RANDOM DISTORTION TESTING AND APPLICATIONS

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ABSTRACT

We address Random Distortion Testing (RDT), that is, the problem of testing whether the Mahalanobis distance between a random signal Θ and a known deterministic model θ_0 exceeds some given $\tau \geqslant 0$ or not, when Θ has unknown probability distribution and is observed in additive independent Gaussian noise with positive definite covariance matrix. A suitable optimality criterion for RDT is presented and theoretical results on optimal tests for this criterion are given. Several applications of these results are presented and analyzed. They address the detection of signals in case of model mismatch and the detection of deviations from model θ_0.

***Index Terms*—** Event testing, hypothesis testing, invariance, Mahalanobis norm, random distortion testing, test with maximal constant conditional power.

1. INTRODUCTION

In many signal processing applications, the observation captured by a sensor is assumed to be a d-dimensional real random vector $Y = \Theta + X$, where Θ is some random distortion from a known deterministic model θ_0 and X is noise independent of Θ. Small deviations of Θ from its model θ_0 can be of poor interest for the user, who may want to detect big enough ones only. It is thus relevant to test whether Θ lies in a neighborhood of θ_0 or not. If Θ is supposed to be deterministic, the Wald and Rao tests can cope with distortions in a neighborhood of θ_0 under the conditions of [1, p. 478, Theorem VIII] or [2, Sec. 3, p. 53]. Motivated by situations where the degree of uncertainty or the number of unknown parameters is so great that standard likelihood theory, including the holy trinity (generalized likelihood ratio tests [3], Wald and Rao tests) and robust tests derived from uncertainty models [4, III.E.2] may not apply to allow for possible distortions of the model, we address the general case where Θ has unknown distribution. Postponing to further work the case with possibly nuisance parameters, we assume that $X \sim \mathcal{N}(0, \mathrm{C})$ with known positive definite covariance matrix C, which is reasonable in many applications.

By taking into account directional variations induced by the noise covariance matrix, the Mahalanobis norm [5] of $\Theta - \theta_0$ is relevant to evaluate how far Θ deviates from θ_0. This Mahalanobis norm is defined for any $y \in \mathbb{R}^d$ by $\|y\| = \sqrt{y^{\mathrm{T}}\mathrm{C}^{-1}y}$ where A^{T} henceforth stands for the transpose of any matrix or vector A. We then address Random Distortion Testing (RDT), that is, the problem of testing whether $\|\Theta - \theta_0\| \leqslant \tau$ or not, when we observe Y and the probability distribution of Θ is unknown. By analogy with standard terminology in statistical inference, we say that this problem is the testing of the null event $\left[\,\|\Theta - \theta_0\| \leqslant \tau\,\right]$ against the alternative event $\left[\,\|\Theta - \theta_0\| > \tau\,\right]$ on the basis of observation Y and we summarize this problem by writing:

$$\text{RDT:}\begin{cases} \textbf{Observation: } Y = \Theta + X \begin{cases} \Theta \text{ and } X \text{ independent,} \\ \Theta \in \mathcal{M}(\Omega, \mathbb{R}^d), \\ X \sim \mathcal{N}(0, \mathrm{C}), \end{cases} \\ \textbf{Null event: } \left[\,\|\Theta - \theta_0\| \leqslant \tau\,\right], \\ \textbf{Alternative event: } \left[\,\|\Theta - \theta_0\| > \tau\,\right], \end{cases} \tag{1}$$

where $\mathcal{M}(\Omega, \mathbb{R}^d)$ stands for the set of all d-dimensional real random vectors defined on $(\Omega, \mathcal{B}, \mathrm{P})$. In contrast to usual literature on statistical inference, we do not test a deterministic unknown parameter but the random parameter Θ with unknown distribution, whereas standard approaches either assume the parameters of the observation to be deterministic but unknown or consider random parameters with known prior. It must be noticed that RDT reduces to the detection of Θ in noise when $\tau = 0$. Beyond signal detection, RDT is thinkable any time a distortion from a nominal reference must be detected, especially when a model for this distortion is hardly feasible.

The following remark is the starting point of our approach for solving the RDT problem of Eq. (1). Given any $\eta \geqslant 0$, consider any test $\mathcal{T}_\eta$ such that, for any $y \in \mathbb{R}^d$,

$$\mathcal{T}_\eta(y) = \begin{cases} 1 & \text{if} \quad \|y - \theta_0\| > \eta \\ 0 & \text{if} \quad \|y - \theta_0\| \leqslant \eta. \end{cases} \tag{2}$$

Such a test will hereafter be called a *thresholding test with threshold height* η. Basically, the purpose of such a test is to compensate the variations induced by C. It is then expected that the smaller $\|Y - \theta_0\|$, the more probable the null event. Given $\gamma \in (0, 1)$, it can be proved that there exists a threshold value $\lambda_\gamma(\tau) \in [0, \infty)$ such that

$$\mathrm{P}\big[\,\mathcal{T}_{\lambda_\gamma(\tau)}(\Theta + X) = 1 \,\big|\, \|\Theta - \theta_0\| \leqslant \tau\,\big] \leqslant \gamma$$

for any $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$. The quantity in the left hand side (lhs) of this inequality can be proved to be the size of $\mathcal{T}_{\lambda_\gamma(\tau)}$, as defined by Eq. (4) below. In other words, it is not that difficult to devise a test that guarantees a specific size for RDT, whatever the signal distribution. This has already been used in [6]. The actual question is then what optimality, if any, such a test may satisfy. The sequel presents our answer to this question with several comments and applications. The proofs of the results stated below are given in [7].

2. PROBLEM ANALYSIS

Consider any eigenvector decomposition $\mathrm{C} = \mathrm{U}\Delta\mathrm{U}^{\mathrm{T}}$ of C, where the diagonal elements $\xi_1, \xi_2, \ldots, \xi_d$ of the diagonal matrix $\Delta = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_d)$ are the eigenvalues of C and U is a $d \times d$ orthogonal matrix. Put $\Phi = \Delta^{-1/2}\mathrm{U}^{\mathrm{T}}$. Now, given any orthogonal matrix R, consider the affine transform g_{R} such that $g_{\mathrm{R}}(y) = \Phi^{-1}\mathrm{R}\Phi(y - \theta_0) + \theta_0$ for every $y \in \mathbb{R}^d$. The set of all these affine transforms g_{R} associated with orthogonal matrix form a group $\mathcal{G}$. The orbits of $\mathcal{G}$ are then the ellipsoids $\Upsilon_\rho = \{y \in \mathbb{R}^d : \|y - \theta_0\| = \rho\}$ with radius $\rho \geqslant 0$. We denote by $\mathfrak{F}$ the family of all these ellipsoids.

The RDT problem of Eq. (1) is invariant under the action of $\mathcal{G}$ in the sense that, given $g_{\mathrm{R}} \in \mathcal{G}$, it remains unchanged if $g_{\mathrm{R}}(\Theta + X)$ is considered instead of $\Theta + X$. Indeed, $g_{\mathrm{R}}(\Theta + X) = g_{\mathrm{R}}(\Theta) + \Phi^{-1}\mathrm{R}\Phi(X)$, $\Phi^{-1}\mathrm{R}\Phi(X) \sim \mathcal{N}(0, \mathrm{C})$ and $\|g_{\mathrm{R}}(\Theta) - \theta_0\| = \|\Theta - \theta_0\|$. This invariance cannot be exploited directly by using the invariance principle [8] because the signal in Eq. (1) has unknown probability distribution. Thresholding tests are $\mathcal{G}$-invariant in that, given any $\eta \geqslant 0$, $\mathcal{T}_\eta(g_{\mathrm{R}}(\Theta + X)) = \mathcal{T}_\eta(\Theta + X)$ for every $g_{\mathrm{R}} \in \mathcal{G}$. However, this basic property does not say anything about any invariance-based optimality property that thresholding tests may verify. We are going to see that thresholding tests actually satisfy a very strong property that implies the optimality of such tests within several invariance-based classes of tests.

3. THEORETICAL RESULTS

Below, a *test* is any measurable map of $\mathbb{R}^d$ into $\{0, 1\}$. Given any $\theta \in \mathbb{R}^d$ and any $Y \sim \mathcal{N}(\theta, \mathrm{C})$, the *power function* of test $\mathcal{T}$ is hereafter defined as the map that assigns to θ the value

$$\beta_\theta(\mathcal{T}) = \mathrm{P}\big[\mathcal{T}(Y) = 1\big]. \tag{3}$$

The size of a given test $\mathcal{T}$ is defined by:

$$\alpha(\mathcal{T}) = \sup_{\theta \in \mathbb{R}^d : \|\theta - \theta_0\| \leqslant \tau} \beta_\theta(\mathcal{T}). \tag{4}$$

Given $\gamma \in (0, 1)$, $\mathcal{T}$ is said to have level (resp. size) γ if $\alpha(\mathcal{T}) \leqslant \gamma$ (resp. $\alpha(\mathcal{T}) = \gamma$). Throughout, $\mathcal{K}_\gamma$ denotes the class of tests with level γ. To introduce our optimality criterion for RDT, we need the following definition.

Definition 1 *Given any $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$ independent of $X \sim \mathcal{N}(0, \mathrm{C})$, a given test $\mathcal{T}$ is said to have constant conditional power function given $\Theta \in \Upsilon_\rho$ if, for any $\theta \in \Upsilon_\rho$,*

$$\mathrm{P}\big[\mathcal{T}(\Theta + X) = 1 \,\big|\, \Theta \in \Upsilon_\rho\big] = \beta_\theta(\mathcal{T}).$$

The next definition introduces our optimality criterion for RDT. Henceforth, $\mathrm{P}_{\|\Theta - \theta_0\|}$ stands for the probability distribution of $\|\Theta - \theta_0\|$ and, given $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$, a support of $\mathrm{P}_{\|\Theta - \theta_0\|}$ is any borel subset $\mathfrak{D}$ of $[0, \infty)$ such that $\mathrm{P}_{\|\Theta - \theta_0\|}(\mathfrak{D}) = 1$. Given $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$, test $\mathcal{T}$ is hereafter said to satisfy a given property for $\mathrm{P}_{\|\Theta - \theta_0\|}$ – almost every $\rho > \tau$ if there exists some support $\mathfrak{D}$ of $\mathrm{P}_{\|\Theta - \theta_0\|}$ such that the property is verified for any $\rho \in (\tau, \infty) \cap \mathfrak{D}$.

Definition 2 *Given $\tau \geqslant 0$ and $\gamma \in (0, 1)$, test $\mathfrak{T}^*$ is said to have level γ and maximal constant conditional power (mccp) over $\mathfrak{F}$ for RDT — and we simply say that $\mathfrak{T}^*$ is γ-mccp — if:*
[level] *$\mathfrak{T}^* \in \mathcal{K}_\gamma$;*
[mccp] *Given any $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$ and for $\mathrm{P}_{\|\Theta - \theta_0\|}$ – almost every $\rho > \tau$, $\mathfrak{T}^*$ has constant conditional power function given $\Theta \in \Upsilon_\rho$ and $\mathrm{P}\big[\mathfrak{T}^*(\Theta + X) = 1 \,\big|\, \Theta \in \Upsilon_\rho\big] \geqslant \mathrm{P}\big[\mathcal{T}(\Theta + X) = 1 \,\big|\, \Theta \in \Upsilon_\rho\big]$ for any $\mathcal{T} \in \mathcal{K}_\gamma$ with constant conditional power function given $\Theta \in \Upsilon_\rho$.*

The main results of this section are then Theorems 1 and 2 below. In Theorem 1, ϑ_τ stands for the set of all $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$ such that $\mathrm{P}\big[\,\|\Theta - \theta_0\| > \tau\,\big] \neq 0$. Given any test $\mathcal{T}$, the power of $\mathcal{T}$ on ϑ_τ for RDT is the map that assigns, to every given $\Theta \in \vartheta_\tau$, the value

$$\beta_\Theta(\mathcal{T}) = \mathrm{P}\big[\,\mathcal{T}(\Theta + X) = 1 \,\big|\, \|\Theta - \theta_0\| > \tau\,\big]. \tag{5}$$

A given test $\mathfrak{T}^*$ is hereafter said to be UMP (Uniformly Most Powerful) with level (resp. size) γ for RDT among all tests of some class $\mathcal{C}$ of tests if $\mathfrak{T}^* \in \mathcal{K}_\gamma$ (resp. $\alpha(\mathcal{T}) = \gamma$) and $\beta_\Theta(\mathfrak{T}^*) \geqslant \beta_\Theta(\mathcal{T})$ for any $\Theta \in \vartheta_\tau$ and any $\mathcal{T} \in \mathcal{C}$.

Theorem 1 *Given $\gamma \in (0, 1)$ and $\tau \geqslant 0$,*

(i) *Given any $\Theta \in \vartheta_\tau$, any γ-mccp test is UMP for RDT among all tests that have constant conditional power function given $\Theta \in \Upsilon_\rho$ for $\mathrm{P}_{\|\Theta - \theta_0\|}$ – almost every $\rho > \tau$.*

(ii) *Any γ-mccp test with constant power on every $\Upsilon_\rho \in \mathfrak{F}$ is UMP for RDT among all tests with $\mathcal{G}$-invariant power function;*

(iii) *Any γ-mccp and $\mathcal{G}$-invariant test is UMP for RDT among all $\mathcal{G}$-invariant tests;*

(iv) *Any γ-mccp test is UMP with level γ for testing $\|\theta - \theta_0\| \leqslant \tau$ against $\|\theta - \theta_0\| > \tau$ among all tests with constant power function on every $\Upsilon_\rho \in \mathfrak{F}$ with $\rho > \tau$.*

In what follows, given $\rho \geqslant 0$, $\mathcal{R}(\rho, \cdot)$ stands for the cumulative distribution function of the square root of any random variable that follows the non-central χ^2 distribution with d degrees of freedom and non-central parameter ρ^2.

Theorem 2 *Given $\gamma \in (0,1)$ and $\tau \geqslant 0$,*

(i) $\mathcal{T}_{\lambda_\gamma(\tau)}$ *is γ-mccp with*

$$\mathrm{P}\big[\mathcal{T}_{\lambda_\gamma(\tau)}(\Theta + X) = 1 \,\big|\, \Theta \in \Upsilon_\rho\big] = 1 - \mathcal{R}(\rho, \lambda_\gamma(\tau))$$

for any given $\Theta \in \mathcal{M}(\Omega, \mathbb{R}^d)$ and $\mathrm{P}_{\|\Theta-\theta_0\|}$ – almost every $\rho \geqslant 0$.

(ii) *Given any $\Theta \in \vartheta_\tau$, $\beta_\Theta(\mathcal{T}_{\lambda_\gamma(\tau)}) \geqslant 1 - \mathcal{R}(\tau', \lambda_\gamma(\tau))$ where $\tau' \in [\tau, \infty)$ is the supremum of the set of all those real values $t \geqslant \tau$ such that $\mathrm{P}\big[\,\tau < \|\Theta - \theta_0\| \leqslant t\,\big] = 0$. Consequently, $\mathcal{T}_{\lambda_\gamma(\tau)}$ is unbiased for RDT in that $\beta_\Theta(\mathcal{T}_{\lambda_\gamma(\tau)}) \geqslant \gamma$ for every $\Theta \in \vartheta_\tau$.*

(iii) *$\mathcal{T}_{\lambda_\gamma(\tau)}$ is UMP with size γ for RDT among all tests with constant conditional power function given $\Theta \in \Upsilon_\rho$ for $\mathrm{P}_{\|\Theta-\theta_0\|}$ – almost every $\rho > \tau$, among all tests with $\mathcal{G}$-invariant power function and among all $\mathcal{G}$-invariant tests.*

(iv) *For every $\rho > \tau$, $\mathcal{T}_{\lambda_\gamma(\tau)}$ has constant power function $\beta_\theta(\mathcal{T}_{\lambda_\gamma(\tau)}) = 1 - \mathcal{R}(\rho, \lambda_\gamma(\tau))$ on Υ_ρ and $\beta_\theta(\mathcal{T}_{\lambda_\gamma(\tau)}) \geqslant \beta_\theta(\mathcal{T})$ for any $\theta \in \Upsilon_\rho$ and any test $\mathcal{T} \in \mathcal{K}_\gamma$ with constant power on Υ_ρ.*

The next proposition is a direct application of Theorem 2 and concerns the case of a deterministic unknown signal.

Proposition 1 *Given $\gamma \in (0,1)$ and $\tau \geqslant 0$,*

(i) *$\mathcal{T}_{\lambda_\gamma(\tau)}$ is unbiased: $\beta_\theta(\mathcal{T}_{\lambda_\gamma(\tau)}) \geqslant \gamma$ for all $\rho > \tau$;*

(ii) *For testing the null hypothesis $\|\theta - \theta_0\| \leqslant \tau$ against the alternative one $\|\theta - \theta_0\| > \tau$ when the observation is $Y \sim \mathcal{N}(\theta, \mathrm{C})$, $\mathcal{T}_{\lambda_\gamma(\tau)}$ is UMP with size γ among all tests with $\mathcal{G}$-invariant power function, among all $\mathcal{G}$-invariant tests and among all tests that have constant power function on every $\Upsilon_\rho \in \mathfrak{F}$ with $\rho > \tau$.*

For testing the mean of a normal distribution, the reader will easily verify that [1, Proposition III, p. 450] follows from Proposition 1 and, thus, from Theorem 2, by considering the particular case $\tau = 0$.

4. APPLICATIONS

In this section, the detection of a non-null unknown signal via its observation in additive Gaussian noise, a problem of interest in many applications, is considered to illustrate the use of the RDT in practice. It will be shown that, with model mismatch, conventional approaches such as Neyman-Pearson's might fail, whereas the proposed RDT remains functioning in any case. Furthermore, as a concrete real-world application, the mechanical respiratory support monitoring is addressed.

4.1. Signal detection

Let Ξ be the random d-dimensional signal of interest with unknown distribution and $\Xi \neq 0$ (a-s). Assume that Ξ is observed in independent noise $X \sim \mathcal{N}(0, \mathrm{C})$ with known $\mathrm{C} > 0$. The detection of Ξ is described as the binary hypothesis testing where $\mathcal{H}_0$ is that only noise is present and $\mathcal{H}_1$ is that the observation is the sum of signal and noise. Since $\Xi \neq 0$ (a-s), there exists a real value $\tau' \geqslant 0$ such that $\|\Xi\| > \tau'$ (a-s). Let Y be the observation, the problem can be summarized by

$$\begin{cases} \mathcal{H}_0 : Y \sim \mathcal{N}(0, \mathrm{C}), \\ \mathcal{H}_1 : Y = \Xi + X, X \sim \mathcal{N}(0, \mathrm{C}), \mathrm{P}\big[\|\Xi\| > \tau'\big] = 1. \end{cases} \tag{6}$$

By setting $\Theta = \varepsilon\Xi$ — where the random variable ε, valued in $\{0, 1\}$, indicates the presence/absence of the target signal —, the problem of Eq. (6) is thus RDT with $\tau = 0$ and $\theta_0 = 0$. According to Theorem 2, the optimal γ-mccp test provided by Theorem 2 is then $\mathcal{T}_{\lambda_\gamma(0)}$. The size and power of this test are exactly the false alarm and detection probabilities of the detection problem Eq. (6), respectively. We obtain $\mathrm{P_{FA}}\big[\mathcal{T}_{\lambda_\gamma(0)}\big] = \gamma$ and $\mathrm{P_D}\big[\mathcal{T}_{\lambda_\gamma(0)}\big] \geqslant 1 - \mathcal{R}(\tau', \lambda_\gamma(0))$.

However, in practice, it may happen that $\mathcal{H}_0$ does not reduce to the presence of noise alone but that there might still be some signal of no interest. This model mismatch might cause standard likelihood approach to violate the Neyman-Pearson's constraint on the false-alarm probability, but introduces no such a problem in the RDT framework. To illustrate this aspect, the target signal is considered deterministic so that the Neyman-Pearson likelihood test applies.

To begin with, let us consider the ideal model of Eq. (6) with deterministic signal $\Xi = \xi_1$ such that $\|\xi_1\| = \tau'$. With respect to this ideal model, the Neyman-Pearson test is defined for every $y \in \mathbb{R}^d$ by $\mathcal{T}_{\mathrm{NP}}(y) = 1$ if $\Lambda = \xi_1^{\mathrm{T}}\mathrm{C}^{-1}y > \xi$ and $\mathcal{T}_{\mathrm{NP}}(y) = 0$ if $\Lambda = \xi_1^{\mathrm{T}}\mathrm{C}^{-1}y \leqslant \xi$, where threshold ξ is calculated so that the ideal false alarm probability is $\mathrm{P}\big[\xi_1^T\mathrm{C}^{-1}Y > \xi\big] = \gamma$ when $Y \sim \mathcal{N}(0, \mathrm{C})$ in absence of model mismatch. The handling of equality in the definition of $\mathcal{T}_{\mathrm{NP}}$ does not matter for the absolute continuity of the probability distribution of the observation.

Assume now that in $\mathcal{H}_0$, some fluctuation induces the observation to randomly distort from zero, regardless whether noise is present or not. The distribution of such distortion, hereafter denoted by Ξ_0, is unknown in practice. However, $\|\Xi_0\|$ is generally bounded (a-s) by some positive value τ. The problem is actually:

$$\begin{cases} \mathcal{H}_0 : Y = \Xi_0 + X \\ \mathcal{H}_1 : Y = \xi_1 + X \end{cases} \text{with} \begin{cases} X \sim \mathcal{N}(0, \mathrm{C}), \\ \|\Xi_0\| \leq \tau \text{ (a-s)}, \|\xi_1\| = \tau'. \end{cases} \tag{7}$$

If $\mathcal{T}_{\mathrm{NP}}$ is still applied, it might violate the constraint on the false-alarm probability. Indeed, it can easily be shown that, as long as the random variable $\xi_1^T\mathrm{C}^{-1}\Xi_0$ is symmetrically distributed, the aforementioned constraint is always violated, i.e. $\mathrm{P_{FA}}\big[\mathcal{T}_{\mathrm{NP}}\big] > \gamma$, for any $\gamma < 0.5$. On the contrary, with

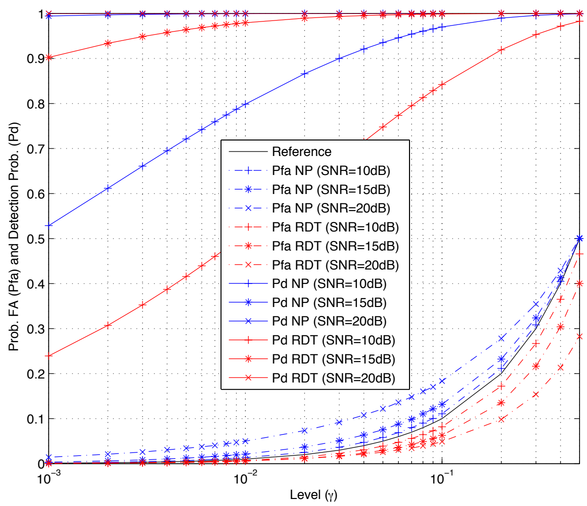

Fig. 1. Detection performance yielded by $\mathcal{T}_{\mathrm{NP}}$ and $\mathcal{T}_{\lambda_\gamma(\tau)}$. The reference represents the Neyman-Pearson's constraint on the false-alarm probability.

the same notation as before, by setting $\Theta = \varepsilon\xi_1 + (1-\varepsilon)\Xi_0$, the detection problem of Eq. (7) is RDT. The γ-mccp optimality criterion is then relevant and, according to Theorem 2, is satisfied by $\mathcal{T}_{\lambda_\gamma(\tau)}$. It then follows from Theorem 2 that $\mathrm{P}_{\mathrm{FA}}\left[\mathcal{T}_{\lambda_\gamma(\tau)}\right] = \mathrm{P}\left[\|\Xi_0 + X\| > \lambda_\gamma(\tau)\right] \leq 1 - \mathcal{R}(\tau, \lambda_\gamma(\tau)) = \gamma$. The constraint on the false-alarm probability is thus respected by $\mathcal{T}_{\lambda_\gamma(\tau)}$.

To illustrate these aspects, numerical simulations with $d = 2$ and $C = \sigma^2 \mathbf{I}_d$ were carried out. Ξ_0 was randomly generated with a normal distribution $\mathcal{N}(0, \sigma_0^2 \mathbf{I}_d)$ and τ was set to $\tau = 2\sigma_0$, which means $\mathrm{P}\left[\|\Xi_0\| < \tau\right] = 86.47\%$. Both the Neyman-Pearson likelihood test (NP) $\mathcal{T}_{\mathrm{NP}}$ and the RDT thresholding test $\mathcal{T}_{\lambda_\gamma(\tau)}$ were employed with different values of Signal-to-Noise Ratio (SNR) $\frac{\tau'}{\sigma}$ (10dB, 15dB, 20dB). The Signal-to-maximum-Distortion Ratio τ'/τ was also set to a similar value $\tau'/\tau = 5$ ($\approx$ 14dB). In other words, the ratio σ_0/σ is -10dB, -5dB, 0dB respectively, which seemingly implies that the distortion caused by unexpected fluctuation is of very small magnitude. Despite such small distortion, $\mathcal{T}_{\mathrm{NP}}$ will fail. The detection results are reported in Fig. 1. On the one hand, Fig. 1 confirms that, for any $\gamma < 0.5$, $\mathcal{T}_{\mathrm{NP}}$ actually yields a false-alarm rate higher than expected. On the other hand, although there is some loss in detection rate due to the unavoidable trade-off between the false-alarm and the detection probability, the detection in the RDT framework guarantees a false-alarm rate lower than the specified level γ.

4.2. Mechanical respiratory support monitoring

Mechanical ventilation is routinely used in emergency wards, operation room or intensive care unit. It is also use at home or in nursing/rehabitation institution. Unfortunately, imperfect interaction between patient and ventilator is very common. AutoPEEP (*Auto-Positve End Expiratory Pressure*) and IEE (*Ineffective Effort during Expiration*) are among the most frequent abnormalities during mechanical respiratory support. On the basis of the respiratory curves (flow, pressure, volume) available on recent ventilators, the automatic detection of such abnormalities can be carried out.

On the one hand, AutoPEEP can be regarded as the non return to zero of the flow signal at the end of expiratory phase to the null value. Let f_t be the clean flow signal and t_k be the end expiration instant of the considered breath. Given some tolerance τ, the detection of AutoPEEP is then testing $|f_{t_k}| \leqslant \tau$ against $|f_{t_k}| > \tau$ based on its observation in noise. The problem is RDT and the optimal test is then $\mathcal{T}_{\lambda_\gamma(\tau)}$ with $d = 1$ and level γ specified by clinician. The assessment on clinical data has shown that the proposed test could provide an good detection of AutoPEEP with an Accuracy of 93% and a recall (sensitivity) of 90% [9].

On the other hand, the presence of IEE introduces a waveform distortion during the expiratory portion of the flow signal. Let $\mathbf{f}_k$ be the clean expiratory flow signal samples of the considered k-th breath and $\mathbf{f}_0$ be the waveform reference which can be estimated from normal expirations. The detection of IEE then resorts to testing $\|\mathbf{f}_k - \mathbf{f}_0\| \leqslant \tau$ against $\|\mathbf{f}_k - \mathbf{f}_0\| > \tau$ based on observation $\mathbf{Y}_k = \mathbf{f}_k + \mathbf{X_k}$ in gaussian noise $\mathbf{X_k}$. The optimal test is also given in the RDT framework by $\mathcal{T}_{\lambda_\gamma(\tau)}$ with d is the number of samples in expiratory portion of a breath. The tolerance τ and level γ are also given by clinician. The experiment on synthetic data has shown that such detection could yield a detection rate of 90% for $\gamma = 0.01$.

5. CONCLUSION AND PERSPECTIVES

This paper has introduced the RDT problem with applications in signal detection and mechanical respiratory support monitoring. The mccp property has been introduced as invariance-based optimality criteria for RDT. To a certain extent, RDT could be considered as a semi-parametric approach since, on the one hand, they guarantee robustness against signal distribution variations and, on the other hand, yield statistical optimality in a sense similar to Neyman-Pearson's. The mere information needed to perform RDT concerns solely the noise covariance matrix, which can often be estimated in practice. No training data is needed, which is relevant for many applications where collecting and annotating a sufficiently large and representative dataset concerning the signal is a laborious task.

In a future work, theoretical extensions could concern the combination of RDT with those of [1]. For instance, since distortion testing enables to overcome the lack of robustness of likelihood ratio tests, elaborating on RDT problems involving nuisance parameters and large sample sizes could be relevant. Other applications such as tracking, anti-collision radar, structural health monitoring could also be investigated.

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