DATA DETECTION USING PARTICLE FILTERING IN RELAY-BASED COMMUNICATION SYSTEM WITH LAPLACE AR CHANNEL MODEL

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ABSTRACT

In this paper, we consider channel modeling and data detection in a communication system where an amplify-and-forward relay is used to transmit data between a receiver and a transmitter in a flat-fading channel. The effective channel, which is the cascade of the transmitter-to-relay and relay-to-receiver Rayleigh channels, is modeled as a first-order Laplace autoregressive (AR) process. We also show that the resulting additive noise of the cascade of the two channels has a normal-Laplace distribution which enables us to derive a better approximation to the likelihood probability distribution function (pdf). Given the channel model and better characterization of the likelihood pdf of the system, we formulate the transmission process as dynamic state-space model, and propose a particle filtering based algorithm for data detection. The effectiveness of the performance of the proposed algorithm is investigated through computer simulations.

Index Terms— Relay-based, Particle filtering, Channel estimation, Laplace AR, normal-Lapalace

1. INTRODUCTION

The increased emergence of new wireless services and demand for high data rates requires the continual enhancement of the capacity of wireless cellular networks. The conventional solution to the need for more capacity in the cellular network is to increase frequency reuse by cell division which requires installing new base stations (BS) [1]. However, the cost of installing new BS and its concomitant infrastructure is high. Recently, relay-based communication system is presented as an alternative solution to enhancing the capacity of cellular networks. Compared to a BS, a relay is a simpler device and it is much less costly.

In relay-based communication network, one or multiple relays forward the data sent from the transmitter before arriving at the receiver. The relay can be a decode-and-forward (DF) relay which essentially acts as a repeater or a simple amplify-and-forward (AF) relay which functions as a regenerator [2].

Relay-based technology promises several benefits over the traditional single-hop cellular network. Since the addition of a relay in a cellular network reduces the transmission distance, it effectively reduces the signal propagation loss which translates to increased signal-to-interference-noise ratio (SINR) [3]. The increased SINR enhances the capacity and improves the coverage of the cellular network. Furthermore, relays can provide spatial diversity to mitigate the effect of fading channels through cooperative communication. Such potential benefits have attracted research interest in the study of of relay-based communication which includes channel modeling and data detection in one or multiple relay systems [4].

In this paper, we consider the problem of channel modeling and data detection in a relay-based communication system where there is one AF relay between the transmitter and the receiver. Such a system is composed of two communication links which are the transmitter-to-relay and relay-to-receiver links. Assuming the receiver and the relay are mobile, the overall channel of the communication system is a cascade of mobile-to-mobile and mobile-to-fixed channels. As a result the effective channel of the communication system has a different statistical characteristics than the single-hop cellular network channel. For example, in Non-Line-Of-Sight (NLOS) transmission, the real and imaginary component of the a wireless channel are modeled as uncorrelated and identically distributed Gaussian pdfs. With similar assumptions, the real and imaginary component of an overall relay channel, however, have uncorrelated identically distributed Laplace pdfs [5]. Furthermore, the second order statistics such as the autocorrelation function of the single-hop channel is different than a relay channel. The time variation of the overall relay channel is faster than the time variation of a single-hop wireless channel, and as a result data detection in a relay-based communication is more challenging.

1.1. Related work

Most of the reported work on data detection in a relay-based communication apply Least Square (LS) or Linear Minimum Mean Square Estimation (LMMSE) methods [2]. These approaches assume a slow-varying channel and utilize pilot data for channel estimation, and, therefore, require significant additional bandwidth for the transmission of pilot data to obtain a reasonable symbol error rate. Other methods model each of the two communication channel as Gaussian AR processes, and employ Extended Kalman Filter (EKF) [2], particle filtering [6] or particle filtering MCMC methods [7] for data detection and channel estimation.

In this paper, we derive a particle filtering based algorithm for blind data detection and joint channel estimation. Our approach has two novel contributions. The first contribution is that we show the effective relay channel can be modeled by one complex Laplace AR process rather that by two separate complex Gaussian AR processes. The second contribution is that we provide a better characterization of the pdf of the noise at the receiver by showing that it is normal-Laplace distribution. Using the derived characterization of the received signal and the developed first-order Laplace AR process channel model, we represent the communication process as a dynamic system in a state-space form, and apply particle filtering techniques for joint data detection and channel estimation without the need for the transmission of pilot signals.

The remaining part of the paper is organized as follows. Section 2 presents the signal model, and Section 3 describes the channel model. The derivation of the proposed algorithms is explained in

Section 4. Computer simulations is provided in Section 5 and, finally, conclusions are given in Section 6.

2. SIGNAL MODEL

Suppose data symbols, $\{s_k\}$, obtained from a finite modulation alphabet, $\mathcal{A}$, are transmitted over a flat-fading relay-based communication system which consists of one AF relay. The discrete-time equivalent of the baseband signal received at the relay at time k, z_k, is given by,

$$z_k = h_{1,k}s_k + u_k, \tag{1}$$

where $h_{1,k}$ denotes the value at time k of the flat-fading transmitter-to-relay channel, h_1, and $u_k \sim \mathcal{N}(0, \sigma_u^2)$ is a zero-mean complex Gaussian noise added by the transmitter-to-relay channel. The relay, in turn, amplifies-and-forwards, the data to the receiver over a flat-fading relay-to-receiver channel, h_2. Thus, the discrete-time equivalent of the baseband signal obtained at the receiver at time k can be written as

$$y_k = A_k h_{1,k} h_{2,k} s_k + h_{2,k} u_k + v_k, \tag{2}$$

where A_k is the amplification factor of the relay and $v_k \sim \mathcal{N}(0, \sigma_v^2)$ is an additive complex zero-mean white Gaussian noise that is introduced by the relay-to-receiver channel. Observing that the first term of (2) is the signal part and the second and third term constitute the noise part of y_k, we compactly rewrite it as,

$$y_k = A_k h_k s_k + n_k, \tag{3}$$

where $h = h_1 h_2$ is the effective channel and n_k is the effective non-Gaussian noise of the system. In the sequel, we derive the pdf of the real and imaginary components of the channel, h, and the noise n.

Proposition 1: If the fading of h_1 and h_2 are Rayleigh processes, then the real and imaginary component of h are uncorrelated zero-mean Laplace pdfs.

The proof of *proposition 1* follows from the observation that, since h_1 and h_2 are Rayleigh processes, h_1 and h_2 can be written in their real and imaginary component as $h_1 = h_{1R} + jh_{1I}$ and $h_2 = h_{2R} + jh_{2I}$ where h_{1R} and h_{1I} are zero mean Gaussian identically and independently distributed (i.i.d.) random variables with a pdf $\mathcal{N}(0, \sigma_1^2)$, and h_{2R} and h_{2I} are i.i.d. random variables with a pdf $\mathcal{N}(0, \sigma_2^2)$. The real and imaginary component of of h are then given by $x = h_{1R}h_{2R} - h_{1I}h_{2I}$ and the $y = h_{1R}h_{2I} + h_{1I}h_{2R}$ respectively.

Let us first consider the real component x, and write it as the difference of two random variables r and s where $r = h_{1R}h_{2R}$ and $s = h_{1I}h_{2I}$. Since r and s are independent random variables, the characteristic function of x can be computed as the product of the characteristic function of r and $-s$, and is given by $E[e^{j\omega x}] = E[e^{j\omega r}]E[e^{-j\omega s}]$. We can compute $E[e^{j\omega r}] = E_{h_{2R}}\left[E_{h_{1R}|h_{2R}}[e^{j\omega h_{1R}h_{2R}}|h_{2R}]\right] = \frac{1}{\sqrt{1+\sigma_1^2\sigma_2^2\omega^2}}$. Similarly, $E[e^{j\omega s}] = \frac{1}{\sqrt{1+\sigma_1^2\sigma_2^2\omega^2}}$. Then, $E[e^{j\omega x}] = \frac{1}{(1+\sigma_1^2\sigma_2^2\omega^2)}$ which represents a Laplace pdf with $\lambda = \sigma_1\sigma_2$.

$$\begin{aligned} p_{h_R}(x) &= \mathcal{L}(x; 0, \lambda) = \frac{1}{2\lambda}e^{-\frac{|x|}{\lambda}} \\ p_{h_I}(y) &= \mathcal{L}(y; 0, \lambda) = \frac{1}{2\lambda}e^{-\frac{|y|}{\lambda}} \end{aligned} \tag{4}$$

where $\lambda = \sigma_1\sigma_2$. Following the same steps, we can show that y also has a Laplace pdf with $\lambda = \sigma_1\sigma_2$. Furthermore, it is straight forward to show that $E[xy] = 0$.

Preposition 2: The real and imaginary components of the noise of the relay-based system, $n = h_2u + v$ have identical uncorrelated symmetric normal-Laplace pdfs.

The proof of *Preposition 2* follows from the observation that the $n_1 = h_2u$ is a product of two independent complex Gaussian random variables. From the proof given for *Proposition 1* , the real and imaginary components of the n_1 are uncorrelated identical Laplace probability density functions with $\lambda_n = \sigma_2\sigma_u$. Since n_1 and v are independent, the characteristic function of the real (imaginary) part of n is given by

$$E[e^{j\omega n_R}] = E[e^{j\omega n_{1R}}]E[e^{j\omega v_R}] = \frac{e^{-\frac{\sigma_v^2\omega^2 v^2}{2}}}{1+\lambda_{n_1}\omega^2}.$$

Taking the inverse of the characteristic function, we obtain the zero-mean symmetric normal-Laplace probability density function [8]. The symmetric normal-Laplace distribution, which we denote by $n_R \sim \mathcal{NL}(n_R; \mu, \sigma_v, \lambda_n)$, has a closed form expression given by

$$p_{n_R}(x) = \frac{1}{2\lambda_n\sqrt(2\pi\sigma_v^2)}e^{-\frac{(x-\mu)^2}{2\sigma_v^2}}\left[R(x_1) + R(x_2)\right]$$

where μ is the mean, $x_1 = \frac{\sigma_v}{\lambda_n} - \frac{(x-\mu)}{\sigma_v}$, $x_2 = \frac{\sigma_v}{\lambda_n} + \frac{(x-\mu)}{\sigma_v}$, and the function $R(\cdot)$ is the Mills' ratio. The Mills' ratio is given by $R(z) = \frac{Q(z)}{1/\sqrt{2\pi}e^{-z^2/2}}$ where $Q(z)$ is the popular Q function of a standard Gaussian density.

Following similar steps, it can be shown that the imaginary part,n_I, has the same pdf as n_R, $n_I \sim \mathcal{NL}(n_I; \mu, \sigma_v, \lambda_n)$. Furthermore, we note that n_R and n_I are uncorrelated, $E[n_R n_I] = 0$, and have equal variance, $\sigma_{n_R}^2 = \sigma_{n_I}^2 = \sigma_v^2 + 2\lambda_n^2$.

3. CHANNEL MODEL

Assuming that $\sigma_1^2 = \sigma_2^2 = 1$, the real and imaginary component of h have marginal standard Laplace distributions. We, therefore, approximately model the overall channel, h, as a complex Laplace AR process.

Following [9], a first order AR process which has a standard Laplace marginal distribution can be generated using

$$h_k = \kappa_k\beta h_{k-1} + \epsilon_k \tag{5}$$

where κ_k is a discrete random variable which takes either 0 or 1 with probability mass function (pmf) given by

$$p_{\kappa_k}(x) = \alpha\delta(x-1) + (1-\alpha)\delta(x)$$

ϵ_k is the driving complex mixture Laplace distribution given by

$$p_{\epsilon_k}(x) = (1-\gamma)\mathcal{L}(x; 0, 1) + \gamma\mathcal{L}(x; 0, \sqrt{(1-\alpha)}\beta)$$

with $p_{\epsilon_0}(x) = \mathcal{L}(x; 0, 1)$, and α and β are constant coefficients, and

$$\gamma = \frac{\alpha\beta^2}{(1-(1-\alpha)\beta^2)}.$$

Multiplying both sides of (5) by h_{k-l}, we obtain the autocorrelation of the Laplace AR process is given by

$$E[h_k h_{k-l}] = 2(\alpha\beta)^l \quad for \quad l \geq 0$$

The Laplace AR coefficients, α and β, in our model are chosen to match the theoretical autocorrelation of the time variation of the effective channel of the relay-based communication system. The theoretical autocorrelation of flat-fading channels depends only on their fading rate which is a product of the Doppler frequencies and sampling period. For relay-based channels which consists of fixed-to-mobile and mobile-to-mobile channels, the theoretical autocorrelation is given by,

$$E[h_k h_{k-l}]_{theoretical} = 2\sigma_1^2\sigma_2^2 J_0(\omega_1 T_s l)^2 J_0(\omega_2 T_s l)$$

where ω_1 and ω_2 are angular doppler frequencies of the mobile relay and mobile receiver respectively, T_s is the sampling period, and $J_0(\cdot)$ is the zeroth-order bessel function of first kind.

Using the received signal (3) as observation equation, and the channel model (5) as a state equation, we can formulate the problem as a dynamic system with a state-space equations given by

$$\text{state space formulation} \quad \begin{cases} h_k = \kappa_k \beta h_{k-1} + \epsilon_k \\ y_k = A_k h_k s_k + n_k \end{cases} \tag{6}$$

In this paper, we assume both, the transmitter-to-relay and the relay-to-receiver, channels are stationary and the their fading rates are known, and, thus, the AR coefficients, α and β, are fixed and pre-determined. Moreover, we assume a fixed gain AF relay with a known amplification factor $A_k = A$.

Having the state-space representation of the problem given by (6), our main objective is to make a real-time estimate of the symbols, s_k, and the channel, h_k, from the available observation until time k, $y_{0:k}$.

4. PARTICLE FILTER METHOD FOR DATA DETECTION AND CHANNEL ESTIMATION

Our approach to the estimation of the transmitted data and channel is based on a Bayesian framework. The framework requires the sequential estimation of the posterior probability distribution function, $p(s_{0:k}, h_{0:k}|y_{0:k})$. Unfortunately, the analytic estimation of the posterior pdf in a closed form is intractable, we, therefore, propose to use Sequential Importance Sampling (SIS) or particle filtering techniques for Monte Carlo approximation of the posterior pdf.

A particle filter can be used to sequentially approximate the posterior pdf, $p(s_{0:k}, h_{0:k}, |y_{0:k})$, by a set of N samples $\{s_{0:k}, h_{0:k}\}_{i=1}^N$, with their associated weights $\{w_k\}_{i=1}^N$ as follows [10],

$$\hat{p}(s_{0:k}, h_{0:k}|y_{0:k}) = \sum_{i=1}^{N} \delta(s_{0:k}, h_{0:k} - s_{0:k}^{(i)}, h_{0:k}^{(i)}) w_k^{(i)},$$

where $\delta(\cdot)$ is a Dirac delta function. The samples are drawn from an importance density $\pi(s_{0:k}, h_{0:k}|y_{0:k})$, and the associated weights are computed as,

$$w_k^{(i)} \propto \frac{p(s_{0:k}, h_{0:k}|y_{0:k})}{\pi(s_{0:k}, h_{0:k}|y_{0:k})}$$

The sequential estimation of $p(s_{0:k}, h_{0:k}|y_{0:k})$ is possible, if we select an importance density that is factorizable as,

$$\begin{aligned} \pi(s_{0:k}, h_{0:k}|y_{0:k}) \quad &= \quad \pi(s_k, h_k|s_{0:k-1}, h_{0:k-1}, y_{0:k}) \\ &\times \quad \pi(s_{0:k-1}, h_{0:k-1}|y_{0:k-1}) \end{aligned} \tag{7}$$

With such importance density, the weights can be sequentially computed as,

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k|s_k^{(i)}, h_k^{(i)}) p(h_k^{(i)}, s_k^{(i)}|h_{k-1}^{(i)}, s_{k-1}^{(i)})}{\pi(s_k, h_k|s_{0:k-1}, h_{0:k-1}, y_{0:k})}$$

After the posterior pdf is approximated, Bayesian estimates of s_k can be obtained by using marginal Maximum A Posteriori (MAP) estimator as

$$s_k^{map} = \arg\max_{s_k} \left\{ \sum_{i=1}^{N} \delta(s_k - s_k^{(i)}) w_k^{(i)} \right\}. \tag{8}$$

and, similarly, the channel, h_k can be estimated using the marginal Minimum Mean Square Error (MMSE) estimator as

$$h_k^{mmse} = \sum_{i=1}^{N} h_k^{(i)} w_k^{(i)} \tag{9}$$

To derive particle filtering algorithms, we consider a prior importance density given by,

$$\pi(s_k, h_k|s_{0:k-1}, h_{0:k-1}, y_{0:k}) = p(s_k|s_{k-1}) p(h_k|h_{k-1}) \tag{10}$$

Assuming the transmitted sequence s_k are i.i.d., we see that $p(s_k|s_{k-1}) = p(s_k) = \frac{1}{|\mathcal{A}|}$ where $|\mathcal{A}|$ is the size of the modulation alphabet. Furthermore, we obtain the prior density of the channel, $p(h_k|h_{k-1})$, is given by

$$p(h_k|h_{k-1}^{(i)}) = \alpha p_{\epsilon_k}(h_k - \beta h_{k-1}^{(i)}) + (1-\alpha) p_{\epsilon_k}(h_k) \tag{11}$$

Therefore, a sample, $(s_k^{(i)}, h_k^{(i)})$, is obtained by attaching a sample of s_k drawn, with equal probability, from the modulation alphabet to a sample of $h_k^{(i)}$ drawn from (11). Note that obtaining samples from the channel prior density requires drawing samples from a Laplace pdf. A sample, $m \sim \mathcal{L}(x; \mu, \lambda)$, can be obtained using the following two steps:

1. Draw two random samples from a uniform distribution $u_1 = U[0,1]$ and $u_2 = U[0,1]$
2. Obtain $m = \mu + \lambda \times (log(u_1) - log(u_2))$

After the samples are obtained using the prior importance density, the corresponding weights are obtained from,

$$\tilde{w}_k^{(i)} \propto w_{k-1}^{(i)} p(y_k|s_k^{(i)}, h_k^{(i)}) \tag{12}$$

where the likelihood function can be approximately computed

$$p(y_k|s_k^{(i)}, h_k^{(i)}) = \mathcal{NL}(y_k; A h_k^{(i)} s_k^{(i)}, \sigma_v, \lambda_w)$$

Before applying (8) and (9) for MAP and MMSE estimation, the weights obtained by (12) should be normalized as,

$$w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{n=1}^{N} \tilde{w}_k^{(n)}} \tag{13}$$

We note that it is shown in [11] that a pdf computed using a SIS algorithm as described above converges to the desired posterior pdf for a sufficiently large number of particles, i.e.,

$$\hat{p}(s_{0:k}, h_{0:k}|y_{0:k}) \overset{N\to\infty}{\longrightarrow} p(s_{0:k}, h_{0:k}|y_{0:k}).$$

A practical implementation of the particle filtering algorithms require a resampling procedure which avoids the degeneracy of the particles. The basic idea of resampling is to generates a new of set of particles at time $k+1$ by randomly selecting from particles at time k with a probability equal to the size of their weights, therefore, effectively replicating those particles with large weights while eliminating those with insignificant weights.

The proposed algorithm for data detection and data estimation is summarized in Table 1.

Initialization $\quad h_0 \sim \mathcal{L}(h; 0, 1)$ (real and imaginary)
For $k = 0$ to K *(total number of symbols)*
For $i = 1$ to N *(total number of particles)*
Draw a sample $s_k^{(i)}$
$s_k^{(i)} \propto \frac{1}{|\mathcal{A}|}$
Draw a sample of h_k (real and imaginary) using the steps
Draw samples $u_1 \sim \mathcal{U}[0, 1]$ and $u_2 \sim \mathcal{U}[0, 1]$
If $u_1 \leq \alpha$ then $\kappa_k = 1$ else $\kappa = 0$
If $u_2 \leq p$ draw a sample h_k from
$h_k^{(i)} \propto \mathcal{L}(h; \kappa_k \beta h_{k-1}^{(i)}, 1)$
Else draw a sample h_k from
$h_k^{(i)} \propto \mathcal{L}(h; \kappa_k \beta h_{k-1}^{(i)}, \sqrt{(1-\alpha)}\beta)$
Update weights $w_k^{(i)}$
$\tilde{w}_k^{(i)} \propto w_{k-1}^{(i)} \mathcal{NL}(y_k; A h_k^{(i)} s_k^{(i)}, \sigma_v, \lambda_w)$
Normalize weights $w_k^{(i)} = \left(\sum_{n=1}^{N} \tilde{w}_k^{(n)}\right)^{-1} \tilde{w}_k^{(i)}$
Resample if $N_{eff} = \frac{1}{\Sigma_{i=1}^{N}\left(w_k^{(i)}\right)^2} < N/2$
$\hat{s}_k = \arg\max_{s_k} \left\{ \sum_{i=1}^{N} \delta(s_k^{(i)} - s_k) w_k^{(i)} \right\}$

Table 1. Algorithm for data detection in relay channels.

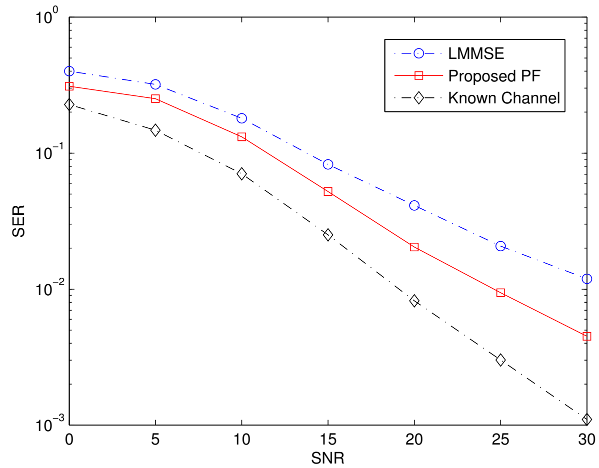

Fig. 1. SER as a function of SNR.

5. SIMULATION RESULTS

We conducted computer simulation experiments to verify the effectiveness our proposed algorithm. To combat the phase ambiguity problem that arises in blind equalization methods, in our experiments, we have used a differentially encoded BPSK modulation scheme with a symbol alphabet $\mathcal{A} = \{+1, -1\}$. We have considered the transmitter-to-relay channel to be a fixed-to-mobile flat-fading channel, and the relay-to-receiver channel to be mobile-to-mobile flat-fading channel. We assume both channels have the same fading rate $f_d T_s = 0.01$ by considering a TDMA protocol in which the transmission between transmitter-to-relay and relay-to-receiver use the same carrier frequency but different time-slots. The fading characteristics of the channels were modeled with a first-order Laplace AR process provided in section 2 with coefficient values $\alpha = 0.9990$ and $\beta = 0.9990$. The values of α and β were selected by matching the autocorrelation of the Laplace AR processes to the theoretical autocorrelation of the overall channel with the fading rate $f_d T = 0.01$.

We have used a particle size of $M = 100$ in all the experiments, and resampling was applied whenever the weights degenerated. The weights were determined degenerated when the effective sample size becomes less than half of the total particle size (M). The particle size was fixed by experimentation, and it was observed that increasing the particle size over $M = 100$ did not provided significant increase in performance.

In Figure 1, we have plotted the performance, Symbol-Error-Rate (SER) versus Signal-to-Noise Ratio (SNR), of a receiver that is based on our proposed algorithm. We have compared our algorithm with an LMMSE receiver that employs one pilot symbol interspersed every 10 data symbols for channel estimation. Moreover, we have simulated a receiver that assumes the channel known to serve as a lower bound benchmark for our algorithm. As seen from the figure, the receivers based on our proposed algorithm performed better than the LMMSE receiver.

6. CONCLUSIONS

In this paper, we have considered the problem of data detection in relay-based communication system in which an AF relay is used to transmit data over a flat-fading channel. We have developed a channel model which is a first-order Laplace AR process. With such channel model, we have designed a new algorithms for blind joint data detection and channel estimation based on Bayesian estimation of the transmitted symbols using particle filtering techniques. The effectiveness of the algorithm in terms of symbol error rate is demonstrated using computer simulations. The algorithm is also compared with an LMMSE receiver, and have been shown that it has superior performance.

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