FUSION OF ALGORITHMS FOR COMPRESSED SENSING

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ABSTRACT

Numerous algorithms have been proposed recently for sparse signal recovery in Compressed Sensing (CS). In practice, the number of measurements can be very limited due to the nature of the problem and/or the underlying statistical distribution of the non-zero elements of the sparse signal may not be known *a priori*. It has been observed that the performance of any sparse signal recovery algorithm depends on these factors, which makes the selection of a suitable sparse recovery algorithm difficult. To take advantage in such situations, we propose to use a fusion framework using which we employ multiple sparse signal recovery algorithms and *fuse* their estimates to get a better estimate. Theoretical results justifying the performance improvement are shown. The efficacy of the proposed scheme is demonstrated by Monte Carlo simulations using synthetic sparse signals and ECG signals selected from MIT-BIH database.

Index Terms— Compressed Sensing, Fusion, Sparse Recovery, Support Recovery, Signal Reconstruction

1. INTRODUCTION

Though numerous algorithms have been proposed recently for sparse signal recovery in Compresses Sensing (CS), the performance of any algorithm depends on many parameters like signal dimension, sparsity level, number of measurements, and the underlying statistical distribution of the non-zero elements of the signal. It has been observed that none of the algorithms outperforms others in all ranges of these parameters [1, 2]. In many applications, the number of measurements are very limited and/or the underlying statistical distribution of the non-zero values may not be known *a priori*. The sparse signal recovery in this context carries significant interest in CS. To enhance the sparse signal recovery in such situations, we propose to employ multiple sparse recovery algorithms, working with different principles, and fuse their estimates to get a better signal estimate which is often better than the best in the group.

The idea of combining information from several sources in order to form a better unified picture has been extensively studied in the context of *data fusion* [3]. Fusion of multiple estimators that use different dictionaries were discussed in [4] and [5]. Recent Machine-learning and Statistics literature [6, 7] suggested fusion of a group of competing estimators using exponential weights leading to an estimator better than the best in the group. For signal denoising applications, Orthogonal Matching Pursuit (OMP) with randomization was proposed in [8] to get several signal representations and fusion was performed by plain averaging.

Relation to prior work: To the best of our knowledge, [9] was the first to discuss a fusion approach for sparse signal recovery in CS where a fusion framework for greedy pursuits was introduced. The fusion algorithm, Fusion of Greedy Pursuits (FuGP), proposed in [9] has many shortcomings. FuGP was discussed only in the context of greedy pursuits. FuGP first forms the common support-set which contains the common atoms in the estimated support-sets of the two participating greedy algorithms. By choice, all the atoms in this common support-set are included in the support-set estimated FuGP. No theoretical support was given for this choice and it became a main hurdle to theoretically analyse FuGP. This choice also made FuGP to restrict the number of maximum participating algorithms as two. To alleviate these drawbacks, we propose another algorithm which we referred to as *Fusion of Algorithms for Compressed Sensing (FACS)* in this paper. Unlike FuGP, FACS is general in nature and do not put any restriction on the nature of the participating algorithm or the maximum number of participating algorithms. We provide theoretical guarantees for the proposed algorithm and the reconstruction performance is verified using numerical experiments.

2. COMPRESSED SENSING: BACKGROUND

Consider a standard CS measurement setup where a K-sparse signal $\mathbf{x} \in \mathbb{R}^N$ is acquired through linear measurements via

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ represents the measurement matrix, $\mathbf{b} \in \mathbb{R}^M$ represents the measurement vector, and $\mathbf{w} \in \mathbb{R}^M$ denotes the additive measurement noise present in the system. CS exploits the sparse structure of the signal and deals with methods which help to reduce the number of measurements without sacrificing the signal reconstruction performance. In CS setup, we assume $K < M \ll N$. The sparse signal reconstruction task in CS involves: (i) estimation of the sparsity level, (ii) identification of the indices of non-zero elements (also known as support-set), and (iii) estimation of the non-zero magnitudes. In this paper, we assume that the sparsity level K is known. Note that if we can estimate the support-set, the non-zero values can be efficiently found by solving a least-squares (LS) problem. The properties of a sparse recovery algorithm can be analysed using Restricted Isometry Property (RIP) [10, 11] of the measurement matrix which is defined as follows.

Definition 1. *[10, 11] Assume that for $\mathbf{A} \in \mathbb{R}^{M \times N}$, there exist a constant $\delta_K \in (0, 1)$ with $0 < K < M$ which satisfies*

$$(1 - \delta_K)\|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1 + \delta_K)\|\mathbf{x}\|_2^2 \tag{2}$$

Notations: Bold upper case and bold lower case Roman letters denote matrices and vectors, respectively. Calligraphic letters and upper case Greek alphabets are used to denote sets. $\|.\|_p$ denotes the p^{th} norm. $\mathbf{A}_{\mathcal{T}}$ denotes the column sub-matrix of $\mathbf{A}$ where the indices of the columns are the elements of the set $\mathcal{T}$. $\mathbf{x}_{\mathcal{T}}$ denotes the sub-vector formed by those elements of $\mathbf{x}$ whose indices are listed in the set $\mathcal{T}$. $\mathcal{T}^c$ denotes the complement of the set $\mathcal{T}$ with respect to the set $\{1, 2, \ldots, N\}$. $\mathtt{supp}(\mathbf{x})$ denotes the set of indices (coordinates) of non-zero elements of $\mathbf{x}$, i.e., $\mathtt{supp}(\mathbf{x}) = \{i : x_i \neq 0\}$. For a set $\mathcal{T}$, $|\mathcal{T}|$ denotes its cardinality (size), and for a scalar c, $|c|$ denotes the magnitude of c. $\mathbf{A}^\dagger$ denotes the transpose and pseudo-inverse of the matrix $\mathbf{A}$, respectively.

for all K-sparse $\mathbf{x}$. Then $\mathbf{A}$ is said to satisfy Restricted Isometry property (RIP) with order K. Restricted Isometry Constant (RIC) is defined as the smallest δ_K which satisfies (2).

3. FUSION FRAMEWORK

The motivation for our proposed *fusion framework* was shown by using an exploratory experiment in Section III of [9]. The experiment and observations are not repeated here for brevity of space. However, it is strongly recommended to read Section III of [9] for a detailed discussion on the fusion framework.

3.1. Fusion of Algorithms for Compressed Sensing (FACS)

Assume that we are employing P sparse recovery algorithms (*algorithm-i*, $i = 1, 2, \ldots, P$), which work with different principles, to recover the sparse signal $\mathbf{x}$ using (1). Let $\hat{\mathbf{x}}_i$ and $\hat{\mathcal{T}}_i$ represent the sparse signal and support-set estimated by the *algorithm-i*, $(1 = 1, 2, \ldots, P)$. Note that many sparse recovery algorithms (e.g., l_1-minimization methods) may return a signal estimate with more than K non-zero elements. In such cases, $\hat{\mathbf{x}}_i$ is taken as the best K-sparse approximation of the estimated signal and $\hat{\mathcal{T}}_i = \mathtt{supp}(\hat{\mathbf{x}}_i)$ such that $|\hat{\mathcal{T}}_i| = K$. Let us define the joint support-set $\Gamma \triangleq \bigcup_{i=1}^{P} \hat{\mathcal{T}}_i$ and we assume that $R \triangleq |\Gamma| \leq M$. Now, note that the joint support-set Γ will be at least as rich, in terms of correct atoms, as the support-set estimated by the best participating algorithm in the group. Since we choose algorithms working with different principles as participating algorithms, it is not surprising that Γ can often contain more correct atoms than the support-set estimated by the best algorithm in the group. The exploratory experiment in Section III of [9] also justifies this. Hence, if we can recover all the correct atoms in Γ, we are likely to improve the signal reconstruction. Keeping this in mind, in FACS we estimate the K support-atoms only from Γ. The main steps involved in FACS are as follows.

1. *Support Merging:* $\Gamma = \bigcup_{i=1}^{P} \hat{\mathcal{T}}_i$
2. *Proxy signal estimation:* Since we have $R = |\Gamma| \leq M$, using LS we can form an efficient estimator for signal from Γ.
3. *Support identification:* The indices of the proxy signal with K prominent magnitudes constitute the estimated support-set denoted by $\hat{\mathcal{T}}$.
4. *Sparse signal estimation:* The sparse signal estimate $\hat{\mathbf{x}}$ is estimated by solving an overdetermined LS problem using $\hat{\mathcal{T}}$.

The proposed FACS algorithm is summarized in Algorithm 1. FACS

Algorithm 1 FACS

Inputs: $\mathbf{A} \in \mathbb{R}^{M \times N}$, $\mathbf{b} \in \mathbb{R}^{M}$, K, and $\hat{\mathcal{T}}_1, \hat{\mathcal{T}}_2, \ldots, \hat{\mathcal{T}}_P$.
Ensure: $|\bigcup_{i=1}^{P} \hat{\mathcal{T}}_i| \leq M$.
Initialization: $\hat{\mathbf{x}} = \mathbf{0} \in \mathbb{R}^N$, $\mathbf{v} = \mathbf{0} \in \mathbb{R}^N$.
1: $\Gamma = \bigcup_{i=1}^{P} \hat{\mathcal{T}}_i$;
2: $\mathbf{v}_\Gamma = \mathbf{A}_\Gamma^\dagger \mathbf{b}$, $\mathbf{v}_{\Gamma^c} = \mathbf{0}$;
3: $\hat{\mathcal{T}} = \mathtt{supp}(\mathbf{v}^K)$; ▷ ($\mathbf{v}^K$ is the best K-sparse approximation of $\mathbf{v}$)
4: $\hat{\mathbf{x}}_{\hat{\mathcal{T}}} = \mathbf{A}_{\hat{\mathcal{T}}}^\dagger \mathbf{b}$, $\mathbf{x}_{\hat{\mathcal{T}}^c} = \mathbf{0}$;
Outputs: $\hat{\mathbf{x}}$ and $\hat{\mathcal{T}}$.

do not explicitly put any restriction on the number of algorithms, P, that can be used as participating algorithms. However, an efficient LS (step 2 in Algorithm 1) requires a constraint $R \leq M$.

<u>*Theoretical Guarantees:*</u> Next, we theoretically analyse the FACS (Algorithm 1) using Restricted Isometry Property (RIP) [11]. The performance analysis is characterized by a measure viz. *Signal-to-Reconstruction-Error Ratio* (SRER) [12] which is defined as

$$\text{SRER} \triangleq \frac{\|\mathbf{x}\|_2^2}{\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2}, \tag{3}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ denote the actual and reconstructed signal vector, respectively. Theorem 1 provides a sufficient condition for FACS to provide SRER improvement over *algorithm-i* $(i = 1, 2, \ldots, P)$.

Theorem 1. *Assume that we have employed $P\,(\geq 2)$ participating algorithms to reconstruct the K-sparse signal $\mathbf{x}$ from* (1). *Let $\hat{\mathbf{x}}_i$ and $\hat{\mathcal{T}}_i$ denote the sparse signal and support-set estimated by 'algorithm-i' $(i = 1, 2, \ldots, P)$. Let the CS measurement matrix $\mathbf{A}$ holds RIP with the Restricted Isometry Constant (RIC) δ_{R+K}. Assuming $\left\|\mathbf{x}_{\hat{\mathcal{T}}_i^c}\right\|_2 \neq 0$, $\|\mathbf{x}_{\Gamma^c}\|_2 \neq 0$, define $\eta_i = \frac{\|\mathbf{x}_{\Gamma^c}\|_2}{\left\|\mathbf{x}_{\hat{\mathcal{T}}_i^c}\right\|_2}$ and $\zeta = \frac{\|\mathbf{w}\|_2}{\|\mathbf{x}_{\Gamma^c}\|_2}$. Let $\hat{\mathbf{x}}$ and $\hat{\mathcal{T}}$ denote the sparse signal and support-set estimated by FACS algorithms by fusing the estimates of $P\,(\geq 2)$ participating algorithms. Then, FACS provides at least SRER gain of $\left(\frac{(1-\delta_{R+K})^2}{(1+\delta_{R+K}+3\zeta)\eta_i}\right)^2$ over the 'algorithm-i' if $\eta_i < \frac{(1-\delta_{R+K})^2}{1+\delta_{R+K}+3\zeta}$.*

Proof: Due to lack of space, only an outline of the proof is given here. A detailed proof can be found in an extended version of this work [13]. We have,

$$\|\mathbf{x} - \hat{\mathbf{x}}\|_2 \leq \|\mathbf{x}_{\hat{\mathcal{T}}} - \hat{\mathbf{x}}_{\hat{\mathcal{T}}}\|_2 + \|\mathbf{x}_{\hat{\mathcal{T}}^c}\|_2. \tag{4}$$

Using Proposition 3.1 in [14], which is due to RIP, we can show that

$$\|\mathbf{x}_{\hat{\mathcal{T}}} - \hat{\mathbf{x}}_{\hat{\mathcal{T}}}\|_2 \leq \frac{\delta_{R+K}}{1-\delta_{R+K}} \|\mathbf{x}_{\hat{\mathcal{T}}^c}\|_2 + \frac{1}{\sqrt{1-\delta_{R+K}}} \|\mathbf{w}\|_2. \tag{5}$$

Substituting (5) in (4), we get

$$\|\mathbf{x} - \hat{\mathbf{x}}\|_2 \leq \frac{1}{1-\delta_{R+K}} \|\mathbf{x}_{\hat{\mathcal{T}}^c}\|_2 + \frac{1}{\sqrt{1-\delta_{R+K}}} \|\mathbf{w}\|_2. \tag{6}$$

Now, defining $\hat{\mathcal{T}}_\triangle \triangleq \Gamma \setminus \hat{\mathcal{T}}$, we can show that

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c}\|_2 \leq \|\mathbf{x}_{\Gamma^c}\|_2 + \|\mathbf{x}_{\hat{\mathcal{T}}_\triangle}\|_2. \tag{7}$$

Using reverse triangle inequality, we get

$$\left\|\mathbf{x}_{\hat{\mathcal{T}}_\triangle}\right\|_2 \leq \left\|(\mathbf{v}_\Gamma)_{\hat{\mathcal{T}}_\triangle}\right\|_2 + \|\mathbf{v}_\Gamma - \mathbf{x}_\Gamma\|_2. \tag{8}$$

Using the fact $\left\|(\mathbf{v}_\Gamma)_{\hat{\mathcal{T}}_\triangle}\right\|_2 \leq \|(\mathbf{v}_\Gamma - \mathbf{x}_\Gamma)\|_2$ in (8), we get

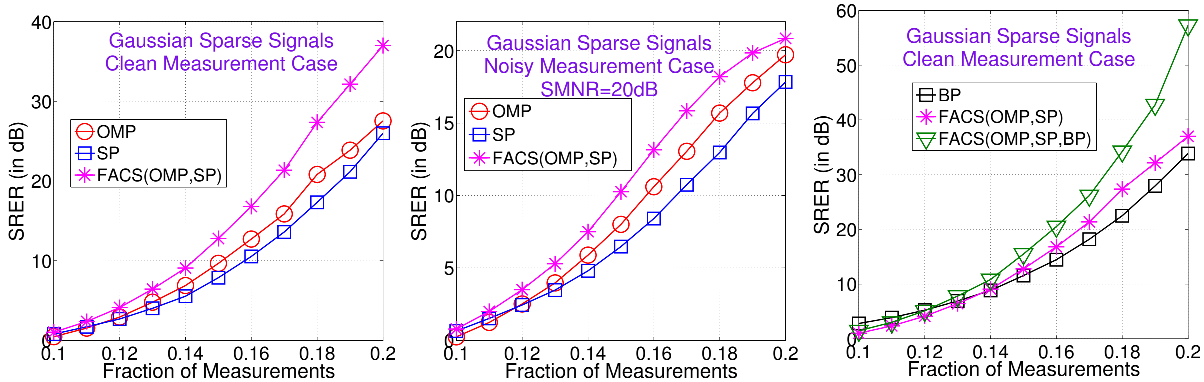
$$\left\|\mathbf{x}_{\hat{\mathcal{T}}_\triangle}\right\|_2 \leq 2\,\|(\mathbf{v}_\Gamma - \mathbf{x}_\Gamma)\|_2. \tag{9}$$

Using Proposition 3.1 in [14], we can show that

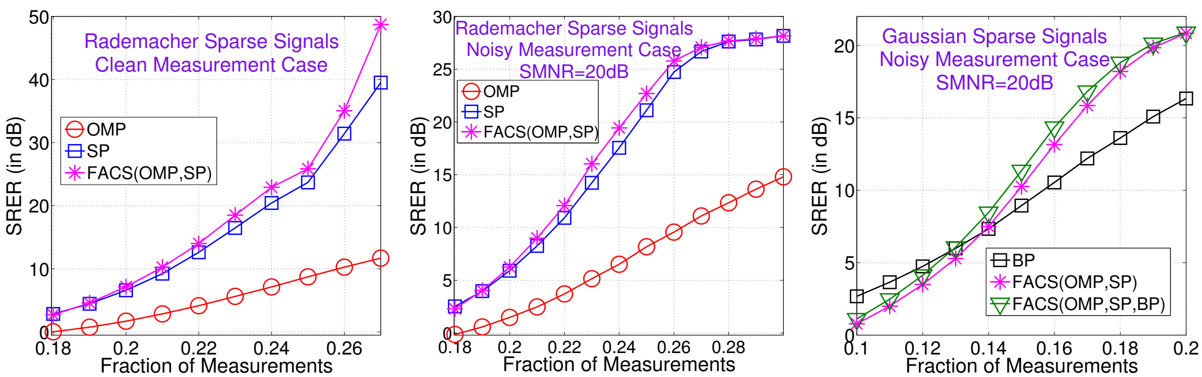
$$\|(\mathbf{v}_\Gamma - \mathbf{x}_\Gamma)\|_2 \leq \frac{\delta_{R+K}}{1-\delta_{R+K}} \|\mathbf{x}_{\Gamma^c}\|_2 + \frac{1}{1-\delta_{R+K}} \|\mathbf{w}\|_2. \tag{10}$$

Now, using (9) and (10) in (7), we get

$$\|\mathbf{x}_{\hat{\mathcal{T}}^c}\|_2 \leq \frac{1+\delta_{R+K}}{1-\delta_{R+K}} \|\mathbf{x}_{\Gamma^c}\|_2 + \frac{2\,\|\mathbf{w}\|_2}{1-\delta_{R+K}}. \tag{11}$$



(a) FACS(OMP,SP): GSS, Clean (b) FACS(OMP,SP): GSS, SMNR = 20 dB (c) FACS(OMP,SP,BP): GSS, Clean



(d) FACS(OMP,SP): RSS, Clean (e) FACS(OMP,SP): RSS, SMNR = 20 dB (f) FACS(OMP,SP,BP): GSS, Clean

Fig. 1: Synthetic sparse signals: Performance of FACS in terms of Signal-to-Reconstruction-Noise-Ratio (SRER) vs. Fraction of Measurements, averaged over $10,000$ trials, for Gaussian sparse signals (GSS) and Rademacher sparse signals (RSS) in clean and noisy measurement cases. Signal dimension $N = 500$, sparsity level $K = 20$.

Substituting (11) in (6) and using definitions of η_i and ζ, we get

$$\|\mathbf{x} - \hat{\mathbf{x}}\|_2 \leq \frac{1 + \delta_{R+K} + 3\zeta}{(1 - \delta_{R+K})^2} \, \eta_i \, \|(\mathbf{x} - \hat{\mathbf{x}}_i)\|_2 \, . \tag{12}$$

Finally using (12) we get,

$$\begin{aligned} \mathrm{SRER}|_{\mathrm{FACS}} &= \frac{\|\mathbf{x}\|_2^2}{\|\mathbf{x} - \hat{\mathbf{x}}\|_2^2} \\ &\geq \frac{\|\mathbf{x}\|_2}{\|\mathbf{x} - \hat{\mathbf{x}}_i\|_2} \times \left(\frac{(1 - \delta_{R+K})^2}{(1 + \delta_{R+K} + 3\zeta)\eta_i} \right)^2 \\ &= \mathrm{SRER}|_{\textit{algorithm-i}} \times \left(\frac{(1 - \delta_{R+K})^2}{(1 + \delta_{R+K} + 3\zeta)\eta_i} \right)^2 . \blacksquare \end{aligned}$$

Now, let us assume that $\left\|\mathbf{x}_{\hat{\mathcal{T}}_i^c}\right\|_2 = 0$ for some $i \in \{1, 2, \ldots, P\}$ which is not considered in Theorem 1. In such case, all the correct atoms are already identified by *algorithm-i*. Hence, further improvement over *algorithm-i* is not possible by FACS. Next, consider $\|\mathbf{x}_{\Gamma^c}\|_2 = 0$ which implies that all the correct atoms are included in the joint support-set Γ. In such cases, with the restriction $R \leq M$, the LS step (step 2 in Algorithm 1) can efficiently estimate the correctly estimate the support atoms.

Since FACS uses multiple participating algorithms the computational complexity of FACS is a little more than the sum of computational complexities of individual participating algorithms. The additional complexity is mainly due to the LS used in Algorithm 1.

4. NUMERICAL EXPERIMENTS AND RESULTS

In CS, the strive is to reduce the number of measurements and hence the sparse recovery in lower measurement cases carry significant interest. For numerical experiment in such cases we define the fraction of measurements, $\alpha \triangleq M/N$. We used OMP, SP, and BP (Basis Pursuit)/BPDN (Basis Pursuit DeNoising) as the participating algorithms fro sparse signal recovery. BP was used for clean measurement cases whereas BPDN was used in noisy measurement cases. For notation brevity, we use BP to denote both BP and BPDN throughout the paper. OMP and SP were implemented in Matlab and for BP/BPDN, we used l_1-magic toolbox [15]. It may be noted that BP/BPDN will not directly estimate the support-set. We choose the indices corresponding to the K-largest magnitudes of the signal estimate as the estimated support-set of BP/BPDN, in our simulations. We use FACS(OMP,SP) to denote FACS using OMP and SP as the participating algorithms. FACS(OMP,SP,BP) denote FACS with OMP, SP, and BP as the participating algorithms.

4.1. Synthetic Sparse Signals

For simulations with synthetic sparse signals, we followed the simulation setup described in detail in Section IV.B of [9]. For noisy measurement simulations, we define Signal-to-Measurement-Noise-Ratio (SMNR) as $\mathrm{SMNR} \triangleq \mathcal{E}\{\|\mathbf{x}\|_2^2\}/\mathcal{E}\{\|\mathbf{w}\|_2^2\}$, where $\mathcal{E}\{\|\mathbf{w}\|_2^2\} = \sigma_w^2 M$. As in [9], we used Gaussian sparse signals (GSS) and Rademacher sparse signals (RSS) with dimension 500 and sparsity level 20. This 4% level of sparsity closely resembles many real application scenarios [16]. We used 100 realizations of $\mathbf{A}$ and for each realization of $\mathbf{A}$, we randomly generated 100 sparse signals and average SRER is calculated. To evaluate the performance of FACS we conducted two experiments which are explained below.

Experiment 1: In the first experiment, we used FACS with OMP and SP as participating algorithms and the results are shown in Fig. 1(a), Fig. 1(b), Fig. 1(d), and Fig. 1(e). The performance of FACS and FuGP were similar and the results of FuGP are not shown here. It is interesting to note that OMP performed better than SP for GSS and *vice-versa* for RSS. This clearly shows that if the sparse signal distribution is not known *a priori*, we will not be able to achieve the best performance. As proposed, for both GSS (refer Fig. 1(a) and Fig. 1(b)) and RSS (refer Fig. 1(d) and Fig. 1(e)), FACS showed a better SRER than the best participating algorithm in clean as well as noisy measurement cases. For example, for GSS, in the clean measurement case (refer Fig. 1(a)), at $\alpha = 0.18$, FACS(OMP,SP) gave 10 dB improvement over SP and 6.5 dB improvement over OMP. In the noisy measurement case (refer Fig. 1(b)), at $\alpha = 0.18$, FACS(OMP,SP) showed 5 dB improvement over SP and 2.5 dB improvement over OMP, for Gaussian sparse signals. For RSS in clean measurement case, at $\alpha = 0.24$ FACS(OMP,SP) resulted in 15.8 dB and 2.5 dB SRER improvement respectively over OMP and SP. The same trend can be seen noisy measurement case also and at $\alpha = 0.24$ FACS(OMP,SP) showed 12.9 dB and 1.9 dB SRER improvement over OMP and SP respectively.

Experiment 2: To show the scalability of FACS we extended Experiment 1 to reconstruct the sparse signal using FACS with OMP, SP, and BP as the participating algorithms. Note that FuGP do not support fusion of more than two participating algorithms. To save space, we have shown only the results for GSS which are shown in Fig. 1(c) and Fig. 1(f). It can be seen that FACS(OMP,SP,BP) further improved SRER as compared to FACS(OMP,SP). For example, for $\alpha = 0.18$, FACS(OMP,SP,BP) improved the performance by 4.9 dB and 0.6 dB as compared to FACS(OMP,SP) in clean and noisy measurement case respectively.

Reproducible Research: In the spirit of reproducible research [17], we provide necessary Matlab codes publicly available from *http://www.ece.iisc.ernet.in/~ssplab/Public/FACS.tar.gz*. The code reproduces the simulation results shown in Fig. 1. We have also done simulations for FACS with four participating algorithms, the fourth algorithm being Compressive SAmpling Matching Pursuit (CoSAMP) [14]. The downloadable folder also contains codes for simulating FACS using all eleven combinations ($\binom{4}{2} + \binom{4}{3} + \binom{4}{4}$) of the four participating algorithms OMP, SP, BP, and CoSAMP. The code also include performance comparison Another performance metric called Average Support Cardinality Error (ASCE) [18] was also used for performance evaluation.

4.2. Real Compressible Signals

To evaluate the performance of FACS on real-world signals, we conducted experiments on ECG signals selected from MIT-BIH Arrhythmia Database [19]. ECG signals are compressible and have a good structure for sparse decompositions. We used the same simulation setup as used in [20] and [21]. Gaussian measurement matrices with appropriate sizes were used to vary the number of measurements, M, from 256 to 480 with an increment of 32. As earlier, here also we employed OMP, SP, and BP as the participating algorithms for FACS. We assumed a sparsity level 128, and the reconstruction results are shown in Fig. 2.

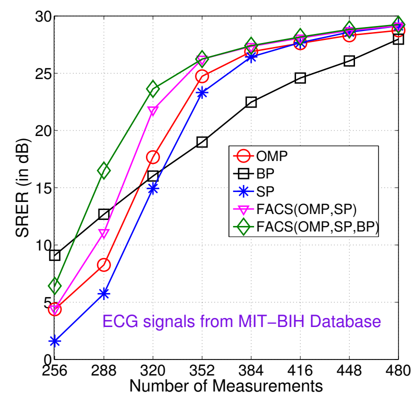


Fig. 2: Real-world signals: Performance of FACS (Signal-to-Reconstruction-Error Ratio (SRER) vs. Number of Measurements) for ECG signals selected from MIT-BIH Arrhythmia Database [19].

Similar to the case of synthetic signals, for ECG signals also FACS(OMP,SP) resulted in a better SRER as compared to both the participating algorithms OMP, and SP. For $M = 288$, FACS(OMP,SP) gave 6.8 dB and 5.8 dB SRER improvement over OMP and SP respectively. By using BP as the third participating algorithm, FACS(OMP,SP,BP) further improved the SRER by 1.8 dB than FACS(OMP,SP) for $M = 288$. A similar trend can be observed for other values of M in Fig. 2, showing the advantage of using FACS in real-life applications.

5. CONCLUSIONS

Using a fusion framework we proposed an algorithm, FACS, for sparse signal recovery which fuses the estimates of multiple sparse recovery algorithms. FACS is general in nature and can accommodate any sparse signal recovery algorithm as a participating algorithm. Using RIP, theoretical guarantees of FACS were also discussed. Numerical simulations showed that the sparse signal estimate of FACS often outperforms the sparse signal estimate of the best algorithm in the group. The advantage of using FACS on real life applications was shown by experiments conducted with ECG signal records from MIT-BIH database.

6. ACKNOWLEDGEMENTS

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