IRREGULARLY SAMPLED SEISMIC DATA INTERPOLATION USING ITERATIVE HALF THRESHOLDING REGULARIZATION

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ABSTRACT

The recent progress of compressive sensing (CS) theory shows that perfect reconstruction is possible when the data are sampled randomly and compressed greatly. This revolutionary theory strongly advocates the irregular sampling pattern in the process of seismic data acquisition, leading to a highly reduced acquisition cost. The complete seismic record can be reconstructed from the sparsely few sampled seismic data. In this paper, an analysis version of iterative half thresholding algorithm is introduced to interpolate the seismic data, using the tight frame dictionary. Inspired by the newly proposed spectral compressive sensing theory and the favorable characteristics of tight frame, we employ the redundant Fourier transform to sparsely represent the oscillating seismic data. A double zero-padding strategy in spatial direction of seismic data is suggested to further enhance the reconstruction quality, allowing for the improved interpolation performance and the increased computational cost and storage requirement. The validity of the proposed method is demonstrated by experimental result.

***Index Terms*—** Compressive sensing, iterative shrinkage-thresholding, half thresholding, seismic trace interpolation, tight frame

1. INTRODUCTION

1.1. Background

Acquisition of reflection seismic data is a multidimensional resource-intensive process. However, fully recording the seismic data is unrealistic for many reasons: a finite number of active recording channels, surface obstacles, as well as some other physical and financial constraints [1]. Therefore, the prestack trace interpolation of these uniformly or nonuniformly sampled seismic records prior to migration is seriously needed [2].

Seismic data interpolation is an important and ongoing research area in seismic signal processing community. There exist many effective methods to recover the missing traces from a regularly or irregularly sampled seismic data. These methods can be roughly classified into two categories: (1) Using the linear, spatial prediction filters to interpolate regularly sampled seismic data [3, 4, 5, 6], just to name a few. These methods find the missing traces by buiding an auto regressive (AR) model, based on the principle that the missing traces spaced equally can be exactly interpolated by a set of linear equations [3]. (2) Reconstruction with certain types of transforms such as the Fourier transform [7], the Radon transform [8], and the curvelet transform [9]. Some well known algorithms which fall under this umbrella can handle the non-uniformly sampled seismic records, including iterative soft thresholding method [9], minimum weighted norm interpolation (MWNI) method [10], anti-leakage Fourier transform (ALFT) method [11], projection onto convex sets (POCS) method [12, 13, 14] etc.

1.2. This paper

The recent progress of the theory of compressive sensing (CS) [15, 16] provides a nice way to understand sparseness-based interpolation. The data are allowed to be sampled randomly and compressed greatly [17, 18, 19]. It leads to a highly reduced cost in the process of data acquisition, barely losing the information of geophysical data. Curvelet-based seismic interpolation using iterative soft thresholding algorithm have obtained convincing results [9].

This article will follow the line of sparse and redundant representations to interpolate the irregularly sampled seismic traces. It is organized as follows. In Section 2, we give the mathematical framework of sparsity-promoting seismic trace interpolation problem. The popular iterative shrinkage-thresholding (soft and hard thresholding) algorithm is presented. In Section 3, we give the analysis-based iterative half thresholding based on newly proposed $\ell_{1/2}$ regularization theory. Inspired by the spectral compressive sensing theory and the favorable characteristics of tight frame, we propose the redundant Fourier transform with double over zero-padding strategy to sparsely represent the oscillating seismic data, allowing for the improved reconstruction performance and the increased computational cost and storage requirement. The experimental results and the concluding remarks are given in Section 4 and in Section 5, respectively.

2. SPARSITY-PROMOTING SEISMIC TRACE INTERPOLATION

2.1. Problem statement

Now let us formulate the problem of seismic trace interpolation. The observed seismic record d_{obs}, including many missing samples, is connected with the complete seismic data d to be recovered via the relation

$$d_{obs} = Md, \tag{1}$$

in which M denotes a diagonal matrix with diagonal entries 1 for the observed samples and 0 otherwise. For the convenience of mathematical expression, a 2-D discrete seismic data set $\mathbf{d}_0 = [d_{t_1,t_2}]_{1\leq t_1\leq n_1, 1\leq t_2\leq n_2} \in \mathbb{R}^{n_1\times n_2}$ can be reordered as a vector $d \in \mathbb{R}^n$, $n = n_1 \times n_2$ through lexicographic ordering $d_{t_1,t_2} \rightarrow d_{(t_2-1)n_1+t_1}$. This kind of expression can be easily generalized to higher dimensional cases. To recover the missing traces in the seismic data, one needs to project the seismic data into certain transform domain, that is,

$$d = Ax, \tag{2}$$

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where $A \in \mathbb{R}^{n \times m}$ corresponds to a synthetic operator of an orthogonal basis ($n = m$) or a frame ($n \neq m$). This leads to

$$d_{obs} = Md = MAx = Kx, \tag{3}$$

where x indicate the representation coefficients in the transform domain, $K = MA$. Combining some priori knowledge, the problem of seismic trace interpolation can be written as

$$\min_x J(x) = \frac{1}{2}\|d_{obs} - Kx\|_2^2 + \tau R(x), \tag{4}$$

in which the priori constraint term can be $R(x) = \|x\|_p^p$, and $p = 2$ is exactly the classical Tikhonov regularization [20].

2.2. Iterative shrinkage-thresholding algorithm

The sparsity in certain transform domain always holds well from a large number of informed studies, and can be taken as a reasonable constraint [21], which corresponds to the ℓ_0-norm of the unknown x, i.e., $R(x) = \|x\|_0 = \#\{x_i \neq 0\}$. Due to the nonconvex complexity of the ℓ_0-norm minimization, the theory of compressive sensing (CS) shows that ℓ_1-norm can be taken as an alternative to ℓ_0-norm, and proves to be able to achieve the same sparseness as the ℓ_0-norm [16, 15]. With these sparsity constraints, the general form of the corresponding algorithm, namely the iterative shrinkage-thresholding (IST) algorithm, can be specified as

$$x^{k+1} = T_{\lambda(\tau,p)}(x^k + K^*(d_{obs} - Kx^k)) = T_{\lambda(\tau,p)}(B(x^k)), \tag{5}$$

where $B(x^k) = x^k + K^*(d_{obs} - Kx^k)$; k denotes the iteration number; K^* indicates the adjoint of K. $T_{\lambda(\tau,p)}(x)$ is the thresholding operator performed elementwise with threshold $\lambda(\tau, p)$ in the sense that

$$T_{\lambda(\tau,p)}(x) = (t_{\lambda(\tau,p)}(x_1), t_{\lambda(\tau,p)}(x_2), \ldots, t_{\lambda(\tau,p)}(x_m))^T \tag{6}$$

and

$$t_{\lambda(\tau,p)}(u) := \begin{cases} f_{\lambda(\tau,p)}(u) & |u| > \lambda(\tau, p) \\ 0 & |u| \leq \lambda(\tau, p), \end{cases} \tag{7}$$

When $p = 1$, $R(x) = \|x\|_1$, we have

$$f_{\lambda(\tau,1)}(u) = u - \lambda\frac{u}{|u|}, \lambda(\tau, 1) = \tau. \tag{8}$$

When $p = 0$, $R(x) = \|x\|_0$, we have

$$f_{\lambda(\tau,0)}(u) = u, \lambda(\tau, 1) = \sqrt{2\tau}. \tag{9}$$

(8) and (9) are the so-called soft and hard thresholding [22, 23].

3. PROPOSED ANALYSIS-BASED ITERATIVE HALF THRESHOLDING METHOD USING REDUNDANT FOURIER TRANSFORM

3.1. Iterative half thresholding algorithm and regularization parameter-setting strategy

Recently, researchers show that exact reconstruction is possible with substantially fewer measurements using nonconvex ℓ_p-norm minimization, $0 < p < 1$ [24]. Particularly, an $\ell_{1/2}$ thresholding representation theory has been developed [25], and the corresponding iterative half thresholding algorithm is also suggested. This algorithm can also be considered as a special case of IST algorithm (5). The only difference lies in the thresholding function. That is to say,

$$\begin{cases} p = 1/2, R(x) = \|x\|_{1/2}^{1/2} \\ \lambda(\tau, 1/2) = \frac{3}{2}\tau^{2/3} \\ f_{\lambda(\tau,1/2)}(u) = \frac{2}{3}u\left(1 + \cos(\frac{2}{3}\pi - \frac{2}{3}\arccos(\frac{\tau}{8}(\frac{|u|}{3})^{-\frac{3}{2}}))\right). \end{cases} \tag{10}$$

Suppose the solutions of the sparsity-promoting seismic data interpolation problem (4), $x^* \in \mathbb{R}^m$, are of q-sparsity. Without loss of generality, we assume: $|[B(x^*)]_1| \geq |[B(x^*)]_2| \geq \ldots \geq |[B(x^*)]_m|$. Thus, according to (10), in iterations we require:

$$\begin{cases} |B(x^*)|_i > \lambda(\tau, 1/2) = \frac{3}{2}\tau^{2/3}, i \in \{1, 2, \ldots, q\} \\ |B(x^*)|_j \leq \lambda(\tau, 1/2) = \frac{3}{2}\tau^{2/3}, j \in \{q + 1, \ldots, m\}, \end{cases}$$

which implies

$$\left(\frac{2}{3}|[B(x^*)]_{q+1}|\right)^{3/2} \leq \tau < \left(\frac{2}{3}|[B(x^*)]_q|\right)^{3/2}.$$

Therefore, a natural regularization parameter-setting strategy can be obtained with ease:

$$\tau^k = (1-\alpha)\left(\frac{2}{3}|[B(x^k)]_{q+1}|\right)^{3/2} + \alpha\left(\frac{2}{3}|[B(x^k)]_q|\right)^{3/2}, \tag{11}$$

where $\alpha \in [0, 1)$.

3.2. The analysis-based IST algorithm: a general formulation

It is interesting to note that (5) is a synthesis formulation since the complete seismic data d can be synthesized from its representation coefficients, $\hat{d} = A\hat{x}$, where $\hat{x}$ is the minimizer of problem (4) [26]. In what follows we will present its analysis formulation.

Consider a tight frame A such that $A^*A = I$. Recall that $K = MA$, M is a diagonal matrix such that $M^* = M = M^2$, $M^*d_{obs} = M^2d = d_{obs}$. From (5) we obtain

$$\begin{aligned} d^{k+1} &= Ax^{k+1} \\ &= AT_{\lambda(\tau,p)}(x^k + K^*(d_{obs} - Kx^k)) \\ &= AT_{\lambda(\tau,p)}(A^*d^k + (MA)^*(d_{obs} - MAx^k)) \\ &= AT_{\lambda(\tau,p)}(A^*(d^k + M^*(d_{obs} - Md^k))) \\ &= AT_{\lambda(\tau,p)}(A^*(d_{obs} + (I - M)d^k)). \end{aligned}$$

This updating rule analyzes the complete seismic data to be reconstructed as the unknown directly. Thus we call

$$d^{k+1} = AT_{\lambda(\tau,p)}(A^*(d_{obs} + (I - M)d^k)) \tag{12}$$

the analysis-based IST algorithm. The differences among ℓ_1, ℓ_0 and $\ell_{1/2}$ constraints only correspond to different thresholding functions, as shown in (8), (9) and (10). The analysis version implies smaller memory requirement in iterations in cases that the number of transforming coefficients is much larger than the number of data elements when a tight frame based transform A is used [27].

3.3. Excessively zero-padded FFT: an ideal tight frame dictionary with a spectral compressive sensing perspective

As can be seen from the previous subsection above, our analysis-based iterative half thresholding algorithm only requires A to be a tight frame. The informed studies has shown that a redundant tight frame preserves energy and ensures an isometric relation between the input signal and the output coefficients, allowing us to use its adjoint as its inverse. More importantly, its redundancy, resulting from its overlapping supports, leads to robust signal representation for the complex structure of the signals [28]. Thus, partial loss of data can be tolerated without adverse effects [29]. These characteristics stimulate researchers to utilize the tight frame based transforms (e.g., undecimated discrete wavelet transform (UDWT) [30], curvelet transform [27], framelet transform [29]) in image denoising, inpainting and many other important applications.

Different from natural images, the seismic data can be considered as the convolved result of the underground layer structure and the seismic wavelet (always a ricker wavelet). Thus the oscillating features in seismic data requires the Fourier transform as an ideal dictionary for sparse representation. Inspired by the newly proposed spectral compressive sensing (SCS) theory [31] and the intriguing properties of frames, in this article we suggest using the redundant Fourier transform as the sparsifying dictionary A, which can be easily constructed by zero-padding. Conventionally, the Fourier transform is preferred to pad to the length of powers of 2 to reduce the computational cost and memory requirement, using the fast and in-place implementation. For the seismic data $\mathbf{d}_0 \in \mathbb{R}^{n_1 \times n_2}$, $2^{p_1-1} \leq n_1 < 2^{p_1}$, $2^{p_2-1} \leq n_2 < 2^{p_2}$, a practical and frequently adopted strategy is padding zeros to the size $2^{p_1} \times 2^{p_2}$.

To combat the spectral leakage and improve the quality of the reconstructed seismic data,in this paper we propose an excessive zero-padding scheme for FFT: The seismic data should be zero-padded to the size $L_1 = N_1 \times 2^{p_1}$, $L_2 = N_2 \times 2^{p_2}$,yielding the augmented data

$$\mathbf{d} = \begin{pmatrix} \mathbf{d}_0 & \mathbf{0}_{n_1 \times (L_2 - n_2)} \\ \mathbf{0}_{(L_1-n_1)\times n_2} & \mathbf{0}_{(L_1-n_1)\times(L_2-n_2)} \end{pmatrix}. \tag{13}$$

As will be shown in our experiment result, double zero-padding in spatial (offset) direction of the seismic data ($N_2 = 2$), is a nice tradeoff allowing for the improved reconstruction performance and the increased computational cost and storage requirement.

4. EXPERIMENTAL RESULTS

We also give the optimal regularization parameter-setting scheme for soft and hard thresholding for the convenience of comparison in our experiment:

$$(soft) : \tau^k = (1-\alpha)|[B(x^k)]_{q+1}| + \alpha|[B(x^k)]_q|,$$

$$(hard) : \tau^k = (1-\alpha)\left(\frac{1}{2}|[B(x^k)]_{q+1}|^2\right) + \alpha\left(\frac{1}{2}|[B(x^k)]_q|^2\right),$$

which can be easily derived in the similar way as (11). The q-sparsity is controlled by the ratio q/m, which is the proportion of coefficients to be thresholded in all resulting coefficients under frame A. To evaluate the quality of our recovery, we define signal-to-noise-ratio (SNR) as $\mathrm{SNR} = 10\,\mathrm{lg}(\|d\|^2/\|d-\hat{d}\|^2)$ in decibel (dB).

We utilize one shot seismic data produced using Marmousi model (a benchmark model in geophysical community), as show in Fig. 1a. It is randomly sampled with a decimating rate 30%, see Fig. 1b. The algorithms are set to perform 100 iterations to obtain a stable converged solution. In all our experiment, the parameters are set: $\alpha = 0$, $q = 10\%m$. The first experiment is dedicated to show the validity of the iterative half thresholding algorithm. The interpolated result of this method is also compared with that of the popular soft and hard thresholding method, see Fig. 1b–d. The difference between the reconstructed data and the original complete one are

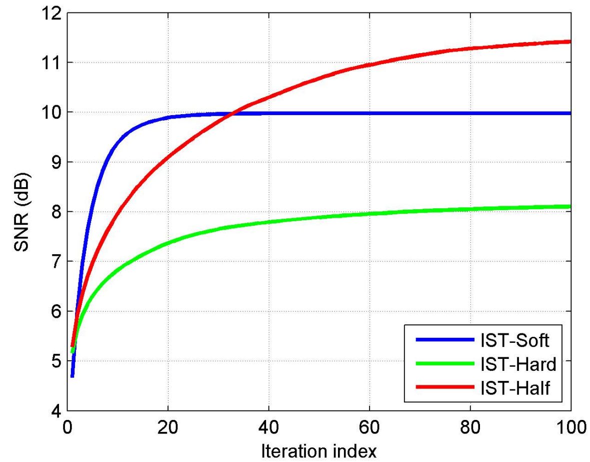

Fig. 2. The SNRs of soft, hard and half thresholding method

Table 1. The SNRs of different zero-padding strategies

(N_1, N_2)	(1,1)	(1,2)	(1,4)	(2,1)	(4,1)	(2,2)
SNR (dB)	14.28	14.67	14.68	14.28	14.28	14.67

shown in Fig. 1b'–d'. clearly, our method do no harm to the structure of the seismic data. We also plot the SNR curve in Fig. 2 to monitor the varying reconstruction performance. As can be seen from this figure, half thresholding is indeed superior to soft and hard thresholding, according to the final result of SNR.

Note that the above result is obtained without zero-padding (data size $n_1 \times n_2$). The next experiment is devoted to find the proper zero-padding parameters $N_i (i = 1, 2)$. In view of the increased computational cost and memory requirement, we only consider the cases: $N_i = 1, 2, 4$. From Table 1, we surprisingly find that: (1) Zero-padding in temporal direction of the seismic data does nothing to improve the reconstruction performance; (2) Double over zero-padding in spatial (offset) direction can obtain better SNR, and more zero-padding improves the reconstruction very little.

5. CONCLUSION

This paper addresses seismic trace interpolation as a $\ell_{1/2}$ regularization problem. The analysis-based iterative half thresholding algorithm combined with the redundant Fourier transform is utilized. Compared with soft and hard thresholding, the validity of the half thresholding method is demonstrated by numerical example, using a shot produced by Marmousi model. A double over zero-padding strategy in spatial direction of the seismic data can further enhance the reconstruction quality. Even though this paper only presents the result of the 2-D seismic data, multidimensional implementation of the proposed methods will definitely enhance the interpolated results, and is a subject of further research.

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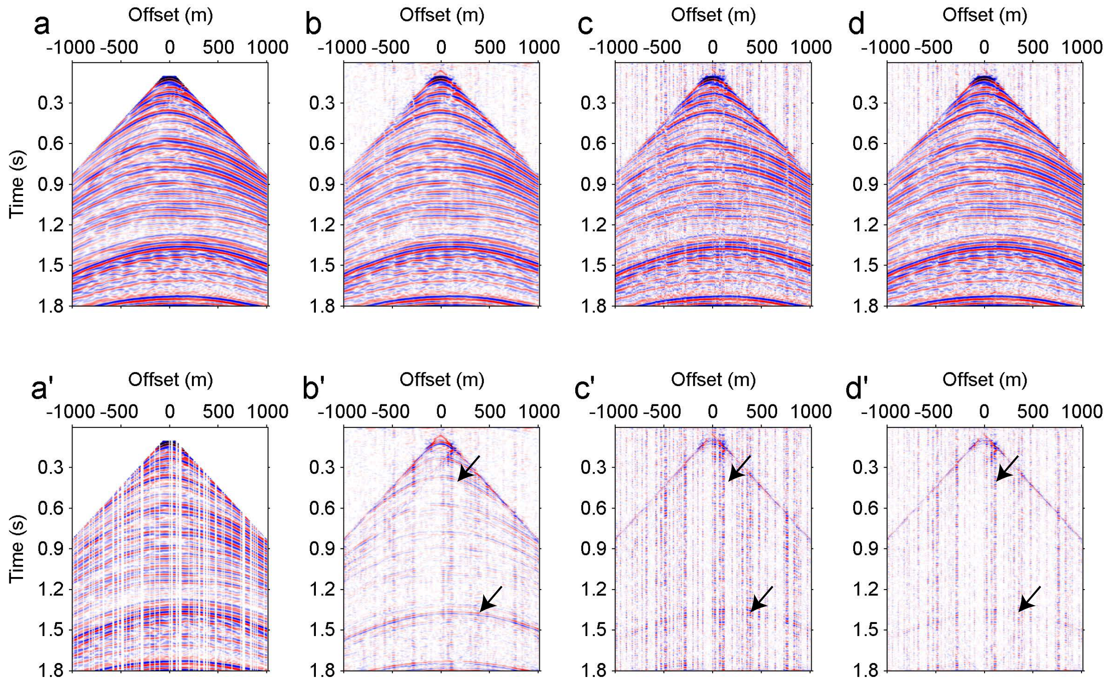

Fig. 1. (a) The complete synthetic data (one shot produced using Marmousi model, first arrival muted): 201 traces, 600 samples in each trace, sampling interval=0.003 s. (a') Irregularly sampled data of (a), random decimating rate=30%, SNR=5.19 dB. (b–d) Interpolated result using iterative soft, hard and half thresholding algorithm, SNR= 9.98 dB, 8.10 dB and 11.41 dB. (b'–d') The difference between (b), (c), (d) and (a).

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