GOLAY SEQUENCE FOR PARITAL FOURIER AND HADAMARD COMPRESSIVE IMAGING

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ABSTRACT

This paper introduces Golay sequence for partial Fourier and Hadamard compressive imaging. In the proposed system, the signal is pre-modulated by a binary Golay sequence before applying a random subsampled Fourier or Hadamard transform. The correpsonding sampling operator has been proved to be incoherent with the (block) DCT or the Haar wavelet transform. Empirical results show that they are also incoherent with the Daubechies wavelets. It is well known that natural images are sparse in the DCT and the wavelet basis. Hence, the proposed sampling operators are promising in many Fourier or Hadamard compressive imaging applications. In fact, they can achieve near-optimal reconstruction performance with small memory requirement and simple hardware implementation. Some simulation results and proof-of-concept experimental results are included to demonstrate the validity of the theory and the potential of the proposed sampling operators.

***Index Terms*—** Compressed sensing, Fourier transform, DCT, Hadamard matrix, Golay sequence.

1. INTRODUCTION

Over the past few years, there have been increased interests in the study of compressive imaging, in which the total number of measurements is much smaller than that of pixels in the reconstructed image [1, 2]. These systems hold great potential for dramatic reduction of sampling rates, imaging time, power consumption and computational complexity for applications such as magnetic resonance imaging (MRI) [3] and shortwave infrared imaging [4] etc. Consider an image with N-pixels in total and let $\mathbf{x}$ represent its vector version. The non-adaptive measurement process can be described as [5, 6]

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{e}, \quad (1)$$

in which $\mathbf{y}$ represents an $M \times 1$ ($M < N$) sampled vector, $\mathbf{\Phi}$ is an $M \times N$ measurement/sensing matrix, and $\mathbf{e}$ is a noise vector. The reconstruction of $\mathbf{x}$ from $\mathbf{y}$ replies on the assumption that $\mathbf{x}$ has a sparse representation under a certain transform $\mathbf{\Psi}$, i.e., $\mathbf{f} = \mathbf{\Psi}\mathbf{x}$ can be approximated by $K \ll N$ coefficients [5, 6]. For natural images, popular choices of Ψ include the DCT and the wavelet transform. It is known that stable recovery of $\mathbf{x}$ can be achieved if Φ is incoherent with Ψ. In particular, when $\mathbf{\Phi}$ is a full random Gaussian or Bernoulli operator, it is incoherent with any $\mathbf{\Psi}$. Thus, $\mathbf{x}$ can be recovered from $\mathcal{O}(K \log(N/K))$ measurements stably.

For large-scale compressive imaging applications, fast computable Φ is often preferred. Due to wide applications of Fourier and Hadamard imaging systems, many people have investigated the use of partial FFT or the Walsh-Hadamard transform (WHT). Although these operators work well for signals in the canonical basis (i.e., $\mathbf{\Psi} = \mathbf{I}$), they lack universality. To address this issue, randomized partial FFT or WHT have been developed [7, 8], in which the signal is pre-processed through either random permuatation or random sign flipping.

In this paper, we propose to use deterministic binary Golay sequence to pre-modulate the signal before applying partial FFT or WHT. It can be viewed as a derandomized version of the random FFT or WHT operators. By exploiting the spectral property of Golay complementary sequence, we show that the resulting sampling operators are maximally incoherent with the (block) DCT transform and the Haar transform. Numerical results also suggest that they are incoherent with the Daubechies wavelet. As most natural images can be sparsified by the DCT and the wavelet, the proposed system holds great potential in high resolution compressive imaging applications such as MRI [3] and single pixel camera based on digital micromirror device (DMD) [2]. It can be shown that $M \geq \mathcal{O}(K \log^4 N)$ measurements are required for uniform sparse reconstruction, while only $M \geq \mathcal{O}(K \log N)$ samples are needed for non-uniform reconstruction. Some experimental results are included to demonstrate the potential of the proposed system. The rest of the paper is organized as follows. In Section II, we briefly review compressed sensing systems using partial (randomized) unitary measurement matrices. In Section III, we propose Golay sequence for Fourier and Hadamard imaging. We also present analytical coherence bounds between the proposed systems and the DCT or the wavelet transforms. Experimental results are shown in Section IV, followed by conclusions in Section V.

Notations: Throughout this paper, vectors are denoted by boldfaced lowercase letters and matrices by boldfaced uppercase characters. If their sizes are not clear from the context, subscripts are provided. $\mathbf{I}$, $\mathbf{F}$ and $\mathbf{C}$ represent the identity matrix, the normalized FFT matrix and the Type-II DCT matrix, respectively. $\mathbf{A}^T$ represents the transpose of $\mathbf{A}$. For an $N \times N$ matrix $\mathbf{A}$, let $\mu(\mathbf{A})$ denote its coherence parameter [9], i.e., the maximum magnitude of its elements,

$$\mu(\mathbf{A}) = \max_{0 \leq p,q \leq N-1} |\mathbf{A}(p, q)|.$$

For two $N \times N$ matrices $\mathbf{A}$ and $\mathbf{B}$, their mutual coherence $\mu(\mathbf{A}, \mathbf{B})$ is defined as

$$\mu(\mathbf{A}, \mathbf{B}) = \mu(\mathbf{AB}) = \max_{0 \leq p,q \leq N-1} |\mathbf{A}(p, :)\mathbf{B}(:, q)|,$$

where $\mathbf{A}(p, :)$ and $\mathbf{B}(:, q)$ correspond to the p-th row of $\mathbf{A}$ and q-th column of $\mathbf{B}$, respectively.

2. REVIEW

In this section, we provide an overview of subsampled unitary matrix for compressed sensing, in which $\mathbf{\Phi}$ takes the following form

$$\mathbf{\Phi} = \frac{1}{\sqrt{M}}\mathbf{R}\mathbf{U}, \quad (2)$$

where $\mathbf{R}$ is a random sampling operator which selects M samples out of N ones uniformly at random, $\mathbf{U}$ is an $N \times N$ fast-computable (random or deterministic) unitary matrix satisfying $\mathbf{U}^*\mathbf{U} = N\mathbf{I}_N$. As mentioned before, to reconstruct the signal from sparse optimisation algorithms, $\mathbf{\Phi}$ needs to be incoherent with $\mathbf{\Psi}^T$. Specifically, its reconstruction performance depends on $\mu(\mathbf{U}\mathbf{\Psi}^T)$. It is well known that when Ψ is an orthonormal matrix, *all* K-sparse signals can be recovered uniformly from l_1-based optimisation provided that [10] $M \geq \mathcal{O}(\mu^2(\mathbf{U}\mathbf{\Psi}^T)K\log^4 N)$. In addition to l_1-based optimisation, one can also use some iterative reconstruction methods such as orthogonal matching pursuit [11], CoSaMP [12] and their variants. While for a given signal, $\mathbf{x}$ can be reconstructed using l_1-based optimisation if M satisfies [9] $M \geq \mathcal{O}(\mu^2(\mathbf{U}\mathbf{\Psi}^T)K\log N)$. This is also called as the bound for *non-uniform* reconstruction.

It is obvious that if Ψ is an orthonormal matrix, then

$$1 \leq \mu(\mathbf{U}\mathbf{\Psi}^T) \leq \sqrt{N}.$$

We say that $\mathbf{U}$ and Ψ are *maximally incoherent* if $\mu(\mathbf{U}\mathbf{\Psi}^T) = \mathcal{O}(1)$. Note that if the signal is sparse in the time or spatial-domain (i.e., when $\Psi = \mathbf{I}_N$), we can achieve the optimal coherence bound of 1 if $\mathbf{U}$ is chosen as the FFT or the WHT matrix. But most natural images are sparse in the DCT or the wavelet domain, and the FFT (or the WHT) is coherent with these transforms. To address this issue, randomized FFT or WHT has been proposed [7, 8], in which $\mathbf{U}$ can be written as

$$\mathbf{U} = \mathbf{T}\mathbf{D}, \tag{3}$$

where $\mathbf{T}$ is an $N \times N$ normalized FFT or WHT matrix, $\mathbf{D}$ is either a random permutation operator or a random sign flipping operator. For any orthonormal basis Ψ, $\mu(\mathbf{T}\mathbf{D}\Psi^T)$ satisfies

$$\mu(\mathbf{T}\mathbf{D}\mathbf{\Psi}^T) \leq \mathcal{O}(\sqrt{\log N})$$

with high probability. Note that the above statistical coherence bound is sub-optimal.

In this paper, we propose to construct the diagonal matrix $\mathbf{D}$ from the Golay sequence. By using a deterministic sequence, it is more memory efficient, which simplifies the hardware design. As a trade-off, it lacks universality. But we shall prove that the resulting sampling operator is maximally incoherent with the (block) DCT and the Haar transform. Hence, it could be used in practice for compressive sampling of natural images.

3. GOLAY SEQUENCE FOR COMPRESSIVE IMAGING

3.1. Golay Complementary Sequence

As Golay sequence holds the key for our development, we first provide a brief introduction of Golay's complementary pair (GCP), Golay sequence and their constructions [13, 14].

Definition 1. *Consider a pair of length-N bipolar sequences* $\mathbf{a} = \big[a(0), a(1), \cdots, a(N-1)\big]$ *and* $\mathbf{b} = \big[b(0), b(1), \cdots \; b(N-1)\big]$. *Let the aperiodic correlation of a length-N sequence* $\mathbf{s}$ *be defined as*

$$r_{\mathbf{s}}(l) = \sum_{k=0}^{N-l-1} s(k)s(k+l). \tag{4}$$

$\mathbf{a}$ *and* $\mathbf{b}$ *are said to be a* Golay complementary pair *[13] if*

$$r_{\mathbf{a}}(l) + r_{\mathbf{b}}(l) = 0, \quad 1 \leq l \leq N-1. \tag{5}$$

$\mathbf{a}$ *(or* $\mathbf{b}$*) is called as a* Golay sequence.

Note that (5) suggests that the Golay sequence is nearly flat in the spectral domain. To see this, define two polynomials $A(z) = \sum_{n=0}^{N-1} a(n)z^n$ and $B(z) = \sum_{n=0}^{N-1} b(n)z^n$. From (5), it can be shown that [13]

$$|A(z)|^2 + |B(z)|^2 = 2N, \quad \text{for all } |z| = 1, \tag{6}$$

which implies that

$$|A(z)| < \sqrt{2N} \quad \text{and} \quad |B(z)| < \sqrt{2N} \quad \text{for all } |z| = 1. \tag{7}$$

In the next subsection, we will use this property to derive the coherence bound.

When $N = 2^n$, Golay [13, 14] proposed a method for explicit construction by using algebraic normal forms (ANF). For an integer i ($0 \leq i \leq 2^n - 1$), let $(i_0, i_1, \cdots, i_{n-1})$ denote the binary representation of i, that is, $i = \sum_{l=0}^{n-1} i_l 2^l$. Define two boolean functions $f_a(i)$ and $f_b(i)$ as follows

$$f_a(i) = \sum_{l=0}^{n-2} i_{\pi(l)} i_{\pi(l+1)} + \sum_{l=0}^{n-1} c_l i_l + c \tag{8}$$

$$f_b(i) = f_a(i) + i_{\pi(0)} + c' \tag{9}$$

in which π represents any permutation of $\{0, 1, \cdots, n-1\}$, and c_j ($j = 0, \cdots, n-1$), c and c' are any choice of constant in $\mathbb{Z}_2$. Then, a length-N ($N = 2^n$) GCP can be given as [13]

$$a(i) = (-1)^{f_a(i)}, \quad b(i) = (-1)^{f_b(i)} \tag{10}$$

Using the explicit construction, one can obtain $n!2^n$ Golay sequences [14]. In fact, all the currently known binary Golay sequences of length $N = 2^n$ can be constructed in this way. Note that in (8), when $\pi(i) = i$, $c_l = 0$ ($0 \leq l \leq n-1$) and $c = 0$, sequence $\mathbf{a}$ boils down to the famous Golay-Rudin-Shapiro sequence [13]. To generate a length-$2N$ GCP from a length-N GCP, one can use the following Golay-Rudin-Shapiro recursion formula [13]

$$(\mathbf{a}, \mathbf{b}) \to (\mathbf{a}|\mathbf{b}, \mathbf{a}| - \mathbf{b}), \tag{11}$$

where $'|'$ means concatenation, and $\mathbf{a}$ and $\mathbf{b}$ can be initialized as ± 1.

3.2. Coherence Analysis

In this subsection, we investigate an $M \times N$ ($N = 2^n$) sampling operator $\mathbf{\Phi}$ with the following form

$$\mathbf{\Phi} = \frac{1}{\sqrt{M}}\mathbf{R}\mathbf{T}\mathbf{D}, \tag{12}$$

in which $\mathbf{R}$ is the same as given in (2), $\mathbf{T}$ is an $N \times N$ normalized FFT ($\mathbf{T} = \mathbf{F}$) or the WHT ($\mathbf{T} = \mathbf{W}$) matrix, and $\mathbf{D} = \text{diag}(\mathbf{d})$ is a diagonal matrix whose main diagonal $\mathbf{d}$ is a length-N Golay sequence. As mentioned before, the performance of $\mathbf{\Phi}$ in compressed sensing depends on the coherence parameter $\mu(\mathbf{T}\mathbf{D}\Psi^T)$. In what follows, we will focus on the case when $\mathbf{\Psi}$ is a (block) Type-II DCT or the Haar wavelet, both of which are well-known sparsifying transforms for natural images.

The following Lemma presents the coherence bound when Ψ is the Type-II DCT.

Lemma 1. *Consider an $M \times N$ ($N = 2^n$) sampling operator Φ given by (12). Let $\mathbf{C}$ represent an $N \times N$ Type-II DCT transform. For any existing Golay-sequence $\mathbf{d}$ constructed from (8)-(10), we have*

$$\mu(\mathbf{T}\mathbf{D}\mathbf{C}^T) \leq 2. \tag{13}$$

Proof. When $\mathbf{T}$ is the FFT matrix ($\mathbf{T} = \mathbf{F}$), the proof can be obtained from (7) through simple math manipulations. When $\mathbf{T}$ is the WHT matrix ($\mathbf{T} = \mathbf{W}$), let us consider the matrix $\mathbf{P} = \mathbf{WD}$. Note that for a WHT, its element $\mathbf{W}(j, i)$ can be written as

$$\mathbf{W}(j, i) = (-1)^{\sum_{l=0}^{n-1} j_l i_l}, \tag{14}$$

where $(j_0, j_1, , \cdots, j_{n-1})$ and $(i_0, i_1, \cdots, i_{n-1})$ denote the binary representation of j and i, respectively. It is clear that $\mathbf{P}(j, i) = \mathbf{W}(j, i)d(i)$, where $d(i)$ is the i-th element of a Golay-sequence $\mathbf{d}$ with the form of (8). Hence, we have

$$\mathbf{P}(j, i) = (-1)^{g(i)}, \tag{15}$$

$$g(i) = \sum_{l=0}^{n-2} i_{\pi(l)} i_{\pi(l+1)} + \sum_{l=0}^{n-1} \hat{c}_l i_l + c, \tag{16}$$

in which $\hat{c}_l = (c_l + j_l) \bmod 2$. Note that $g(i)$ in (16) takes the same form as (8). As a result, each row $\mathbf{P}(j, :)$ forms a length $N = 2^n$ Golay sequence. The coherence bound can then be easily derived by exploiting (7). □

Note that Lemma 1 implies that we can use any existing Golay sequence when the sparsifying transform is chosen as the DCT. As mentioned before, when $N = 2^n$, there are $2^n n!$ Golay sequences. How to select the best Golay sequence for practical imaging systems will be left for our future work.

We now move on to consider the case when Ψ is the block DCT or the Haar wavelet transform. Let $\hat{\mathbf{C}}$ represent a block diagonal DCT matrix as follows

$$\hat{\mathbf{C}} = \text{diag}\left(\mathbf{C}_L, \mathbf{C}_L, \cdots, \mathbf{C}_L\right), \tag{17}$$

in which $\mathbf{C}_L$ ($L = 2^l$) is the $L \times L$ Type-II DCT matrix. An $N \times N$ Haar wavelet transform matrix $\mathbf{H}_N$ can be constructed iteratively

$$\mathbf{H}_N = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{H}_{N/2} \otimes \begin{bmatrix} 1 & 1 \end{bmatrix} \\ \mathbf{I}_{N/2} \otimes \begin{bmatrix} 1 & -1 \end{bmatrix} \end{bmatrix}, \tag{18}$$

in which H_1 is initialized as $H_1 = 1$. Then, the following lemma holds

Lemma 2. *Consider an $M \times N$ sampling operator $\mathbf{\Phi}$ in (12). Suppose that the main diagonal of $\mathbf{D}$ is a Golay sequence constructed from (8) with $\pi(i) = i$. Then, for the block DCT $\hat{\mathbf{C}}$ in (17) and the Haar wavelet transform $\mathbf{H}$ in (18),*

$$\mu(\mathbf{TD}\hat{\mathbf{C}}^T) \leq 2 \tag{19}$$

$$\mu(\mathbf{TDH}^T) \leq \sqrt{2} \tag{20}$$

Proof of the above lemma will be given in the journal version of this paper. As a quick check, Table 1 lists the coherence bounds for $N = 256$ through numerical calculations. One can see that when $\mathbf{\Psi}$ is the (block) DCT or the Haar wavelet, these numerical solutions agree very well with our analytical results. Although currently, we couldn't derive the analytical coherence bounds for other transforms, numerical simulations in Table 1 suggest that $\mathbf{TD}$ is also maximally incoherent with the Daubechies wavelet. It should be noted that for block DCT transform, Lemma 2 suggests that the coherence bound is independant of the block size of the DCT. In fact, the above result can be easily extended to $\mathbf{\Psi}$ with variable-length of DCT [15], which has been used for H.264/AVC as a sparsifying transform. In addition, although our discussions are here on 1D signals, the results can be easily generalized to multidimensional signals. It should be pointed out that unlike Lemma 1, Lemma 2 requires that $\mathbf{d}$ is constructed from a subset of existing Golay sequences with $\pi(i) = i$ in (8). There are only $N = 2^n$ such sequence. At this stage, we are not sure whether the above result can be generalized to any other Golay sequences.

Table 1. Coherence value $\mu(\mathbf{TD\Psi}^T)$ for $N = 256$, where the main diagonal of $\mathbf{D}$ is a length-256 Golay sequence

$\mathbf{\Psi}$	$\mathbf{T}$	
	F	**W**
DCT	1.9875	1.9975
Block DCT8	1.8450	1.8123
Haar	1.4142	1.4142
Daubechies 4	2.3998	2.4430
Daubechies 8	2.3042	2.1554

Up until now, we have shown that $\mathbf{TD}$ is maximally incoherent with the Type-II DCT and the wavelet. From the previous discussion, we know that for the proposed system, $M \geq K \log^4 N$ and $M \geq K \log N$ incomplete Fourier (or Hadamard) measurements are required respectively, for uniform and non-uniform reconstruction using l_1-based optimisation. Furthermore, recent work [16] has shown that if the sampling operator is incoherent with the Haar transform, one can also reconstruct the signal stably using total-variation minimization algorithm with $M \geq \mathcal{O}(K \log N)$ measurements. This suggests the great potential of Golay sequence in compressive imaging applications.

Connections with existing work Note that deterministic sequence has also been investigated for partial Fourier and Hadamard imaging in the previous work. In [17], chirp sequence has been used for MRI with $\mathbf{T} = \mathbf{F}$. The coherence bound is obtained through empirical calculations. In our previous work [18], we have proposed to use the Rudio-Shapiro sequence for Hadamard imaging ($\mathbf{T} = \mathbf{W}$), in which we have derived the analytical coherence bound when Ψ is the DCT or the FFT. Here, the result is extended to a general Golay sequence with length of $N = 2^n$ and to the case when the sparsifying transform $\mathbf{\Psi}$ is the block DCT and the Haar wavelet. Furthermore, we have demonstrated that Golay sequence can be also used for Fourier imaging.

4. EXPERIMENTAL RESULTS

In this section, we present experimental results of the proposed sampling operators. Numerous computer simulations and proof-of-concept hardware measurements have been carried out. Due to lack of space, only two examples are presented.

4.1. Simulation Results

In this subsection, we present the simulation results for the application of Golay-sequence in compressive Fourier imaging. The test image is the 512×512 image *Boat*. The sparsifying transform is chosen as the Daubechies 4 wavelet and the GPSR [19] software package has been used. For comparison purpose, we also present the results of partial randomized Fourier transform, in which the main diagonal of $\mathbf{D}$ is constructed from a Bernoulli sequence. Fig. 1 presents the peak signal to noise ratios of reconstructed images. As can be seen here, by replacing a full random Bernoulli sequence with the proposed Golay sequence, there is almost no performance degradation.

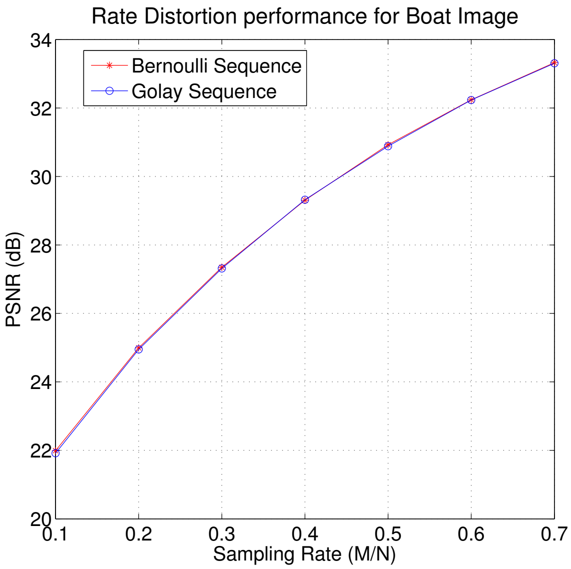

Fig. 1. Rate distortion performance for the boat image.

4.2. Measurement results

This experiment is to investigate the potential application of Golay sequence in Hadamard compressive imaging system. Such operators can be implemented in DMD-based single-pixel camera [2]. Here, we implemented a proof-of-concept measurement system as shown in Fig. 2. A computer monitor is used as the light source. It sequentially displays the 2D binary masks specified by each row of Φ, where "1" and "-1" correspond to "on" and "off", respectively. The light from the screen was focused by a set of lens onto the detector after going through the sample. The photo diode is used as a detector to measure the intensity of the light. Data acquisition (DAQ) card (National Instrument USB-6221) is used to collect the signal recorded from detector and send it to PC for image reconstruction. Just as in [2], to get measurement results of Φ with 1 and -1, each measured sample was subtracted by the mean of the light intensity, which is obtained by setting the whole monitor as on. The window on the screen was of 512 by 512 pixels physically. Images are then then reconstructed using the NESTA [20] package. Fig. 3 shows some reconstructed results of a 128 × 128 image *clock* when the sampling rate is 7.5% (left), 15% (middle) and 30% (right), respectively. This experiment confirms the robustness of the of the proposed system in the noisy measurement. It suggests the proposed sampling operators are very attractive for DMD-based single pixel camera in applications such as short wave infrared imaging [4].

5. CONCLUSIONS

Partial Fourier and Hadamard imaging using compressed sensing technique have attracted great research interests recently. In this paper, we have proposed to use Golay sequence for such systems. In particular, the signal of interest is first multiplied by the Golay sequence before applying partial Fourier or Hadamard transform in compressed imaging. The proposed system features simple design and hard-ware friendly implementation along with fast computation in reconstruction. We have shown that the corresponding sampling operators are maximally incoherent with the (block) DCT and the wavelet transform. As a result, they provide a near-optimal bound for l_1-based optimisation and total variation minimization. Some simulation and proof-of-concept experimental results have been presented to demonstrate the great potential of the proposed system.

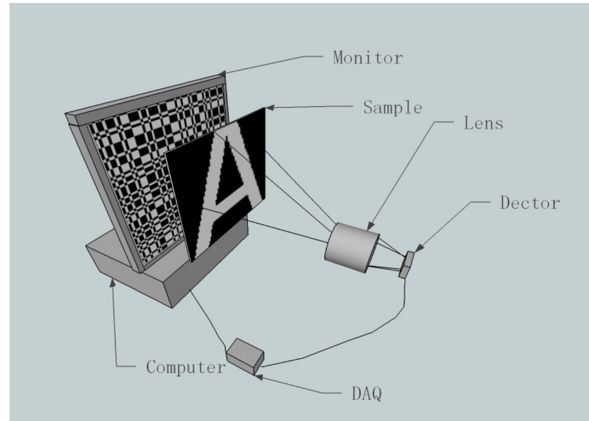

Fig. 2. Schematic diagram for the proof-of-concept measurement.

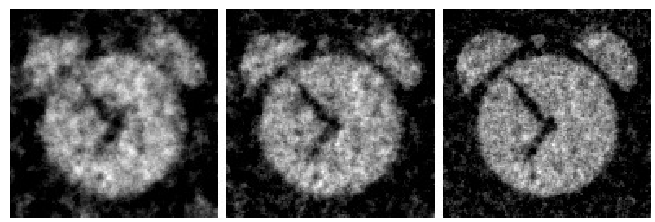

Fig. 3. Reconstructed 128 × 128 images of *clock* from proof-of-concept experiment in Fig. 2. Left: $M/N = 7.5\%$; middle $M/N = 15\%$ and right: $M/N = 30\%$.

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