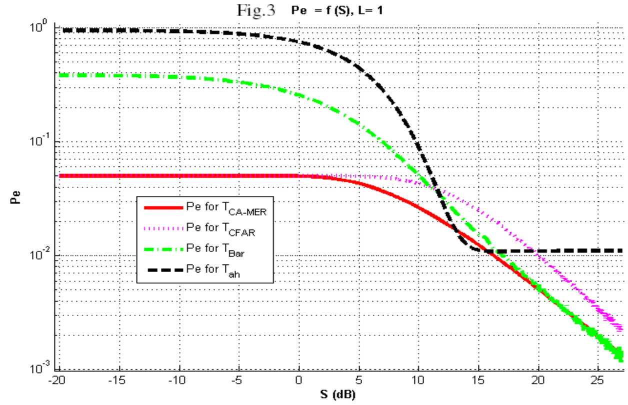


Fig.4 Pe = f (S)

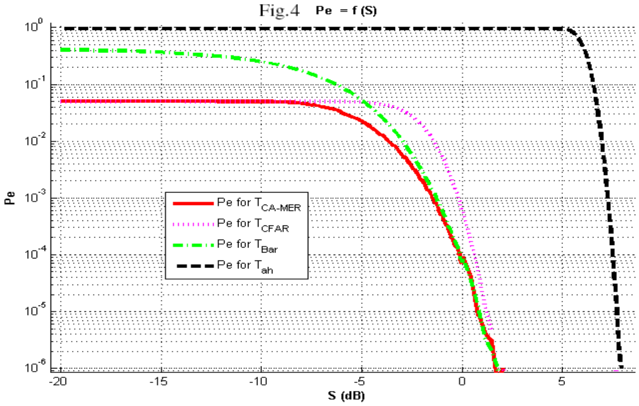
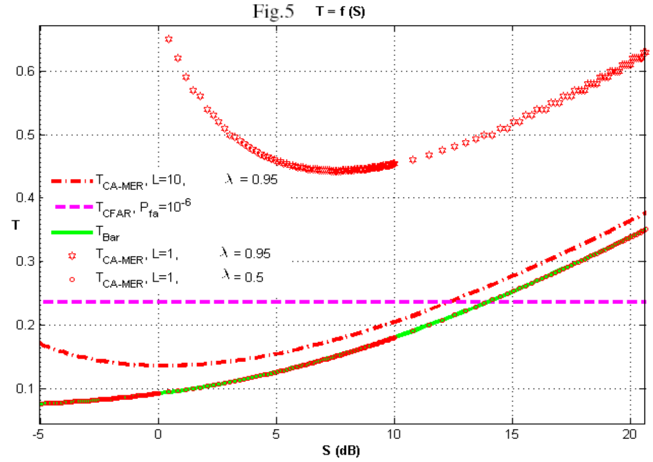


Fig.5 T = f (S)



probability P_e^{exp} and the theoretical one P_e^{th} as a function of the parameter λ. As we can see, the theoretical error probability fits well with the experimental one for the parameter [3] λ that represents the true probability of absence of a peak value.

Fig.3 and Fig.4 show the behaviour of the four aforementioned thresholding techniques versus SNR S for 15 reference cells and $L = 1$ (no integration) or $L = 105$ slots. As we can see in Fig.3, for $S = 15dB$ the error probability of the CA-MER equals $\simeq 0.015$ while it equals $\simeq 0.03$ (the double) for the CA-CFAR and it equals $\simeq 0.1$ (six times higher) when using Bartelmaos thresholding. The same conclusions can be reached in Fig.4 with two additional remarks, first, the error probability decreases quickly for high values of the integration parameter L, and secondly, for low S values the CA-MER is similar to CA-CFAR detector while for high S values it is similar to the Bartelmaos thresholding method.

For the last carried out simulation, the set of curves presented in Fig.5 depicts the behaviour of the optimal T_{CA-MER} versus S for $L = 1$ and $L = 10$. For $L = 1$ and $\lambda = 0.5$ both CA-MER and Bartelmaos methods give the same optimal threshold, thus that latter is a specific case of our proposed method. This figure shows clearly that the optimal threshold is function of L, λ, and S.

[3] In practice, the probability of absence of a peak value λ is approximated by the ratio of the peaks' number in the analyzing window to its total number of cells.

6. CONCLUSION

In this paper, a new cell averaging thresholding technique based on minimizing the error probability has been introduced. The proposed method takes into account the hypothesis of presence or absence of a peak value, and the fact that reference cell's input are resulting from a non-coherent integration process. Through a set of simulation results, the developed theoretical derivation has been validated. The presented results clearly demonstrate the efficiency of the proposed method, i.e. the CA-MER, as compared to the classical ones especially for low SNR values. The CA-MER can be used for any detection problem in which a false alarm and a non-detection events have the same cost.

7. REFERENCES

[1] A. Boudjellal, A. Belouchrani, and K. Abed-Meraim, "A New ToAs' CACFAR Wiener Rake Estimator for Downlink Mobile Positioning in UMTS-FDD System", *in proceedings WOSSPA*, pp. 343-347, 2011.

[2] S. Bartelmaos, K. Abed-Meraim and F. Soltani, "An Efficient Rake-CFAR Method for Downlink Mobile Positioning in UMTS-FDD mode", *in proceedings SPAWC*, pp. 1-5, 2007.

[3] Xiu-Ying Hou, N. Morinaga, T. Namekawa, "Direct Evaluation of Radar Detection Probabilities", *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-23, no 4, pp. 418-424, 1987.

[4] E. Grosicki, K. Abed-Meraim, P. Loubaton and J. Chaufray, "Comparison of Downlink Mobile Positioning Methods for the UMTS-FDD Mode without Using IPDL Periods", *in Proceedings ISSPA*, vol. 2, pp. 347-350, 2003.

[5] E. Grosicki and K. Abed-Meraim, "A New Trilateration Method to Mitigate the Impact of NLOS Er-

rors in TOA for Mobile Localization", *in Proceedings ICASSP*, vol. 4, pp. 1045-1048, 2005.

[6] J-M. Chaufray, Ph. Loubaton, and P. Chevalier, "Consistent estimation of Rayleigh fading channel in the context of the wideband CDMA mode of the UMTS", *IEEE Transaction on Signal Processing*, vol. 49, no. 12, pp. 3055-3064, Dec. 2001.

[7] A. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location", *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 24-40, July 2005.

[8] 3GPP TR 101.112, "Selection procedures for the choice of radio transmission technologies of the UMTS", v 3.2.0, Avril, 1998.

[9] TS 25.213, "Spreading and Modulation (FDD)", 3GPP Technical Specication, v 3.2.0, March 2000.

[10] J. Caffery and S. Venkatraman, *Signal Processing for Mobile Communications Handbook, Chp. 21 Geolocation Techniques for Mobile Radio Systems*, CRC PRESS, 2005.

[11] I. C. Libertil and T. S. Rappaport, *Smart Antenna for Wireless Communications, Chp. 6 Propagation Channel Characteristics*, Prentice Hall, 1999.

[12] R. Bassem and P. Mahafza, *Radar Systems Analysis and Design Using MatLab*, Chapman and Hall/CRC, 2000.

[13] M. Barkat, *Signal Detection and Estimation*, Artech House, INC., 2005.

[14] B. Sklar, *Mobile Communications Handbook, Chp. 18 Rayleigh Fading Channels*, CRC Press LLC 1999.