SPECTRAL ESTIMATION OF PHASE NOISE BASED ON COMPLEMENTARY AUTOCORRELATION

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ABSTRACT

In this paper, we propose a novel estimation technique[1] of the power spectral density (PSD) of phase noise based on complementary autocorrelation (CAC). Utilizing the fact that the CAC of phase noise is nonzero, this technique offers a distinct advantage that the PSD of phase noise is estimated while canceling other circular-symmetric noises. This property is most useful when estimating phase noise directly from digitally-modulated signals because they typically are accompanied by various kinds of circular noises that easily mask the phase noise of interest. A practical method to compute the phase noise PSD is given. In addition, an expression for a noise suppression factor of the method is given. Numerical simulations demonstrate the effectiveness of the proposed method, and verifies the noise suppression factor.

Index Terms— phase noise, complementary autocorrelation, power spectral density, noise canceling

1. INTRODUCTION

Phase noise is random fluctuation in the phase of a signal, and is often presented in the frequency domain using its power spectral density (PSD). Since phase noise represents spectral purity and frequency stability of a signal, it plays a crucial role in evaluation of radio-frequency and microwave oscillators. A variety of methods are known for a long time [1][2].

In communication systems, phase noise degrades the performance through various mechanisms. Among today's digital modulation schemes, the impact of phase noise is more noticeable and serious in the OFDM systems. Since in OFDM many subcarriers are placed with a narrow frequency spacing, those subcarriers are susceptible to intercarrier interference (ICI) induced by phase noise [3], for which various compensation schemes are considered [4][5]. Thus, accurate estimation of phase noise in oscillators is extremely important when designing and characterizing OFDM systems.

It is convenient if oscillator phase noise can be estimated directly from modulated signals because, especially in small systems such as hand-held devices, oscillator outputs are typically not accessible. Indeed, such direct estimation methods are found in literature [6]. However, since modulated signals are typically accompanied by various other kinds of noises related to the generation, propagation and reception of them, the phase noise of oscillators is easily masked. This disables the use of the existing methods in noisy settings.

In this paper, to overcome the difficulty, we propose a novel phase noise estimation technique using the complementary autocorrelation (CAC). The CAC of circular-symmetric complex noise is zero; however, since the oscillator phase noise is a phase-domain signal, its CAC is nonzero. This property has been utilized to develop the new estimation technique. Under some mild conditions, the technique can approximately estimate the PSD of only phase noise while suppressing most (quasi) circular-symmetric noises that accompany the modulated signals, which include additive noises induced by linear distortion of paths, IQ imbalances, and additive spurious signals. To the best of the author's knowledge, this is the only estimation technique being capable of extracting and estimating the phase noise PSD.

The unmodulated signal case is treated first, then the result is extended onto the modulated case. A possible practical method is presented based on the complementary spectral density, for which a theoretical suppression factor of circular noises is presented. In addition, since not all modulation-induced noises are circular, reducibility of them are studied briefly. Numerical simulation demonstrates the proposed method, and verifies the noise suppression factor.

2. COMPLEMENTARY AUTOCORRELATION

Let a discrete-time complex random signal be denoted as $z(n)$. Assuming $z(n)$ is wide-sense stationary (WSS), the (ordinary) autocorrelation of $z(n)$ at the lag k is defined as

$$r_z(k) = E[z(n)z^*(n-k)], \tag{1}$$

where $E[\cdot]$ denotes the mathematical expectation. The complementary autocorrelation (CAC) of $z(n)$ is defined as

$$c_z(k) = E[z(n)z(n-k)], \tag{2}$$

[1]Patent pending (US)

which appears to be the non-conjugate version of $r_z(k)$. The CAC has the general property that, for a circular-symmetric signal, its CAC is zero, $c_z(k) = 0$, for all k. In addition, for a particular case when $z(n)$ is expressed as

$$z(n) = u(n)e^{j\varphi}, \tag{3}$$

where $u(n)$ is a real random process and $\varphi \in [0, 2\pi)$ is a known deterministic phase offset, from (1) and (2), the following equality holds:

$$c_z(k) = r_z(k)e^{j2\varphi}. \tag{4}$$

From this relation, we see that, for $z(n)$ having the form as in (3), the autocorrelation $r_z(k)$ can be obtained by calculating the CAC. This simple relation is the basis of the following developments in this paper.

3. SPECTRAL ESTIMATION OF PHASE NOISE

The unmodulated signal case is discussed first, then the result is extended onto the modulated case. For discussion in this section, phase noise is treated as a zero-mean, wide-sense stationary (WSS) process, to present the idea cleanly.

3.1. Unmodulated signal case

Letting the phase noise be denoted by $\theta(n) \in \mathbb{R}$, we consider an unmodulated signal $x(n) \in \mathbb{C}$ having the simple form:

$$x(n) = e^{j\theta(n)} + v(n), \tag{5}$$

where $v(n) \in \mathbb{C}$ is a circular-symmetric noise signal uncorrelated with $\theta(n)$. Assuming that $|\theta(n)|$ is sufficiently small, the first term in (5) can be approximately written as $j\theta(n)+1$. Thus, defining a new signal as $y(n) = x(n) - 1$, it follows that

$$y(n) \simeq j\theta(n) + v(n). \tag{6}$$

The CAC of $y(n)$ is calculated as

$$\begin{aligned} c_y(k) &= E[y(n)y(n-k)] \\ &\simeq -c_\theta(k) + c_v(k) \\ &= -r_\theta(k). \end{aligned} \tag{7}$$

To obtain the third equality, since $\theta(n)$ is real, the relation (4) is used (corresponding to the case of $\varphi = 0$), and the fact that $v(n)$ is circular-symmetric. We see from (7) that the CAC of $y(n)$ is approximately equal to the autocorrelation of $\theta(n)$ multiplied by -1, and the noise effects are canceled out.

Since the PSD is defined as the discrete-time Fourier transform of an autocorrelation, the PSD of the phase noise $\theta(n)$ can then be estimated approximately as,

$$P_\theta(\omega) \simeq -\sum_{k=-\infty}^{\infty} c_y(k)\, e^{-j\omega k}, \tag{8}$$

where the effects of $v(n)$ are completely suppressed.

The IEEE standard 1139 [7] recommends reporting the phase noise spectrum using the quantity defined as $\mathscr{L}(f)$. Letting the sampling rate be denoted by T, the relation between the two can be written as

$$\mathscr{L}(f) = \frac{1}{T} P_\theta(2\pi f T). \tag{9}$$

Also used frequently is the quantity known as the one-sided PSD of phase noise, customarily denoted as $S^I(f)$ [8], which relates to $P_\theta(\omega)$ as

$$S^I(f) = \frac{2}{T} P_\theta(2\pi f T). \tag{10}$$

3.2. Modulated signal case

We now discuss the modulated signal case in which the baseband symbols $s(n) \in \mathbb{C}$ are circular-symmetric. The following observation model at the baseband is considered:

$$x(n) = As(n)e^{j\theta(n)} + v(n), \tag{11}$$

where $A \neq 0$ denotes a complex gain. For an idealized coherent receiver, the symbols and the gain can be recovered exactly. Thus, we can treat $s(n)$ and A as known quantities. Under the assumption that $s(n) \neq 0$, we redefine the new signal $y(n)$ by removing modulation from $x(n)$ as

$$y(n) = \frac{x(n)}{As(n)} - 1. \tag{12}$$

Inserting (11) into (12) with the approximation $j\theta(n) + 1$ for sufficiently small $|\theta(n)|$ gives

$$y(n) \simeq j\theta(n) + v'(n), \tag{13}$$

where $v'(n) = v(n)/(As(n))$. Since $s(n)$ is assumed to be circular-symmetric, so is $v'(n)$. This enables the same treatment of (13) as the unmodulated case, that is, the phase noise PSD can be obtained by (8).

For the modulated signal case, there are several points that need special attention. The assumption $s(n) \neq 0$ is easily violated in practice (e.g., OFDM signals), which calls for a special treatment of those points. Furthermore, since the variance of $v'(n)$ is generally different from that of $v(n)$, noise enhancement can occur depending on the probability density distribution of $s(n)$. These two points pose us a performance trade-off between the noise enhancement and estimation bias, which is not detailed in this introductory conference paper.

3.3. Reducibility of modulation-induced noise

In modulated signals, while most modulation-induced noises are circular-symmetric, there exist others that are noncircular. Noncircular noises are irreducible by taking CAC. Thus, it is important to inspect reducibility of each kind of noise associated with the corresponding noise-inducing mechanism.

We first consider linear distortion. Linear distortion is modeled by a linear filter and, in wireless communication systems, accounts for nonideal path responses (amplitude nonflatness and phase nonlinearity), nonideal transmission filter (truncation of coefficients, etc.), timing misalignment in the receiver, and the multipath effects, to name a few. Under linear distortion, the observation, denoted as $x_{\mathrm{ld}}(n)$, can be written as

$$x_{\mathrm{ld}}(n) = A \sum_{l=0}^{\infty} h(l) s(n-l) e^{j\theta(n-l)}, \tag{14}$$

where $\{h(l)\}$ are the filter coefficients. Let $h(0) = 1$ then $y(n)$ in (12) takes the form

$$y(n) \simeq j\theta(n) + v_{\mathrm{ld}}(n), \tag{15}$$

where

$$v_{\mathrm{ld}}(n) = \sum_{l=1}^{\infty} h(l) \frac{s(n-l)}{s(n)} e^{j\theta(n-l)}. \tag{16}$$

For nonzero circular $\{s(n)\}$, $v_{\mathrm{ld}}(n)$ can be shown to be circular. Thus, we see that all the effects due to linear distortion can be canceled out by taking the CAC.

Similar analysis reveals that the effects of additive colored noise, additive line spectra (complex exponentials), and the IQ origin offset can be also canceled out. For IQ impairments including IQ gain imbalance and quadrature skew, another analysis shows that the reducibility depends on the modulation scheme. Nonlinear distortion can be shown to be irreducible in general.

4. PROPOSING A PRACTICAL METHOD

In practice, phase noise is not zero-mean or WSS though assumed so in Section 3, which disables direct use of the equations there. However, if a long sequence is segmented with the mean subtracted, the phase noise in each segment is approximately a zero-mean WSS process. This suggests a frequency-domain method based on the discrete Fourier transform (DFT) of each segment. Thus, the proposed method takes a basic form of the complementary version of Welch's method (Periodogram averaging over overlapped segments).

Based on the signal model (11), the modulation-removed version of $x(n)$ is obtained by

$$\tilde{x}(n) = x(n)/(As(n)). \tag{17}$$

Segmentation is done in such a way that its i-th, length-L segment is given by $\tilde{x}(l+iD), l = 0, \ldots, L-1$, where D is the inter-segment offset. The segment DFT of the phase noise is approximately obtained by

$$\Theta_i(k) \simeq \sum_{l=0}^{L-1} w(l) \left\{ \tilde{x}(l+iD) e^{-j\bar{\phi}_i} - 1 \right\} e^{-j\frac{2\pi}{L}kl}, \tag{18}$$

for $k = 0, \ldots, L-1$, where $\bar{\phi}_i$ is the phase average in the i-th segment defined by

$$\bar{\phi}_i = \frac{1}{L} \sum_{l=0}^{L-1} \arg\{\tilde{x}(l-iD)\}, \tag{19}$$

with $\arg\{\cdot\}$ representing the complex angle. Window $w(l)$ is optional. The phase noise PSD is approximately obtained by averaging the complementary spectral density over all segments, as

$$P_\theta(k) \simeq \frac{1}{KLU} \left| \sum_{i=0}^{K-1} \Theta_i(k) \Theta_i(L-k) \right|, \tag{20}$$

where K is the number of segments, and

$$U = \frac{1}{L} \sum_{l=0}^{L-1} w^2(l). \tag{21}$$

The suppression factor of the additive circular-symmetric noise by the above-proposed method has been derived assuming the Gaussian distribution. Just to show the result, it is

$$F_S = 2\sqrt{K/\pi}, \tag{22}$$

for the case of $D = L$ (0% overlapping).

5. SIMULATIONS

Estimation of the phase noise PSD and suppression of other modulation-induced noises using the proposed method are demonstrated by numerical simulations for two different kind of modulation schemes, 8PSK and Gaussian. For the both schemes, $N = 10^5$ symbols $s(n), n = 1, \ldots, N$ are generated, which are then corrupted by phase noise $\theta(n)$ and the circular noise $v(n)$ to construct the observations $x(n)$ as in (11). To generate $v(n)$, we consider such impairments as the IQ gain imbalance, IQ quadrature skew, IQ origin offset, linear distortion (nonflat gain and nonlinear phase), and AWGN. Each of these factors are balanced so that they all have approximately equal contribution to the total noise power. The signal $\tilde{x}(n)$ in (17) is segmented into $L = 500$ unoverlapped segments, thus, there are $K = N/L = 200$ segments, for which the suppression factor becomes $F_S = 15.96$ or $10 \log F_S = 12.03$ (dB).

Fig. 1 shows the results with an 8PSK-modulated signal. The bottom trace is the PSD of the true phase noise with a line spectrum at the frequency of 0.25 (normalized to the sampling rate). The overall PSD, the top trace, includes contributions of both the phase noise and the additive modulation-induced noise $v(n)$. Our task is to extract and estimate only the phase noise PSD by using the proposed method. The resultant estimate of phase noise PSD is the middle trace. The true phase noise PSD is recovered well in frequencies up to

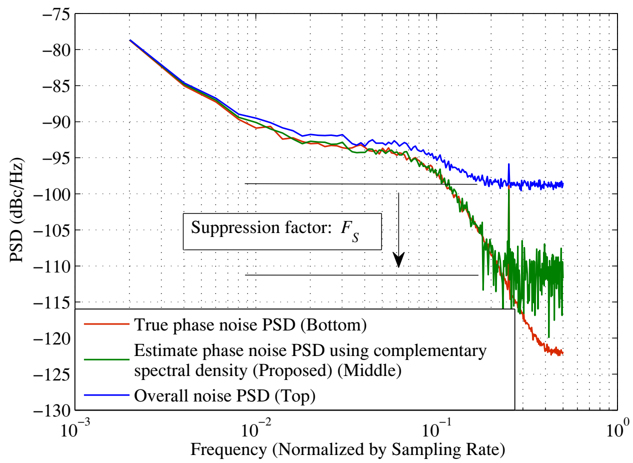

Fig. 1. Estimation of phase noise PSD of an 8PSK-modulated signal by suppressing other circular-symmetric noises.

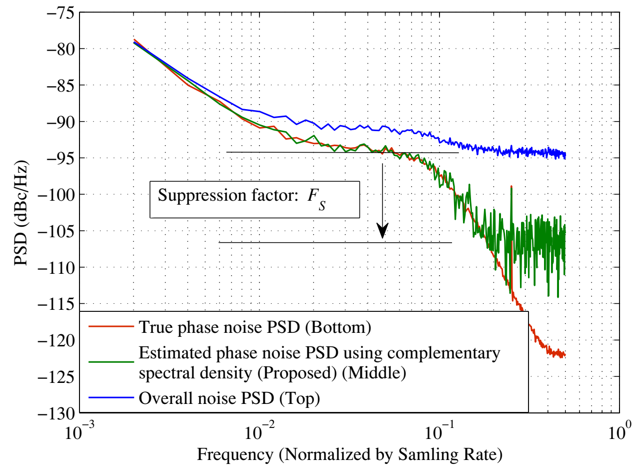

Fig. 2. Estimation of phase noise PSD of a Gaussian-modulated signal by suppressing other circular noises.

0.2 or so. In addition, the amount of circular noise suppression seems to roughly agree with the theoretical suppression factor, $F_S = 12.03$ (dB), because for this simulation $v(n)$ is composed of only circular noises and is expected to follow the theory. The line spectrum successfully remains, and it now can be observed much more cleanly.

The results with a Gaussian-modulated signal are shown in Fig. 2. The Gaussian modulation generally represents well a CDMA signal with more-than-a-few active channels and an OFDM signal. Here it must be noted that the noise enhancement does occur in the calculation $v'(n) = v(n)/s(n)$ as mentioned in Section 3.2 because, in a Gaussian modulation, many symbols are located near the origin causing division by near-zero values. To get around the issue, in this simulation, for points $\{s(n) \mid |s(n)| < 0.1\sigma_s^2, \sigma_s^2 = E[|s(n)|^2\}$, the corresponding $\{v'(n)\}$ are generated not by the division, but by interpolating nearby points. The amount of noise enhancement by this relief scheme can be roughly estimated at 5dB by visually inspecting two top traces in Fig. 1 and 2 at frequencies > 0.3.

6. CONCLUSIONS

Estimation of the PSD of phase noise based on the complementary autocorrelation is presented. It offers the distinct advantage to extract phase noise while suppressing circular-symmetric noises induced by modulated signals. A practical method is presented based on the complementary spectral density with a theoretical expression of suppression factor. Numerical simulations demonstrate the idea, and confirms the noise suppression factor.

7. REFERENCES

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