CORRELATION-AWARE SPARSE SUPPORT RECOVERY: GAUSSIAN SOURCES

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ABSTRACT

Consider a multiple measurement vector (MMV) model given by $\mathbf{y}[n] = \mathbf{A}\mathbf{x}_s[n], \quad 1 \leq n \leq L$ where $\{\mathbf{y}[n]\}_{n=1}^{L}$ denote the L measurement vectors, $\mathbf{A} \in \mathcal{R}^{M\times N}$ is the measurement matrix and $\mathbf{x}_s[n] \in \mathcal{R}^N$ are the unknown vectors with same sparsity support denoted by the set S_0 with $|S_0| = D$. It has been shown in a recent paper by the authors that when the elements of $\mathbf{x}_s[n]$ are uncorrelated from each other, one can recover sparsity levels as high as $O(M^2)$ for suitably designed measurement matrix. The recovery is exact when support recovery algorithms are applied on the ideal correlation matrix. When we only have estimates of the correlation, it is still possible to probabilistically argue the recovery of sparsity levels (using a coherence based argument) that is much higher than that guaranteed by existing coherence based results. However the lower bound on the probability of success is found to increase rather slowly with L (as $1-C/L$ for some constant $C > 0$) without any further assumption on the distribution of the source vectors. In this paper, we demonstrate that when the source vectors belong to a Gaussian distribution with diagonal covariance matrix, it is possible to guarantee the recovery of original support with overwhelming probability. We also provide numerical simulations to demonstrate the effectiveness of the proposed strategy by comparing it with other popular MMV based methods.

Index Terms **—** Support Recovery, LASSO, Block Sparsity, Multiple Measurement Vector (MMV), Correlation.

1. INTRODUCTION

Joint sparsity recovery, or block sparsity recovery from multiple measurement vectors (MMV) is an active area of research [2,4,5,9] that seeks to recover the common support shared by a set of sparse signal vectors. Recently, the role of statistical correlation among the multiple measurement vectors in recovery of their common sparsity support has been investigated, possibly for the first time, in [1]. In our recent paper [10], we also proposed a correlation aware approach to sparse support recovery in MMV problems. However, while the work in [1] considers the case where the vectors actually have temporal correlation and proposes algorithms to learn the correlation by sparse Bayesian learning techniques, we actually consider the case when the non zero entries of the source vectors $\mathbf{x}_s[l]$ have *no correlation* among themselves. In [10], it was shown that for an uncorrelated signal model, it is possible to recover the support for much higher levels of sparsity by using the ideal correlation matrix. On the other hand, when we only have estimates of the correlation (from the multiple measurement vectors), one can probabilistically argue the recoverability of higher sparsity levels [11] where the probability $\rightarrow 1$ as $L \rightarrow \infty$. However, in [11], no specific assumption regarding the distribution of the source vectors was made, and so the lower bound on the probability of support recovery increased rather slowly with L (as $1 - C/L$ for constant $C > 0$). In this paper, we show that when the source vectors are assumed to belong to a multivariate Gaussian distribution with a diagonal covariance matrix, it is possible to guarantee support recovery with probability that increases as $1 - \beta^{-L}, \beta > 1$. Using a coherence based argument, it is thereby shown that one can recover much higher levels of sparsity (than that guaranteed by existing coherence based bounds) using the proposed technique, with overwhelming probability as L increases. It is to be noted that unlike usual probabilistic results in literature which compute the probability over an ensemble of measurement matrices, in our formulation, the measurement matrix is fixed and the probability is taken over the ensemble of source vectors.

In [3], a very similar problem of joint sparse recovery in the MMV model has been considered. However, the main difference between this work and [3] is that the size of the non zero indices (say D) guaranteed to be recovered by the approach in [3] is restricted by $D = O(M)$. However, we have shown [10] that under the assumption of uncorrelated sources, one can fundamentally overcome this limitation and recover sparsity levels as large as $D = O(M^2)$. In this paper, we perform analysis of the framework proposed in [10] for case of finite snapshots. As it is discussed later, this analysis can also apply to the case when there is "partial correlation" among the sources (instead of them being completely uncorrelated).

Notations: Matrices are denoted by boldface uppercase symbols (such as $\mathbf{A}$) and vectors are denoted by boldface lowercase symbols (such as $\mathbf{a}$). The (m, n) th element of a matrix $\mathbf{A}$ is denotes $\mathbf{A}_{m,n}$ while ith element of the vector $\mathbf{a}$ is denoted as $[\mathbf{a}]_i$. The $N \times 1$ vector of all 1s is denoted as $\mathbf{1}_{N\times 1}$. The symbol $\mathbf{A} \odot \mathbf{B}$ denotes the Khatri-Rao product (column-wise Kronecker product) between matrices $\mathbf{A}, \mathbf{B}$ with same number of columns.

2. REVIEW OF CORRELATION AWARE LASSO

In a typical MMV problem, $\mathbf{y}[l] \in \mathcal{R}^{M\times 1}, 1 \leq l \leq L$ denotes a set of L multiple measurement vectors, $\mathbf{A} \in \mathcal{R}^{M\times N} = [\mathbf{a}_1 \quad \mathbf{a}_2 \cdots \mathbf{a}_N]$ is the measurement matrix, and $\mathbf{x}_s[l] \in \mathcal{R}^{N\times 1}, 1 \leq l \leq L$ are L unknown vectors with the same sparsity $||\mathbf{x}_s[l]||_0 = D$ and common support denoted by the set $S_0 = \{i_0, i_1, \cdots, i_{D-1}\}$. We have

$$\mathbf{y}[l] = \mathbf{A}\mathbf{x}_s[l], \quad 1 \leq l \leq L. \tag{1}$$

We introduce the following assumptions and notations:

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A1 The sub vectors consisting of non zero entries of $\mathbf{x}_s[l]$ are i.i.d Gaussian random vectors with a diagonal correlation matrix $\mathbf{\Lambda}$ with σ_i^2 denoting the ith diagonal element.

A2 $N > M^2$.

The goal is to recover the support S_0 using these ML measurements and determine what is the maximum D upto which all sparsity supports with cardinality D can be recovered. In [10], it was demonstrated that with the perfect knowledge of the autocorrelation matrix of the vectors $\mathbf{y}[l]$, one can recover sparsity supports of much higher cardinality by applying Basis Pursuit (BP) on an appropriate model. The recoverable sparsity can be as high as $O(M^2)$ for suitably designed physical measurement matrices, which give rise to Vandermonde matrices in an appropriate model derived from the autocorrelation matrix [10]. Without such correlation awareness, one could only recover sparsity supports upto [5] $D < \frac{\text{rank}(\mathbf{X})+\text{Spark}(\mathbf{A})-1}{2}$ which gives a maximum of $D < M - 1$. Most existing approaches [1], [3] to joint support recovery in MMV problems can therefore only recover supports of size $D = O(M)$. In practice, we do not have ideal knowledge of the correlation matrix and only have an estimate of the sample correlation matrix. The estimated sample correlation matrix is given by $\hat{\mathbf{R}}_{\mathbf{yy}}^{[L]} \triangleq \frac{1}{L}\sum_{l=1}^{L}\mathbf{y}[l]\mathbf{y}[l]^T = \mathbf{A}\hat{\mathbf{R}}_{xx}^{[L]}\mathbf{A}^T$ where $\hat{\mathbf{R}}_{xx}^{[L]} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}_s[l]\mathbf{x}_s[l]^T$ On vectorization, we can write

$$\mathbf{z}^{[L]} \triangleq \text{vec}(\mathbf{R}_{yy}^{[L]}) = (\mathbf{A}\odot\mathbf{A})\mathbf{r}_{xx}^{[L]} + \mathbf{e}^{[L]} \quad (2)$$

where $\mathbf{r}_{xx}^{[L]} \in \mathcal{R}^{N\times 1}$ is a D sparse vector *with the same support* S_0 and non zero elements given by

$$\sigma_i^{2[L]} \triangleq \frac{1}{L}\sum_{l=1}^{L}[\mathbf{x}_s[l]]_i^2, i \in S_0 \quad (3)$$

Hence $\mathbf{r}_{xx}^{[L]}$ is a non negative vector for all values of L. The elements of the vector $\mathbf{e}^{[L]} \in \mathcal{R}^{M^2\times 1}$ are given by

$$\frac{1}{L}\sum_{\substack{i,j=1\\ i\neq j}}^{D}\sum_{l=1}^{L}\mathbf{A}_{m,i}\mathbf{A}_{n,j}[\mathbf{x}_s[l]]_i[\mathbf{x}_s[l]]_j, 1 \le m, n \le M. \quad (4)$$

They denote the estimates of cross correlation between the elements of $\mathbf{y}[l]$ which are nonzero for finite L. We aim to recover the support S_0 of the unknown vector $\mathbf{r}_{xx}^{[L]}$ from this model (2).

As shown in [11], the problem for recovering the sparse support becomes finding the support of a vector $\hat{\mathbf{r}} \in \mathcal{R}^{N\times 1}$ which is the solution to the following constrained LASSO:

$$\min_{\mathbf{r}}\Big(\frac{1}{2}||(\mathbf{A}\odot\mathbf{A})\mathbf{r} - \mathbf{z}^{[L]}||_2^2 + h||\mathbf{r}||_1\Big) \quad \text{subject to } \mathbf{r} \succeq \mathbf{0}. \quad (5)$$

Relation of Coherence to Recoverable Sparsity: The coherence of the measurement matrix $\mathbf{A}$, defined as $\mu_A = \max_{i\neq j}\frac{|\mathbf{a}_i^T\mathbf{a}_j|}{\|\mathbf{a}_i\|_2\|\mathbf{a}_j\|_2}$, directly guarantees the recovery of up to a certain level of sparsity. Given a measurement matrix $\mathbf{A}$, if the sparsity $D < \frac{1}{2}(1+\frac{1}{\mu_A})$, then all D-sparse vectors can be successfully recovered [7] (in the noiseless case). In our model (2), the effective measurement matrix is $\mathbf{A}\odot\mathbf{A}$ whose coherence is given by μ_A^2 [10]. Hence if a sparsity level of $D \le D_{max}$ is guaranteed to be recovered using conventional MMV based methods (which use $\mathbf{A}$), we can guarantee the recovery of sparsity levels of $D \sim D_{max}^2$ using our framework for the case of ideal correlation matrix. We will show that under assumptions (A1-A2), even for the sample correlation matrix (which is a random variable), we can argue this to be true with overwhelming probability as L increases.

3. ANALYSIS OF LASSO

In [11], we performed analysis of the LASSO (5) by computing the probability of successful support recovery. However the analysis was done without assuming any specific probability distribution for the source vectors. Accordingly, the lower bound on the probability of successful recovery (denoted $\mathcal{P}_s$) was found to be rather loose, and it increases with L as $\mathcal{P}_s \ge 1 - \frac{C}{L}$. As evident, the bound increases rather slowly with increasing L. In this paper, we demonstrate that assuming a specific distribution on the unknown source vectors, namely, multivariate Gaussian, it is possible to guarantee support recovery with overwhelming probability with increasing L. In particular, we will show that the probability of support recovery increases as $\mathcal{P}_s \ge 1 - C\beta^{-L}, \quad C > 0, \beta > 1$.

We will derive the aforementioned result by first considering a set of sufficient conditions (given by the following Lemma 1) for the optimal solution of (5) to yield the true support of the sparse vectors, and then computing the probability with which these conditions will hold true. The following lemma is proved in [11] using techniques similar to [8] and [7].

Lemma 1. *[11] If $D < \frac{1}{2}(1+\frac{1}{\mu_A^2})$, and the following events denoted by E_1 and E_2 hold true:*

$$E_1: \quad \|\mathbf{e}^{[L]}\|_2 < h\frac{1+\mu_A^2 - 2\mu_A^2 D}{1+\mu_A^2-\mu_A^2 D}, \quad (6)$$

$$E_2: \quad \sigma_i^{2[L]} > \frac{\|\mathbf{e}^{[L]}\|_2 + h}{1+\mu_A^2-\mu_A^2 D}, \quad \forall i \in S_0 \quad (7)$$

then the optimal solution $\mathbf{r}^*$ to the LASSO satisfies $\text{Supp}(\mathbf{r}^*) = S_0$.

Notice that the above set of sufficient conditions involves the random variables $\sigma_i^{2[L]}$ and $\mathbf{e}^{[L]}$. Since E_1 and E_2 are sufficient conditions for support recovery, if we can develop a lower bound on the probability of joint occurrence of the events E_1 and E_2, then the probability of successful support recovery by solving LASSO can also be lower bounded by the same quantity. In what follows, we will derive individual concentration inequalities that will finally help us lower bound the probability of joint occurrence of E_1 and E_2.

3.1. Concentration Inequalities

We first state a concentration inequality ([6], Lemma 6) that will be directly used in our derivation:

Lemma 2. *Let each of x_i and y_i, $i = 1, \cdots, k$ be real sequences of i.i.d zero mean Gaussian random variables with variance σ_x^2 and σ_y^2 respectively. Then*

$$\mathcal{P}\Big(|\sum_{i=1}^{k}x_i y_i| \ge t\Big) \le 2\text{exp}\Big(-\frac{t^2}{2\sigma_x\sigma_y(2\sigma_x\sigma_y k + t)}\Big)$$

Using above lemma, we can get the following result. The notation $\sigma_{max}^{(k)}$ denotes the kth largest element in the set of non negative numbers $\{\sigma_i\}_{i=1}^{D}$.

Lemma 3. *Under the assumptions (A1-A2), the following holds* $\forall 1 \leq i \leq M^2$*:*

$$\mathcal{P}\Big(|[\mathbf{e}^{[L]}]_i| \geq C\Big) \leq 2\exp\Big(-\frac{Lt^2}{2\sigma_{max}^{(1)}\sigma_{max}^{(2)}(2\sigma_{max}^{(1)}\sigma_{max}^{(2)}+t)}\Big),$$

where $t \triangleq \frac{C}{\|\mathbf{A}\|_{\infty,\infty}^2 D(D-1)}$

Proof. Notice from (4) that

$$|[\mathbf{e}^{[L]}]_i| \leq \frac{\|\mathbf{A}\|_{\infty,\infty}^2}{L}\sum_{\substack{k,j=1\\ i\neq j}}^{D}\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_k[\mathbf{x}_s[l]]_j\Big| \tag{8}$$

Therefore

$$|[\mathbf{e}^{[L]}]_i| \geq C \Longrightarrow \sum_{\substack{k,j=1\\ i\neq j}}^{D}\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_k[\mathbf{x}_s[l]]_j\Big| \geq \frac{CL}{\|\mathbf{A}\|_{\infty,\infty}^2} \tag{9}$$

Also, notice that

$$\sum_{\substack{k,j=1\\ i\neq j}}^{D}\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_k[\mathbf{x}_s[l]]_j\Big| \geq \frac{CL}{\|\mathbf{A}\|_{\infty,\infty}^2} \Longrightarrow$$
$$\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_{k_0}[\mathbf{x}_s[l]]_{j_0}\Big| > \frac{CL}{\|\mathbf{A}\|_{\infty,\infty}^2 D(D-1)} \tag{10}$$

for some $k_0, j_0 \in \{1,\cdots,D\}, k_0 \neq j_0$. Using (9) and (10), we can say that

$$\mathcal{P}\Big(|[\mathbf{e}^{[L]}]_i| \geq C\Big) \leq \mathcal{P}\Big(\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_{k_0}[\mathbf{x}_s[l]]_{j_0}\Big| > \frac{CL}{\|\mathbf{A}\|_{\infty,\infty}^2 D(D-1)}\Big) \tag{11}$$

Under assumption (A1), $\{[\mathbf{x}_s[l]]_j\}_{l=1}^{L}$ and $\{[\mathbf{x}_s[l]]_k\}_{l=1}^{L}$ denote sequences of i.i.d. zero mean Gaussian random variables which are independent of each other. Hence, given $k \neq j$, we can use Lemma 2 to obtain

$$\mathcal{P}\Big(\sum_{l=1}^{L}\Big|[\mathbf{x}_s[l]]_{k_0}[\mathbf{x}_s[l]]_{j_0}\Big| > \frac{CL}{\|\mathbf{A}\|_{\infty,\infty}^2 D(D-1)}\Big)$$
$$\leq 2\exp\Big(-\frac{Lt^2}{2\sigma_{k_0}\sigma_{l_0}(2\sigma_{k_0}\sigma_{l_0}+t)}\Big)$$
$$\leq 2\exp\Big(-\frac{Lt^2}{2\sigma_{max}^{(1)}\sigma_{max}^{(2)}(2\sigma_{max}^{(1)}\sigma_{max}^{(2)}+t)}\Big)$$

The desired result directly follows from above and (11). $\square$

We now prove a second concentration inequality which directly provides a bound on the probability of occurrence of event E_2.

Lemma 4. *Let* $x_i, i = 1,\cdots,L$ *denote i.i.d zero mean Gaussian random variables with variance* σ_x^2*. Also, assume* $0 < C < \sigma_x^2$*. Then, there exists* $\beta > 1$ *such that*

$$\mathcal{P}\Big(\frac{1}{L}\sum_{i=1}^{L}x_i^2 > C\Big) \geq 1-\beta^{-L} \tag{12}$$

Proof. The proof is based upon Chernoff Bound. Denote $p_x^{[L]} \triangleq \frac{1}{L}\sum_{i=1}^{L}x_i^2$. Then

$$\mathcal{P}(p_x^{[L]} > C) = \mathcal{P}\Big(\sum_{i=1}^{L}x_i^2 > CL\Big) = \mathcal{P}\Big(\sum_{i=1}^{L}z_i^2 > \frac{CL}{\sigma_x^2}\Big) \tag{13}$$

where z_i denote i.i.d zero mean standard Normal variables. Therefore $\sum_{i=1}^{L}z_i^2$ is a Chi-Squared random variable with L degrees of freedom. Observe

$$\mathcal{P}(p_x^{[L]} > C) = 1-\mathcal{P}\Big(\sum_{i=1}^{L}z_i^2 \leq \frac{CL}{\sigma_x^2}\Big)$$
$$\geq 1-\exp(\frac{sCL}{\sigma_x^2})(1+2s)^{-L/2} \tag{14}$$

for $s > 0$. Equation (14) follows from the Chernoff Bound and also from the fact that the Moment Generating function of a Chi-Squared random variable with L degrees of freedom is given by $(1-2s)^{-L/2}, s < 1/2$. Now define the function

$$\beta(s) = \exp(-\frac{2sC}{\sigma_x^2})(1+2s) \tag{15}$$

we can write from (14) that

$$\mathcal{P}(p_x^{[L]} > C) \geq 1-\Big(\beta(s)\Big)^{-L/2}. \tag{16}$$

We want to ensure that $\exists s > 0$, such that $\beta(s) > 1$. Now $\beta(s) > 1 \Leftrightarrow C < \gamma(s)$ where $\gamma(s) \triangleq \frac{\sigma_x^2}{2s}\log(1+2s)$. It can be verified that $\gamma(s)$ is a decreasing function in s for $s > 0$ and $\gamma(0) = \sigma_x^2$. Since, it is given that $C < \sigma_x^2$, then indeed $\exists s_0 > 0$ such that $C < \gamma(s_0)$. This in turn implies, $\beta(s_0) > 1$. Hence, we conclude from (16) that $\mathcal{P}\Big(\frac{1}{L}\sum_{i=1}^{L}x_i^2 > C) \geq 1-\beta^{-L}$ for $\beta = \sqrt{\beta(s_0)} > 1$. $\square$

3.2. Probability of Support recovery by Solving the LASSO

Armed with the inequalities provided by Lemmas 3 and 4, we now state our main result on the probability of support recovery by solving the LASSO (5), given by the following theorem:

Theorem 1. *Consider the MMV model given by (1) which satisfies the assumptions (A1-A2).If*

$$D < \frac{1}{2}(1+\frac{1}{\mu_A^2}), \quad 0 < h < \sigma_{min}^2\frac{(1+\mu_A^2-\mu_A^2D)^2}{2(1+\mu_A^2)-3\mu_A^2D},$$

then the common support S_0 of size D can be recovered by solving the proposed LASSO given by (5), with probability greater than $1 - \alpha\gamma^{-L}$ for some $\gamma > 1$.

Proof. Using Lemma 1, we can say that the probability of successful recovery of sparse support by solving the LASSO (5), denoted as P_s, satisfies:

$$P_s \geq \mathcal{P}(E_1 \bigcap E_2) \tag{17}$$

Using a technique similar to the proof of Lemma 2 in [11], it can be shown that

$$\mathcal{P}(E_1 \bigcap E_2) \geq \prod_{i=1}^{D} \mathcal{P}(\sigma_i^{2[L]} > c_2) - \sum_{i=1}^{M^2} \mathcal{P}(| [\mathbf{e}^{[L]}]_i | \geq \frac{c_1}{M}) \tag{18}$$

Now using the expression for $\sigma_i^{2[L]}$ from (3), we can say, from Lemma 4 that

$$\prod_{i=1}^{D} \mathcal{P}(\sigma_i^{2[L]} > c_2) \geq \prod_{i=1}^{D} (1 - \beta_i^{-L}) \tag{19}$$

and using Lemma 3, we get

$$\sum_{i=1}^{M^2} \mathcal{P}\Big(| [\mathbf{e}^{[L]}]_i | \geq \frac{c_1}{M} \Big) \leq 2M^2 e^{-\delta L} \tag{20}$$

where $\delta \triangleq \frac{t^2}{2\sigma_{max}^{(1)}\sigma_{max}^{(2)}(2\sigma_{max}^{(1)}\sigma_{max}^{(2)}+t)}$ and $t \triangleq \frac{c_1}{M\|\mathbf{A}\|_{\infty,\infty}^2 D(D-1)}$. Using (19) and (20) in (18), we obtain

$$\mathcal{P}_s \geq \prod_{i=1}^{D} (1 - \beta_i^{-L}) - 2M^2 e^{-\delta L} \tag{21}$$

which proves the desired result since each $\beta_i > 1$. □

4. NUMERICAL RESULTS

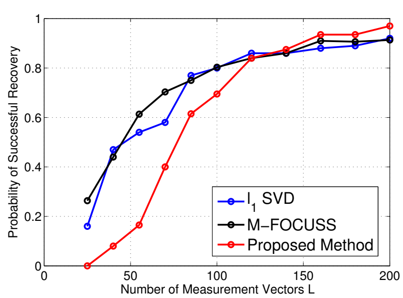

Fig. 1. Comparison of probability of support recovery as a function of the number of measurement vectors (L) by the proposed algorithm, l_1 SVD and M-FOCUSS. Here $M = 20, N = 256, D = 15$.

In this section, we compare the performance of the proposed method with two other well known methods for sparse support recovery in a MMV setting, viz., the l_1 SVD [9] and M-FOCUSS [2], for different values of D. We consider a fixed measurement matrix $\mathbf{A}$ generated as one instance of a Gaussian random matrix with zero mean and unit variance. We first assume $M = 20, \quad N = 256$. In the implementation of M-FOCUSS, we use $p = 1$ to be consistent with the other two methods. We demonstrate the probability of successful support recovery (by generating zero mean i.i.d. random $\mathbf{x}_s[l], \quad 1 \leq l \leq L$ but keeping $\mathbf{A}$ constant) averaged over 500 Monte Carlo runs, for the proposed method, l_1-SVD and M-FOCUSS. We consider $M = 20, N = 256, D = 15$ and plot the probability of successful recovery as a function of L in Figure 1.

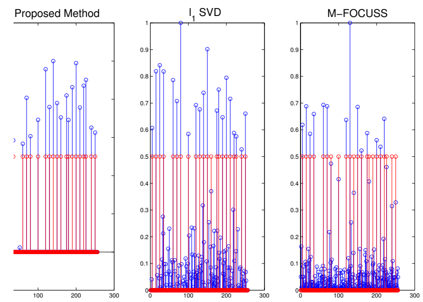

Fig. 2. Comparison of recovered support when $D = 22 > M$ by the proposed algorithm, l_1 SVD, and M-FOCUSS. Red: true support, blue: recovered support. Here $M = 20, N = 256$.

The plot brings an interesting observation: for low snapshots $L \leq 100$, the other two methods show better recovery, however, as L increases beyond 120, the proposed method shows better performance. This can be explained by the fact that as L increases, the cross terms become smaller (which has same effect of lower noise term in the LASSO) and hence the performance improves. We also carried out simulations for a value of D larger than M, viz., $D = 22$. With L around 300, the proposed method could successfully recover the sparsity whereas l_1-SVD and M-FOCUSS failed. In fact, the recovered vectors by these methods did not even turn out to be sparse. A representative plot showing the recovered support by the three methods is shown in Figure 2.

This shows the promise that with reasonably high number of measurement vectors, the proposed method can actually recover sparsity levels which are impossible to recover using existing techniques. We are conducting extensive studies to understand the effect of snapshots L on these methods for various values of D [12].

5. CONCLUSION AND FUTURE WORK

In this paper, we showed that using a correlation aware framework, one can possibly recover higher levels of sparsity with overwhelming probability as L increases, when the source vectors come from a Gaussian distribution with diagonal covariance matrix. The current analysis was based on the coherence of the measurement matrix which can be loose and in future we would like to develop tighter conditions analogous to RIP, null space condition etc. for the proposed framework. Also, more extensive numerical study needs to be performed to compare the performance of proposed method with powerful existing techniques such as l_1 SVD and M-FOCUSS.

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