STATISTICAL ANALYSIS OF THE JOINTLY-OPTIMIZED ACOUSTIC ECHO CANCELLATION BF-AEC STRUCTURE

Marcos H. Maruo *, *José C. M. Bermudez* †, *Leonardo S. Resende*

Department of Electrical Engineering
Federal University of Santa Catarina, 88040-900, Florianópolis, SC, Brasil
Emails: {maruo, j.bermudez, resende}@ieee.org

ABSTRACT

This work presents a statistical analysis of a beamformer-assisted acoustic echo canceler (AEC). A new formulation leads to analytical models that can also be used to predict the transient performance of adaptive wideband beamformers. Monte Carlo simulations illustrate the accuracy of the model, which is then used to provide design guidelines. Application of the new model confirms previous experimental findings that the same cancellation performance of a single-microphone AEC can be achieved with a shorter AEC when the possibility of spatial filtering is available.

Index Terms— Statistical analysis, beamformer-assisted acoustic echo cancellation, constrained least-mean square algorithm

1. INTRODUCTION

Acoustic echoes arise when a microphone picks up the signal radiated by a loudspeaker and its reflections at the borders of a reverberant environment. Without a handset to provide attenuation between loudspeaker and microphone, intelligibility and listening comfort degrade [1, 2]. Typical room reverberation times lead to the need for adaptive acoustic echo cancelers with very long responses [1, 2]. Fast convergence and satisfactory echo cancellation are hard to obtain under these conditions [1, 3–5].

The desired speech signal is usually corrupted by speech from other talkers, noise and echoes in an acoustic environment. Spatial filtering (beamforming) can help attenuate interfering signals in directions other than the direction of arrival (DOA) of the desired speaker. Beamformers (BF) have limited echo suppression capacity due to limits in the array directivity [6] and the large number of microphones necessary to suppress all reflections outside the desired DOA [7].

Acoustic echo cancellation solutions in which BFs and acoustic echo cancelers (AECs) have complementary functions have raised a lot of interest recently [8–13]. BFs and AECs contribute by different means to reduce the residual echo. Hence, using both techniques in a synergistic way can improve the acoustic echo cancellation performance. BFs an AECs are usually combined by means of two basic structures [8, 14]. The AEC first structure (AEC-BF) employs one AEC per microphone [15–17]. The BF then processes the AEC outputs for spatial filtering. It requires several long AECs, leading to very high computational costs [15]. Moreover, signals not in the desired DOA must be treated as double talk, complicating the design. The BF first (BF-AEC) structure does the spatial filtering first, leaving basically the echo in the desired DOA to be cancelled by a single AEC.

Despite the possibilities of combined BF and AEC acoustic echo cancellation systems, we find only few analyses of their transient behavior in the literature. The AEC-BF structure has been studied in [15–17]. A stochastic model has been derived using the power transfer function method for the case of a fixed BF, where just the AEC is adapted. The performance of a system where BF and AEC are jointly adapted has not yet been studied in detail. Only a preliminary mean analysis of the jointly optimized BF-AEC has been presented in [18], with the joint-optimization treated as a combination of a constrained and an unconstrained optimization problem.

This work studies the transient behavior of the jointly optimized BF-AEC structure [9, 11, 13, 18]. We formulate the joint optimization as a single constrained optimization problem, what simplifies the statistical analysis. The analysis yields analytical models for the first and second moment behaviors of the BF and AEC weights, as well as for the mean power of the residual echo. Monte Carlo simulations illustrate the accuracy of the models. We also indicate how the derived models can be used for design.

In this paper, plain lowercase or uppercase letters denote scalars, lowercase boldface letters denote column vectors and uppercase boldface letters denote matrices.

2. PROBLEM STATEMENT

Fig. 1 shows the BF-AEC structure, with M echo impulse response vectors $\mathbf{h}_m$ of length N_{h}, M microphone signals $x_m[n]$, one adaptive wideband beamformer composed of M filters $\mathbf{b}_m[n]$ of length N_{BF} and an adaptive AEC filter $\hat{\mathbf{h}}[n]$ of length N_{AEC}. We assume responses $\mathbf{h}_m$ constant for mathematical tractability. It has been conjectured that the spatial filtering may reduce the required AEC length, as compared to the classical FIR AEC structure [19]. Hence, our analysis admits $N_{\text{AEC}} \leq N_{\text{h}}$.

2.1. Signal Model

The mth microphone signal $x_m[n]$ is the sum of a near-end signal $r_m[n]$ and an echo $e_m[n]$:

$$x_m[n] = e_m[n] + r_m[n]. \tag{1}$$

Each signal $r_m[n]$ is composed of local speech, local interferences and random noise. The echo $e_m[n]$ results from the filtering of $u[n]$ by $\mathbf{h}_m$. We define the microphone array snapshot $\boldsymbol{x}_{\text{s}}[n]$ at time n as

$$\boldsymbol{x}_{\text{s}}[n] = \begin{bmatrix} x_0[n] & x_1[n] & \cdots & x_{M-1}[n] \end{bmatrix}^T. \tag{2}$$

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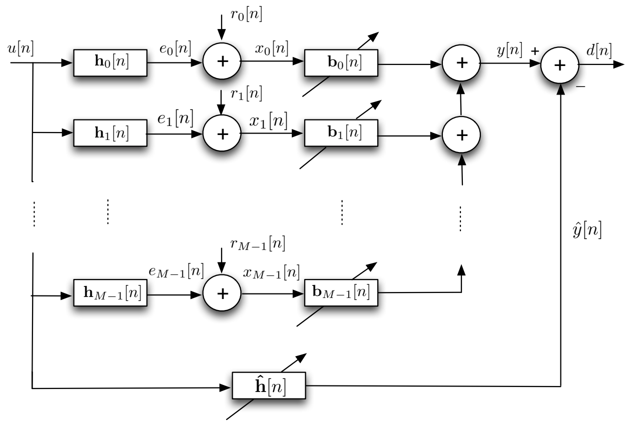


Fig. 1. BF-AEC system configuration [18].

and the combined beamformer input regressor as [20]

$$\boldsymbol{x}_{\mathrm{b}}[n] = \Big[\boldsymbol{x}_{\mathrm{s}}^T[n]\, \boldsymbol{x}_{\mathrm{s}}^T[n-1]\, \cdots\, \boldsymbol{x}_{\mathrm{s}}^T[n-(N_{\mathrm{BF}}-1)]\Big]^T. \tag{3}$$

Defining the vector $\mathbf{b}_{\mathrm{s}_\ell}[n]$ of the ℓth components of all vectors $\mathbf{b}_m[n]$, $m = 0, \ldots, M-1$ at time n as

$$\mathbf{b}_{\mathrm{s}_\ell}[n] = \Big[b_{0_\ell}[n]\, b_{1_\ell}[n]\, \cdots\, b_{M-1_\ell}[n]\Big]^T,\ \ell = 0, \ldots, N_{\mathrm{BF}}-1$$

we write the beamformer output $y[n]$ as

$$y[n] = \sum_{\ell=0}^{N_{\mathrm{BF}}-1} \boldsymbol{x}_{\mathrm{s}}^T[n-\ell]\mathbf{b}_{\mathrm{s}_\ell}[n]. \tag{4}$$

Now, defining the stacked beamformer weight vector

$$\mathbf{b}[n] = \Big[\mathbf{b}_{\mathrm{s}_0}^T[n] \quad \mathbf{b}_{\mathrm{s}_1}^T[n] \quad \cdots \quad \mathbf{b}_{\mathrm{s}_{N_{\mathrm{BF}}-1}}^T[n]\Big]^T \tag{5}$$

and using (3), we write $y[n]$ as the linear filtering $y[n] = \mathbf{b}^T[n]\boldsymbol{x}_{\mathrm{b}}[n]$.

Next, defining the AEC response vector

$$\hat{\boldsymbol{h}}[n] = \Big[\hat{h}_0[n] \quad \hat{h}_1[n] \quad \cdots \quad \hat{h}_{N_{\mathrm{AEC}}-1}[n]\Big]^T \tag{6}$$

and the AEC input vector $\boldsymbol{u}_{\hat{\mathrm{h}}}[n] = [u[n] \cdots u[n-(N_{\mathrm{AEC}}-1)]]^T$ yields $\hat{y}[n] = \hat{\boldsymbol{h}}^T[n]\boldsymbol{u}_{\hat{\mathrm{h}}}[n]$. The residual echo is $d[n] = y[n] - \hat{y}[n]$.

Finally, defining the combined input vector

$$\boldsymbol{s}[n] = \Big[-\boldsymbol{u}_{\hat{\mathrm{h}}}{}^T[n] \qquad \boldsymbol{x}_{\mathrm{b}}{}^T[n]\Big]^T \tag{7}$$

and, from (5) and (6), the combined adaptive coefficient vector

$$\boldsymbol{w}[n] = \Big[\hat{\boldsymbol{h}}^T[n] \qquad \mathbf{b}^T[n]\Big]^T \tag{8}$$

we write the residual echo $d[n]$ as the inner product

$$d[n] = -\boldsymbol{u}_{\hat{\mathrm{h}}}^T[n]\hat{\boldsymbol{h}}[n] + \boldsymbol{x}_{\mathrm{b}}^T[n]\mathbf{b}[n] = \boldsymbol{s}^T[n]\boldsymbol{w}[n]. \tag{9}$$

2.2. Performance Surface

The mean output power (MOP) performance surface J is defined as the mean value of $d^2[n]$ conditioned on $\boldsymbol{w}[n] = \boldsymbol{w}$. From (9),

$$\begin{aligned} J = E\{d^2[n]|\boldsymbol{w}[n] = \boldsymbol{w}\} &= E\Big\{\boldsymbol{w}^T\boldsymbol{s}[n]\boldsymbol{s}^T[n]\boldsymbol{w}\Big\} \\ &= \boldsymbol{w}^T\boldsymbol{R}_{\mathrm{ss}}\boldsymbol{w}. \end{aligned} \tag{10}$$

where $\boldsymbol{R}_{\mathrm{ss}} = E\{\boldsymbol{s}[n]\boldsymbol{s}^T[n]\}$ is the input autocorrelation matrix. A set of $N_{\mathcal{F}}$ linear constraints on the beamformer coefficients implements the spatial filtering. Usually, a constraint matrix $\boldsymbol{C}$ and a vector $\mathcal{F}$ jointly define the desired frequency response information in the specified DOA [20, 21].

To formulate the linear constraints as a function of the combined coefficient vector, we define the extended constrained matrix [11]

$$\boldsymbol{C}_{\mathrm{e}} = \Big[\mathbf{0}_{N_{\mathcal{F}} \times N_{\mathrm{AEC}}} \qquad \boldsymbol{C}^T\Big]^T.$$

Finally, the joint optimization problem can be formulated as

$$\boldsymbol{w}_{\mathrm{opt}} = \arg\min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}_{\mathrm{ss}}\boldsymbol{w} \tag{11a}$$

$$\text{subject to } \boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{w} = \mathcal{F} \tag{11b}$$

Note that (11) has the same form as the LCMV problem studied in [20]. From the results in [20],

$$\boldsymbol{w}_{\mathrm{opt}} = \boldsymbol{R}_{\mathrm{ss}}^{-1}\boldsymbol{C}_{\mathrm{e}}\left(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{R}_{\mathrm{ss}}^{-1}\boldsymbol{C}_{\mathrm{e}}\right)^{-1}\mathcal{F}. \tag{12}$$

Using (12) in (10) yields the minimum MOP

$$J_{\min} = \boldsymbol{w}_{\mathrm{opt}}^T\boldsymbol{R}_{\mathrm{ss}}\boldsymbol{w}_{\mathrm{opt}} = \mathcal{F}^T\left(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{R}_{\mathrm{ss}}^{-1}\boldsymbol{C}_{\mathrm{e}}\right)^{-1}\mathcal{F}. \tag{13}$$

2.2.1. *Feasible space*

Decomposing a feasible $\boldsymbol{w}$ satisfying (11b) into one component in the column space of $\boldsymbol{C}_{\mathrm{e}}$, and other in its orthogonal complementary space [20], and defining $N_{\mathrm{w}} = N_{\mathrm{AEC}} + M \times N_{\mathrm{BF}}$ yields

$$\begin{aligned} \boldsymbol{w} &= (\boldsymbol{I}_{N_{\mathrm{w}}} - \boldsymbol{C}_{\mathrm{e}}(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{C}_{\mathrm{e}})^{-1}\boldsymbol{C}_{\mathrm{e}}^T)\boldsymbol{w} + (\boldsymbol{C}_{\mathrm{e}}(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{C}_{\mathrm{e}})^{-1}\boldsymbol{C}_{\mathrm{e}}^T)\boldsymbol{w} \\ &= \boldsymbol{P}_{\mathrm{e}}\boldsymbol{w} + \boldsymbol{f}_{\mathrm{e}} \end{aligned} \tag{14}$$

where $\boldsymbol{P}_{\mathrm{e}}$ is the $N_{\mathrm{w}} \times N_{\mathrm{w}}$ projection matrix onto the orthogonal complementary space of the columns of $\boldsymbol{C}_{\mathrm{e}}$, and is given by

$$\begin{aligned} \boldsymbol{P}_{\mathrm{e}} &= (\boldsymbol{I}_{N_{\mathrm{w}}} - \boldsymbol{C}_{\mathrm{e}}(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{C}_{\mathrm{e}})^{-1}\boldsymbol{C}_{\mathrm{e}}^T) \\ &= \begin{bmatrix} \boldsymbol{I}_{N_{\mathrm{AEC}}} & \mathbf{0}_{N_{\mathrm{AEC}} \times M.N_{\mathrm{BF}}} \\ \mathbf{0}_{M.N_{\mathrm{BF}} \times N_{\mathrm{AEC}}} & \boldsymbol{P} \end{bmatrix} \end{aligned} \tag{15}$$

where $\boldsymbol{P} = \boldsymbol{I}_{M.N_{\mathrm{BF}}} - \boldsymbol{C}(\boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{C}^T$ is the projection matrix onto the orthogonal complementary space of the columns of $\boldsymbol{C}$ [20], and

$$\boldsymbol{f}_{\mathrm{e}} = \boldsymbol{C}_{\mathrm{e}}(\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{C}_{\mathrm{e}})^{-1}\mathcal{F} = [\mathbf{0}_{1 \times N_{\mathrm{AEC}}} \qquad \boldsymbol{f}^T]^T \tag{16}$$

is the extended quiescent part of the solution with $\boldsymbol{f} = \boldsymbol{C}(\boldsymbol{C}^T\boldsymbol{C})^{-1}\mathcal{F}$ [22]. We used $\boldsymbol{C}_{\mathrm{e}}^T\boldsymbol{w} = \mathcal{F}$ for all feasible $\boldsymbol{w}$ in (14).

3. ADAPTIVE SOLUTION

We now study the behavior of the BF-AEC with coefficients jointly adapted using the constrained LMS algorithm proposed in [20]. Using the stochastic gradient approach in [20, 23] yields the weight update equation

$$\boldsymbol{w}[n+1] = \boldsymbol{P}_{\text{e}}(\boldsymbol{w}[n] - \mu \boldsymbol{s}[n]d[n]) + \boldsymbol{f}_{\text{e}} \tag{17}$$

Exploiting the properties of $\boldsymbol{P}_{\text{e}}$, we split the update equation into two simpler update equations:

$$\begin{aligned} \hat{\boldsymbol{h}}[n+1] &= \hat{\boldsymbol{h}}[n] + \mu d[n]\boldsymbol{u}_{\hat{\text{h}}}[n] \\ \mathbf{b}[n+1] &= \boldsymbol{P}(\mathbf{b}[n] - \mu d[n]\boldsymbol{x}_{\text{b}}[n]) + \boldsymbol{f}. \end{aligned} \tag{18}$$

This significantly reduces the computational complexity, as $\boldsymbol{P}$ is of much lower dimension than $\boldsymbol{P}_{\text{e}}$.

4. STATISTICAL ANALYSIS

4.1. Simplifying Assumptions

We now study the behavior of BF-assisted echo canceler using (18) under the following typical simplifying assumptions:

A1 $\boldsymbol{s}[n]$ is stationary, zero-mean and Gaussian;

A2 $u[n]$ and $r[n]$ are statistically independent;

A3 $\boldsymbol{R}_{\text{ss}}$ is positive-definite, and $\boldsymbol{C}$ has full column rank;

A4 Statistical dependence of $\boldsymbol{s}[n]\boldsymbol{s}^T[n]$ and $\boldsymbol{w}[n]$ can be neglected;

A5 The desired DOA does not change during adaptation.

Though not always valid in practice, these assumptions make analysis viable and frequently lead to results that retain sufficient information to serve as reliable design guidelines [5, pg 315][9, 11]. Simulation results will confirm their reasonability for this analysis.

4.2. Mean Weight Behavior

Define the weight error vector

$$\boldsymbol{v}[n] = \boldsymbol{w}[n] - \boldsymbol{w}_{\text{opt}}. \tag{19}$$

Considering the equivalent forms of (11) and the LCMV problem studied in [20], and assuming the algorithm is initialized at a feasible solution, the mean behavior of $\boldsymbol{v}[n]$ can be obtained from [20] as

$$\begin{aligned} E\{\boldsymbol{v}[n+1]\} &= (\boldsymbol{I}_{N_{\text{w}}} - \mu\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})E\{\boldsymbol{v}[n]\} &(20)\\ &= (\boldsymbol{I}_{N_{\text{w}}} - \mu\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})^{n+1}E\{\boldsymbol{v}[0]\}. &(21) \end{aligned}$$

where it has been verified from (12) and (15) that

$$\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{w}_{\text{opt}} = \mathbf{0}_{N_{\text{w}}} \tag{22}$$

4.3. Weight Error Correlation Matrix

Again due to the equivalent forms of (11) and the LCMV problem, we could use the results in [23] for the behavior of the weight covariance matrix $\boldsymbol{K}_{\text{ww}}[n]$. However, the recursive expression derived in [23] for $\boldsymbol{K}_{\text{ww}}[n]$ depends explicitly on $E\{\boldsymbol{w}[n]\}$, and is not adequate for obtaining design guidelines. We derive here a more useful model using the weight error vector $\boldsymbol{v}[n]$.

Using (19) in (17), $d_0[n] = \boldsymbol{s}^T[n]\boldsymbol{w}_{\text{opt}}$, and $\boldsymbol{P}_{\text{e}}^2 = \boldsymbol{P}_{\text{e}}$ yields

$$\boldsymbol{v}[n+1] = (\boldsymbol{I}_{N_{\text{w}}} - \mu\boldsymbol{P}_{\text{e}}\boldsymbol{s}[n]\boldsymbol{s}^T[n]\boldsymbol{P}_{\text{e}}^T)\boldsymbol{v}[n] - \mu\boldsymbol{P}_{\text{e}}\boldsymbol{s}[n]d_0[n] \tag{23}$$

where

$$\boldsymbol{v}[n] = \boldsymbol{P}_{\text{e}}\boldsymbol{v}[n], \tag{24}$$

Using **A1**-**A5** and (22), calculation as in [4] leads to

$$\begin{aligned} \boldsymbol{R}_{\text{vv}}[n+1] &= \boldsymbol{R}_{\text{vv}}[n] - \mu\Big\{\boldsymbol{R}_{\text{vv}}[n]\boldsymbol{R}_{\text{proj}} + \boldsymbol{R}_{\text{proj}}\boldsymbol{R}_{\text{vv}}[n]\Big\} \\ &+ \mu^2\Big\{2\boldsymbol{R}_{\text{proj}}\boldsymbol{R}_{\text{vv}}[n] + \text{tr}\left(\boldsymbol{R}_{\text{proj}}\boldsymbol{R}_{\text{vv}}[n]\right) + J_{\min}\Big\}\boldsymbol{R}_{\text{proj}} \end{aligned} \tag{25}$$

where $\boldsymbol{R}_{\text{proj}} = \boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$, $\boldsymbol{R}_{\text{vv}}[n] = \boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{vv}}[n] = \boldsymbol{R}_{\text{vv}}[n]\boldsymbol{P}_{\text{e}}$.

As $\boldsymbol{P}_{\text{e}}$ is singular, one cannot analyze (25) in the principal coordinate space of $\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$ as in [4]. Using **A3** we can show that $\mathcal{R}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}) = \mathcal{R}(\boldsymbol{P}_{\text{e}})$ where $\mathcal{R}(\cdot)$ denotes the range of a matrix [24, pg. 102]. Also, $\boldsymbol{w}[n]$ from (17) will be feasible for all n if initialized in the feasible region [20]. From (24), $\boldsymbol{v}[n] \in \mathcal{R}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})$. Then, $\boldsymbol{R}_{\text{vv}}[n] \in \mathcal{R}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})$. Hence, updates of (25) occur only within $\mathcal{R}(\boldsymbol{P}_{\text{e}})$, and its convergence can be studied in this reduced dimension. Using the properties of projection matrices, the eigenvalues of $\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$ are given by

$$\tilde{\boldsymbol{\lambda}}_{\text{ss}} = \begin{bmatrix}\boldsymbol{\lambda}_{\text{ss}}^T & \mathbf{0}_{1\times N_{\mathcal{F}}}\end{bmatrix}^T \tag{26}$$

where $\boldsymbol{\lambda}_{\text{ss}}$ contains the $N_{\text{w}} - N_{\mathcal{F}}$ eigenvalues corresponding to eigenvectors in the non-constrained space and the remaining $N_{\mathcal{F}}$ eigenvalues are zero. We then perform the orthogonalization procedure in [4] using only the eigenvectors associated with $\boldsymbol{\lambda}_{\text{ss}}$. The diagonal entries of $\boldsymbol{R}_{\text{vv}}[n]$ in the principal coordinate space of $\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$ can be grouped in the vector $\boldsymbol{\rho}_{\text{vv}}$, which then obeys

$$\boldsymbol{\rho}_{\text{vv}}[n+1] = \boldsymbol{\Phi}\boldsymbol{\rho}_{\text{vv}}[n] + \mu^2 J_{\min}\boldsymbol{\lambda}_{\text{ss}} \tag{27}$$

where $\boldsymbol{\Phi} = \text{diag}(\rho_1, \rho_2, \ldots, \rho_{N_{\text{w}}-N_{\mathcal{F}}}) + \mu^2\boldsymbol{\lambda}_{\text{ss}}\boldsymbol{\lambda}_{\text{ss}}^T$ and $\rho_k = (1 - \mu\lambda_{\text{s}_k})^2 + \mu^2\lambda_{\text{s}_k}^2$. Matrix $\boldsymbol{\Phi}$ is symmetric and positive definite, and convergence of $\boldsymbol{\rho}_{\text{vv}}[n]$ is sufficient for the convergence of $\boldsymbol{R}_{\text{vv}}[n]$ [4]. Assuming convergence, the solution to (27) is [4, 25]

$$\boldsymbol{\rho}_{\text{vv}}[n] = \boldsymbol{\Phi}^n\boldsymbol{\rho}_{\text{vv}}[0] + \mu^2 J_{\min}\sum_{j=0}^{n-1}\boldsymbol{\Phi}^j\boldsymbol{\lambda}_{\text{ss}} \tag{28}$$

4.4. Stability

Convergence of (28) is determined exclusively by the eigenvalues $\lambda_{\boldsymbol{\Phi}}$ of $\boldsymbol{\Phi}$ [25], which are real and positive. Using Gershgorin theorem [26] we can show that a sufficient condition for all $\lambda_{\boldsymbol{\Phi}} < 1$ is

$$\mu < \frac{2}{3\,\text{tr}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})}. \tag{29}$$

This stability limit is less restrictive than that derived in [20]. It has also the practical advantage that the trace of $\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$ is, by definition, the total average power of $\boldsymbol{P}_{\text{e}}\boldsymbol{s}[n]$, which can be estimated from the available signals.

4.5. Mean Output Power (MOP)

Using A4, (13), (19), (22) and (24) we can rewrite the MOP as

$$\begin{aligned} E\{d^2[n]\} &= E\{\boldsymbol{w}^T[n]\boldsymbol{s}[n]\boldsymbol{s}^T[n]\boldsymbol{w}[n]\} \\ &= \text{tr}(\boldsymbol{R}_{\text{vv}}[n]\boldsymbol{R}_{\text{ss}}) + J_{\min} \\ &= \text{tr}(\boldsymbol{R}_{\text{vv}}[n]\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}) + J_{\min} \\ &= \boldsymbol{\rho}_{\text{vv}}^T[n]\boldsymbol{\lambda}_{\text{ss}} + J_{\min} \end{aligned} \tag{30}$$

In the last line of (30) we performed the orthogonal decomposition of $\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}$ using the eigenvalue structure (26).

4.6. Steady State

When (29) holds, (27) will converge such that $\lim_{n\to\infty} \boldsymbol{\rho}_{\text{vv}}[n+1] = \boldsymbol{\rho}_{\text{vv}}[n] = \boldsymbol{\rho}_{\text{vv}}[\infty]$. Doing as in [5, pg. 326–327] yields

$$J[\infty] = J_{\min}\left[1 + \frac{\frac{1}{2}\sum_{i=1}^{N_{\text{w}}-N_{\mathcal{F}}} \frac{\mu\lambda_{s_i}}{1-\mu\lambda_{s_i}}}{1 - \frac{1}{2}\sum_{i=1}^{N_{\text{w}}-N_{\mathcal{F}}} \frac{\mu\lambda_{s_i}}{1-\mu\lambda_{s_i}}}\right] \tag{31}$$

where λ_{s_i} is the ith eigenvalue in $\boldsymbol{\Lambda}_{\text{ss}}$. For $\mu \max\{\lambda_{s_i}\} \ll 1$, $1 - \mu\lambda_{s_i} \approx 1$ for all i and (31) reduces to

$$J[\infty] \approx J_{\min}\left[1 + \frac{\frac{1}{2}\mu\,\text{tr}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})}{1 - \frac{1}{2}\mu\,\text{tr}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})}\right]. \tag{32}$$

Further assuming $\mu\,\text{tr}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}}) \ll 2$ yields a simpler estimator for the steady state excess output power, as

$$J[\infty] \approx J_{\min}\left[1 + \frac{\mu}{2}\text{tr}(\boldsymbol{P}_{\text{e}}\boldsymbol{R}_{\text{ss}}\boldsymbol{P}_{\text{e}})\right] \tag{33}$$

5. RESULTS

5.1. Model Validation

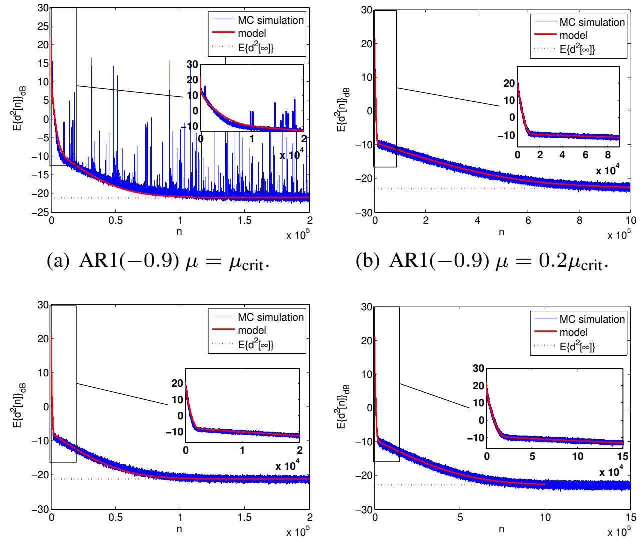
In the following we denote the critical step size in (29) as μ_{crit}. We show simulation results for two unit power first order autorregressive (AR) input signals with different correlation coefficients. We considered 2 microphones. $\mathbf{h}_0$ and $\mathbf{h}_1$ had 128 taps each, generated according to the exponential model in [1]. The desired DOA was assumed orthogonal to the microphone array. The microphone signals were corrupted by a zero-mean Gaussian white noise with variance 10^{-2}. The adaptive BF was designed with $N_{\text{BF}} = 16$, linear phase, and all-pass frequency response with $N_{\mathcal{F}} = 16$. The AEC used $N_{\text{AEC}} = 128$. Fig. 2 shows the predicted (continuous red) and simulated (continuous blue) transient MOP. The predicted steady-state MOP is shown by the red dotted lines. Simulations and theoretical predictions show excellent agreement. Note that $\mu = \mu_{\text{crit}}$ keeps the algorithm stable, and is clearly close the real stability limit for highly correlated signals (Fig. 2(a)).

5.2. Performance Curves

Figs. 3(a) and 3(b) illustrate the use of the derived models for design. They show $E\{d^2[\infty]\}$ obtained for echo responses with 1024 coefficients, an AR(1) far-end signal with $a_1 = -0.9$ and unit power, and white Gaussian noise with variance 10^{-2}. Fig. 3(a) is for $N_{\text{AEC}} = 1024$, and shows using (32) how much performance gain is expected as we increase the number of microphones. Fig. 3(b) shows the influence of the AEC length for $\mu = 0.05\mu_{\text{crit}}$ for different numbers M of microphones. It shows that echo cancellation can be improved with $N_{\text{AEC}} < N_{\text{h}}$ by increasing M. These results confirm the findings in [19] that the same cancellation performance of a single-microphone AEC can be achieved with a shorter AEC when the possibility of spatial filtering is available.

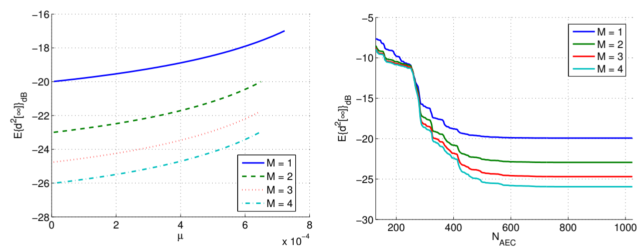
6. CONCLUSIONS

This work presented a new analysis of a beamformer-assisted acoustic echo canceler using the constrained least-mean square algorithm. The analysis considered joint adaptation of both the beamformer and the echo canceler. A new formulation of the problem was introduced, which led to analytical models that can be used to predict also the transient behavior of adaptive wideband linear constraint minimum variance beamformers. Simulation results illustrated the accuracy of the new model. It was shown how the model can be employed to construct useful design guidelines.



(a) AR1(−0.9) $\mu = \mu_{\text{crit}}$. (b) AR1(−0.9) $\mu = 0.2\mu_{\text{crit}}$.
(c) AR1(−0.5) $\mu = \mu_{\text{crit}}$. (d) AR1(−0.5) $\mu = 0.1\mu_{\text{crit}}$.

Fig. 2. Proposed model and Monte-Carlo simulation results based on 300 runs for different far-end signal statistics ($M = 2$, $N_{\text{h}} = 128$, $N_{\text{BF}} = 16$ $N_{\text{AEC}} = 128$)



(a) MOP function of μ and M. (b) MOP function of N_{AEC} and M.

Fig. 3. MOP for an AR1(-0.9) far-end signal in function of (a)μ and M (b)N_{AEC} and M

7. RELATION TO PRIOR WORK

Thanks to a new formulation, the results of this work can be used to predict the behaviors of both the adaptive LCMV broadband BF and the jointly optimized BF-assisted AEC [11]. The only existing BF second moment analysis is for the minimum variance distortionless response (MVDR) beamformer, and has been done under narrowband signal assumptions [23]. The results in [23] can be adapted to the broadband case without loss of generality, but its expressions are not amenable for design. Analyses of BF-assisted AECs are even rarer. The AEC-BF structure has been studied in [15], but for a fixed BF, where just the AEC is adapted. No previous result provides design informations as to how the different parameters can be combined to achieve a given performance.

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