

# A GRADIENT-LIKE VARIATIONAL BAYESIAN APPROACH FOR JOINT IMAGE SUPER-RESOLUTION AND SOURCE SEPARATION, APPLICATION TO ASTROPHYSICAL MAP-MAKING

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## ABSTRACT

In this work, a new unsupervised Bayesian method for joint image super-resolution and component separation is introduced. More precisely, we are interested in super-resolution for astrophysical map-making and separation between extended and point emissions. This is tackled as an inverse problem in a Bayesian framework, where a Markovian model is used as a prior for the extended emission and a student's  $t$ -distribution is attributed for the point sources component. All model and noise parameters are unknown, therefore we chose to estimate them jointly with the images. Nevertheless, both Joint Maximum A Posteriori (JMAP) and Posterior Mean (PM) estimators are intractable. Hence, we propose to approximate the true posterior by free-form separable distribution using a gradient-like variational Bayesian approach, which allows a joint update of the shape parameters of the approximating marginals. Applications on simulated and real datasets, obtained from Herschel space observatory, show a good quality of reconstruction. Furthermore, compared to conventional methods, our method gives a higher resolution while conserving photometry and reducing noise.

**Index Terms**— Super-resolution, source separation, Bayesian methods, astrophysics, Variational Bayesian

## 1. INTRODUCTION

The evolution and the formation of stars and galaxies are key issues in astrophysics since the last decades. Launched in 2009, the space observatory Herschel [1] allows spectacular mapping of nearby star-forming clouds, galaxies and distant galaxies, in the far infrared and sub-millimeter domains. Herschel is the largest telescope in space, but the spatial resolution of the sky maps is limited in these spectral ranges. Moreover, the observed regions contain different components (dust clouds, young stellar objects, unresolved galaxies, ...) which must be separated in order to derive their properties.

For these reasons, we propose to tackle the problem of high resolution map making and source separation jointly as an inverse problem. This is achieved by means of a Bayesian

framework which permits seamless integration of prior information. We focus in this study on separation between extended emission (dust clouds, ...) and point sources (unresolved galaxies, stars, ...). Several works have been devoted for super-resolution like [2, 3] and for two components separation problem like [4],[5], [6] or [7].

In the case of Herschel's data processing, map-making and components separation are done separately. For instance, many methods are proposed for map-making like *Coadd* (normalized retroprojection), *SANEPIC* [8], which lack correction for the instrument optical system, and a Bayesian superresolution mapmaker [9] which is dedicated for extended emission and suffers from artifacts in the presence of point sources. However, the point source extraction is performed after the map-making stage using point-spread function (PSF) fitting like in *DAOPHOT* [10] or in a Bayesian way like in *SUSSEX* [11]. Nevertheless, their source extraction quality is reduced in the presence of variations in the background due to existence of extended emission.

Therefore, we opt for joint superresolution map-making and component separation to overcome drawbacks of previous methods. In our Bayesian framework, a Markovian field model is attributed to the smooth part and sparse  $t$ -distribution prior was assigned to point source component. Moreover for the forward modeling, a state of the art instrument model was chosen so highest resolution is recovered. Furthermore, an unsupervised approach is chosen in order to make the method robust to prior parameters choice and facilitate its use by astrophysicists.

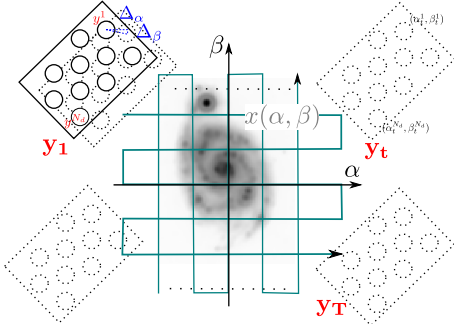
Nevertheless, the joint posterior has a complex expression and neither the joint maximum a posteriori (JMAP) nor the posterior mean (PM) estimators have a tractable form. Thus, an approximation should be used to obtain a practical solution. Several methods were used in literature such as stochastic sampling by Markov Chain Monte Carlo (MCMC) methods [12] or deterministic like the variational Bayesian approach [13] which approximate the true posterior by a separable free-form distribution. Compared to the latter method, MCMC methods are more time demanding especially for

huge data sets since it needs too many samples to explore the space. Therefore, we propose herein a new deterministic method [14] based on the variational Bayesian approach since treated data have generally hundred millions samples per observed field. The novelties of our approach are: the joint super-resolution and component separation framework, the unsupervised approach where almost all the hyper-parameters are estimated jointly with the unknowns, the joint estimation of drift in detectors and the application of the new variational Bayesian technique which provides an efficient estimator for high dimensional datasets.

The paper is organized as follows: We present in section 2 our Bayesian approach with some details over the forward and prior models. Then, the new variational Bayesian approach is introduced and the expression of approximated posterior is driven. Section 4 is dedicated for method tests with simulated and real data from Herschel space observatory. Finally, we conclude this work and give some perspectives.

## 2. BAYESIAN FRAMEWORK

In a map-making problem, one tries to restore the sky  $\mathbf{x}$  given several observations  $\mathbf{y}$  and instrument model  $\mathbf{H}$ . In this work,  $\mathbf{y} = \{\mathbf{y}_1, \dots, \mathbf{y}_T\}$  is composed of several observations  $\mathbf{y}_i$  covering the interest field and mutually shifted by a known translation (figure 1).



**Fig. 1.** Schematic representation of the scanning process in the telescope.

Supposing a white noise  $\mathbf{n}$  adds to the data, the forward model [9] is written as  $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$ . The instrument model  $\mathbf{H} = \mathbf{U}\mathbf{C}$  is a linear operator containing  $\mathbf{U}$  the pointing matrix which determines the field of view (FOV) of the detector and  $\mathbf{C}$  is a convolution matrix accounting basically for the instrument optics. In our joint restoration and separation framework, sky  $\mathbf{x}$  is decomposed into a smooth and a point components (i.e.  $\mathbf{x} = \mathbf{s} + \mathbf{p}$ ), and it should be estimated directly from the data. Then, the forward model reads  $\mathbf{y} = \mathbf{H}(\mathbf{p} + \mathbf{s}) + \mathbf{n}$ . This is a highly ill-posed problem since data does not contain discriminative information between smooth and point sources which should be provided by prior information. In addition, the operator  $\mathbf{H}$  is ill-conditioned. Hence, we

consider a Bayesian approach to reduce the dimension of the admissible solutions space. The ingredients for the posterior distribution (likelihood and prior) are defined next. Then, the problem of estimation is discussed in the following section.

### 2.1. Likelihood

The additive noise is supposed to have a white Gaussian distribution with a unknown variance  $\rho_n^{-1}$ . Furthermore, in physical applications, detectors have different variable offsets  $\mathbf{o}$  which should be estimated for each dataset. Using the forward model, the likelihood is written as  $\mathcal{P}(\mathbf{y}|\rho_n, \mathbf{o}, \mathbf{H}, \mathbf{x}) \propto \exp\left(-0.5\rho_n \|\mathbf{y} - \mathbf{H}\mathbf{x} - \mathbf{o}\|_2^2\right)$ .

### 2.2. Priors

The choice of prior distributions for sky components is a very crucial step since they determine the separation between smooth and point sources. Therefore, a correlated multivariate Gaussian Markov field, which favors small variations, was assigned to  $\mathbf{s}$ ,  $\mathcal{P}(\mathbf{s}|\rho_s) \propto \exp\left(-\frac{\rho_s(\|\mathbf{D}_\alpha \mathbf{s}\|_2^2 + \|\mathbf{D}_\beta \mathbf{s}\|_2^2)}{2}\right)$ , where  $\mathbf{D}_\alpha$  and  $\mathbf{D}_\beta$  are finite differences matrices according to axes  $\alpha$  and  $\beta$  respectively, and  $\rho_s$  is a parameter determining the degree of correlation in the field which is considered unknown.

The point sources  $\mathbf{p}$  are attributed a separable homogeneous t-distribution,  $\mathcal{P}(\mathbf{p}|\rho_p, \nu) \propto \prod_i \left(1 + \frac{\rho_p}{\nu} p_i^2\right)^{-\frac{\nu+1}{2}}$ , with  $\nu$  as number of degrees of freedom and  $\rho_p$  a scale parameter which are set to some fixed values. It can be also rewritten as,  $\mathcal{P}(\mathbf{p}|\rho_p, \nu) = \prod_i \int_{\mathbb{R}} \mathcal{P}(p_i|\rho_p, \nu) \mathcal{P}(\rho_p|\nu) d\rho_p$ , where  $\mathcal{P}(p_i|\rho_p, \nu) = \mathcal{N}(0, (\rho_p \rho_i)^{-1})$  and  $\mathcal{P}(\rho_p|\nu) = \mathcal{G}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$ . The latter form is more convenient to work with since it requires dealing with Gaussian and Gamma distributions. Hence, we adopt the approach proposed by [15] by adding an extra parameter  $\rho$  and estimating it jointly with other unknowns.

For the model hyperparameters  $\boldsymbol{\theta} = \{\rho_n, \rho_s, \mathbf{o}\}$  conjugate prior were assigned, (i.e.  $\rho_n \sim \mathcal{G}(\gamma_n, \phi_c)$ ,  $\rho_s \sim \mathcal{G}(\gamma_s, \phi_s)$  and  $\mathbf{o} \sim \mathcal{N}(\mathbf{m}_o, \mathbf{V}_o)$ ). Other shaping parameters ( $\gamma_n, \phi_n, \gamma_s, \phi_s, \mathbf{m}_o, \mathbf{V}_o$ ) are fixed to have flat distributions.

All the ingredient are now set to obtain the joint posterior distribution  $\mathcal{P}(\mathbf{s}, \mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\theta}|\mathbf{y})$ . However applying the JMAP or PM yields intractable form and an approximation is needed.

## 3. GRADIENT-LIKE VARIATIONAL BAYESIAN APPROACH

The major difficulty faced when working with the previous joint distribution is the mutual dependence between different variables which yields an enormous variable space. Therefore, the variational Bayesian approach, introduced by [16], proposes to approximate the joint posterior  $\mathcal{P}(\mathbf{s}, \mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\theta}|\mathbf{y})$  by a separable free form distribution  $Q(\mathbf{u}) = \prod_i Q(\mathbf{u}_i)$ ,  $\mathbf{u} = \{\mathbf{s}, \mathbf{p}, \boldsymbol{\rho}, \boldsymbol{\theta}\}$  that minimizes the Kullback-Leibler divergence,

$KL(\mathcal{P}||\mathcal{Q}) = \int \mathcal{Q}(\mathbf{u}) \log \left( \frac{\mathcal{P}(\mathbf{u}|\mathbf{y})}{\mathcal{Q}(\mathbf{u})} \right) d\mathbf{u} = \log(\mathcal{P}(\mathbf{y}|\mathcal{M})) + \mathcal{F}(\mathcal{Q})$ , where  $\mathcal{P}(\mathbf{y}|\mathcal{M})$  is the model evidence and  $\mathcal{F}(\mathcal{Q})$  is the free energy. For distributions from exponential family,  $\mathcal{Q}$  has an analytical form with mutually dependent parameters that should be updated singly which might be time consuming. Hence, the new variational approach [17] proposes to update the approximating marginals simultaneously. Like in gradient methods, this approach extends the notion of optimal step gradient to infinite functional space. So, the approximating marginals have an iterative functional form and their form at iteration  $k$  read  $\mathcal{Q}_k(\mathbf{u}_i) \propto (\mathcal{Q}(\mathbf{u}_i))^{1-\lambda} \exp\left(\lambda \langle \log(\mathcal{P}(\mathbf{u}, \mathbf{y})) \rangle_{\prod_{j \neq i} \mathcal{Q}(\mathbf{u}_j)}\right)$ , with  $\lambda$  being the gradient step that can be set to minimize the free energy at each iteration. Herein, a strong separation is chosen,  $\mathcal{Q}(\mathbf{u}) = \mathcal{Q}(\rho_n)\mathcal{Q}(\rho_s) \prod_i \mathcal{Q}(s_i)\mathcal{Q}(p_i)\mathcal{Q}(\rho_i) \prod_j \mathcal{Q}(o_j)$ . By applying approximating marginals equation, we obtain distributions from the same family as the priors,  $\tilde{\mathcal{Q}}(\mathbf{s}) = \mathcal{N}(\tilde{\mathbf{m}}_s, \tilde{\mathbf{V}}_s)$ ,  $\tilde{\mathcal{Q}}(\mathbf{p}) = \mathcal{N}(\tilde{\mathbf{m}}_p, \tilde{\mathbf{V}}_p)$ ,  $\tilde{\mathcal{Q}}(\rho) = \prod \mathcal{G}(\tilde{\gamma}_i, \tilde{\phi}_i)$ ,  $\tilde{\mathcal{Q}}(\rho_n) = \mathcal{G}(\tilde{\gamma}_n, \tilde{\phi}_n)$ ,  $\tilde{\mathcal{Q}}(\rho_s) = \mathcal{G}(\tilde{\gamma}_s, \tilde{\phi}_s)$ ,  $\tilde{\mathcal{Q}}(\mathbf{o}) = \mathcal{N}(\tilde{\mathbf{m}}_o, \tilde{\mathbf{V}}_o)$ ,

with  $\tilde{\mathbf{V}}_s^k = \left[ (1 - \lambda_s)(\tilde{\mathbf{V}}_s^{k-1})^{-1} + \lambda_s \mathbf{Diag}(\bar{\rho}_n \mathbf{H}^t \mathbf{H} + \bar{\rho}_s (\mathbf{D}_\alpha^t \mathbf{D}_\alpha + \mathbf{D}_\beta^t \mathbf{D}_\beta)) \right]^{-1}$ ,  
 $\tilde{\mathbf{m}}_s^k = \tilde{\mathbf{m}}_s^{k-1} + \lambda_s \tilde{\mathbf{V}}_s^k \left( \bar{\rho}_n \mathbf{H}^t (\mathbf{y} - \mathbf{H}(\tilde{\mathbf{m}}_p + \tilde{\mathbf{m}}_s)) - \bar{\rho}_s (\mathbf{D}_\alpha^t \mathbf{D}_\alpha + \mathbf{D}_\beta^t \mathbf{D}_\beta) \tilde{\mathbf{m}}_s \right)$ ,  
 $\tilde{\mathbf{V}}_p^k = \left[ (1 - \lambda_p)(\tilde{\mathbf{V}}_p^{k-1})^{-1} + \lambda_p \mathbf{Diag}(\bar{\rho}_n \mathbf{H}^t \mathbf{H} + \rho_p \tilde{\gamma} \tilde{\phi}^t) \right]^{-1}$ ,  
 $\tilde{\mathbf{m}}_p^k = \tilde{\mathbf{m}}_p^{k-1} + \lambda_p \tilde{\mathbf{V}}_p^k (\rho_n \mathbf{H}^t (\mathbf{y} - \mathbf{H}(\tilde{\mathbf{m}}_s + \tilde{\mathbf{m}}_p)) - \rho_p \tilde{\gamma} \tilde{\phi} \tilde{\mathbf{m}}_p)$ ,  
and  $\bar{\rho}_n = \tilde{\gamma}_n \tilde{\phi}_n$ ,  $\bar{\rho}_s = \tilde{\gamma}_s \tilde{\phi}_s$ . Other shaping parameters and theoretical details are given in [18]. At convergence (iteration  $K$ ), a MAP estimator is used on approximating marginals to obtain the estimates (i.e.  $\hat{\mathbf{s}} = \tilde{\mathbf{m}}_s^K$ ,  $\hat{\mathbf{p}} = \tilde{\mathbf{m}}_p^K$ , ...).

#### 4. RESULTS

The proposed method was tested using simulated and real data from the space observatory Herschel. For simulation, smooth component was generated using a sample from correlated Gaussian field ( $\rho_s = 200$ ) then superposed with 13 point sources with different intensities  $\mathcal{I}_p \in [0.5, 18]$  (fig.2.a). A white Gaussian noise ( $\rho_n = 600$ ) was added to the forward model output. Reconstruction result (fig.2.c)<sup>1</sup> shows a good accordance with the original map and a significant enhancement compared to *Coadd* (fig.2.b) method ( $\mathbf{x}_c = \frac{\mathbf{U}^t \mathbf{y}}{\mathbf{U}^t \mathbf{1}}$ ), used in the official data processing product [19]. The relative error in smooth component estimation for our method is  $\mathcal{E}_s = \sqrt{\frac{\|\mathbf{s} - \hat{\mathbf{s}}\|_2^2}{\|\mathbf{s}\|_2^2}} = 4\%$  and the error in flux estimation in point source reconstruction (fig.2.e)  $\mathcal{E}_p = 0.7\%$ . In comparison

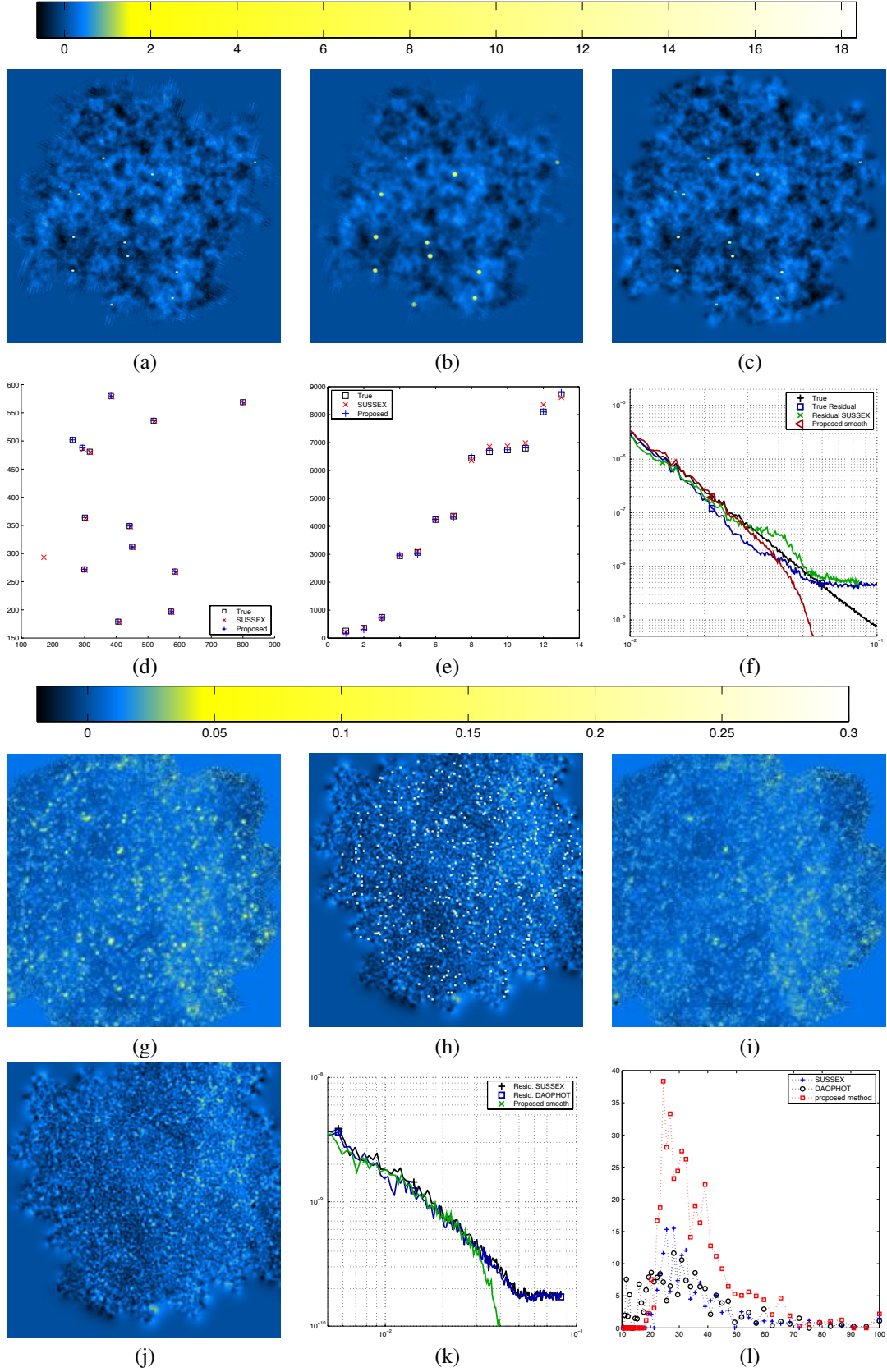
<sup>1</sup>A non-linear color-map was used to show the small variation in background representing smooth component. Meanwhile, high intensities are given almost the same color.

with separation based on *SUSSEX* [11], we find  $\mathcal{E}_{s,s} = 35\%$  and  $\mathcal{E}_{p,s} = 2.2\%$ . Furthermore, the positions (fig.2.d) of sources are accurately estimated by our method. Meanwhile *SUSSEX* missed the weakest source (top-left) and replaced it with a noisy point. This explains, in addition to reduced resolution and noise, the important error in the smooth component for *SUSSEX*. By studying the circular-mean power spectrum (CMPS)[18] for smooth components (fig.2.f), we see that the CMPS of our method corresponds to the true one for a large frequency interval and it cut-offs at the frequency where noise becomes dominant ( $f_n$ ). However, when all sources are removed perfectly from coadded map (*True residual*), its CMPS is below the true value due to the PSF effect for  $f < f_n$ . Beyond this frequency, the noise dominates the CMPS. Interestingly, having residual of point sources in the smooth component (*SUSSEX* case) deforms the CMPS and produces high power in high frequencies.

Real data were treated also for field (17p732) taken by instrument SPIRE for wave band PSW. Our method (fig.2.h) gives a sharper map compared to *Coadd* method (fig.2.g). Furthermore, we can see that the smooth part after applying *SUSSEX* (fig.2.i) still contains some point sources, even with lowering the threshold. In comparison, smooth component of our method  $\hat{\mathbf{s}}$  (fig.2.j) looks more homogeneous. However, by performing a CMPS study (fig.2.k) shows that both components have the same power for  $f < f_n$ . This may be explained by residual point sources in *SUSSEX* residual. Moreover, studying the point source reconstruction shows that our method can detect more sources (around 300 sources compared to 200 for *SUSSEX*). The comparison of a normalized histogram of source fluxes (fig.2.l) proves the increasing number of estimated sources by our method. For high fluxes, both methods estimate the same number of sources. Nevertheless, our method detects more sources for lower fluxes before it cut-off for so low fluxes due to noise. We compared also with *DAOPHOT* [10] with low threshold, many noise point were detected as sources without enhancing detection in middle fluxes range. However, more study is needed to validate the new sources from the astrophysical point of view.

#### 5. CONCLUSIONS

We have presented a Bayesian approach for high resolution map-making with joint separation between smooth and point sources. Discriminative priors (Markov field and t-distribution) were introduced for the separation and a new gradient like variational Bayesian method was applied to obtain the estimators. Method validation against real and simulated data confirms resolution enhancement and good separation capacity. However, noise model can be enhanced by supposing a pink one instead of the white one used. In addition, we are looking in non stationary Markovian models for cases when smooth components exhibit several variation modes.



**Fig. 2.** (a) Original simulated field  $x$ , (b) coaddition mapmaking, (c) our method  $\hat{x} = \hat{s} + \hat{p}$ , (d) point sources position in image index, (e) point sources fluxes [Jy/Beam], (f) circular-mean power spectrum, (g) coaddition map-making for field 17p732, (h) our method  $\hat{x}$ , (i) residual map of *SUSSEX* method (extended emission), (j) estimated extended emission  $\hat{s}$ , (k) circular-mean power spectrum, (l) normalized histogram: normalized number of sources vs flux [Jy/Beam]

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