NON-PARAMETRIC DATA PREDISTORTION FOR NON-LINEAR CHANNELS WITH MEMORY

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ABSTRACT

With the growing application of high order modulation techniques, the mitigation of the non-linear distortions introduced by the power amplification, has become a major issue in telecommunication. More sophisticated techniques to counteract the strong generated interferences need to be investigated in order to achieve the desired power and spectral efficiency. This work proposes a novel approach for the definition of a transmitter technique (predistortion) that outperforms the standard methods with respect to both performance and efficiency.

Index Terms— Predistortion, Non-linear channel, Polynomial channel model solution.

1. INTRODUCTION

Signal power amplification is necessary to achieve the desired SNR at the receiver. However the process is hardly linear due to the inherent characteristics of the amplifier which includes the saturation effect [1]. This non-linear amplification results in inter-symbol interfere (ISI) at the receiver thereby causing degradation [10]. On the other hand, efficient power amplification requires the amplifier to be operated very close to its saturation region where non-linear effects are stronger. Towards exploiting higher power gain, mitigating the non-linear effects of power amplification has been given a priority in both satellite [2] and terrestrial communications [4]. With the increasing and widespread use of high order modulation schemes towards achieving higher throughput, the mitigation of the non-linear interference generated by the power amplifier has become even more challenging. Multilevel modulation schemes are spectrally efficient but excite severe non-linear distortions due to the inherent high peak to average power ratio (PAPR) typical of the non-constant envelope signals (e.g. QAM and APSK). Proper countermeasures need to be put in place to guarantee the required throughput and the power efficiency. In most applications, it is often more desirable to counteract the generated non-linear interferences at the transmitter side with specific signal pre-processing technique generally known as predistortion. Such a processing does not entail a change in the existing user terminals, thereby making it market attractive.

One of the most consolidated approaches defines the predistortion as a nonlinear function that approximates the equivalent channel inverse function. Such an approach is paraterized by a certain number of kernel coefficients. A large number of techniques belong to this channel inverse predistortion function category, for e.g., analytic channel inverse function [3], Volterra series [4], memory polynomials [5, 6, 7] and orthogonal polynomials [8, 9]. The non-parametric approaches for predistortion elaborated in literature, rely on iterative numerical optimization techniques and do not exploit directly any channel information [10, 11].

In this work we propose a novel non-parametric data predistortion method based on the point-wise solution of the non-linear channel equation. The new technique does not suffer of the typical inaccuracies of the channel inverse based predistortion techniques [3]-[9], exploits channel model information in a better way compared to [10, 11] and provides significant gain in performance. A variation of the method suitable for reduced complexity implementation is also developed showing negligible performance loss.

The rest of the paper is organized as follows: Section 2 defines the non-linear channel model, Section 3 describes the novel predistortion technique, Section 4 provides a tailored version of the introduced technique suitable for an efficient implementation, Section 5 numerically compares the designed techniques with memory polynomial predistortion and some conclusions are drawn in Section 6.

Notations : * represents complex conjugation and $\mathbb{E}$ is the ensemble average.

2. SYSTEM MODEL

A general communication chain consisting of a non-linear channel with memory is represented in Fig. 1. The constellation symbols, $\{a_n\}$, drawn from a M- sized constellation set S_M are pre-distorted to obtain the transmitted symbols $\{x_n\}$. The non-linear channel contains linear transmit and receive filters (pulse shaping filter or Input/Ouptut multiplexing filters) and a non-linear amplifier. Denoting $H_{nl}(\cdot)$ as the channel non linear function, μ_n as the noise and $X_{n,K} = [x_n, \ldots, x_{n-K}]$ as a stacking of transmitted symbols, the received symbol at nth instance, r_n, can be expressed as

$$r_n = H_{nl}(X_{n,\infty}) + \mu_n \tag{1}$$

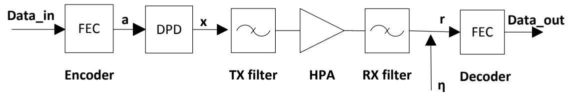

Fig. 1. Non-linear Channel: DPD denotes Digital Predistortion block

$H_{nl}(\cdot)$ can be expressed using the Volterra expansion [12] as

$$\sum_{p=0}^{\infty} \sum_{(k_0,\ldots,k_{2p})=0}^{\infty} h^{(p)}_{k_0,\ldots,k_{2p}} \prod_{j=0}^{p} x_{n-k_j} \prod_{i=p+1}^{2p} [x_{n-k_i}]^*. \quad (2)$$

Here $h^{(p)}_{k_0,\ldots,k_{2p}}$ denotes the Volterra kernel coefficients. The full Volterra model is a highly complex representation of the channel function due to the presence of all cross terms $\{k_{2l}\}$. A reduced complexity version of $H_{nl}(\cdot)$ is the memory polynomial that does not include all the cross terms [13]

$$H_{nl}(X_{n,\infty}) \approx \sum_{\substack{p=0\\(even)}}^{\infty} \sum_{k=0}^{\infty} h^{(p)}_k x_{n-k}|x_{n-k}|^p. \quad (3)$$

In general, both models ((2) and (3)) include only odd polynomial terms in x_n generating the relevant *in-band* distortion. Concerning channel estimation, polynomial channel models of (2) and (3) are linear equations in their kernel parameters $h^{(p)}_{k_0,\ldots,k_{2p}}$. Because of this linear relation, the estimation of $h^{(p)}_{k_0,\ldots,k_{2p}}$ is a linear least-square (LS) problem that can be solved using standard training based techniques [13]. To focus on the predistortion technique, we assume the channel model parameters to be fixed and that they have been computed *off-line*.

3. NON-PARAMETRIC PREDISTORTION BASED ON CHANNEL MODEL

Towards a low-complexity implementation, we use the simplified channel model of (3) limiting channel model degree to $D+1$ and memory to K in (3). We henceforth consider

$$H_{nl}(X_{n,K}) = \sum_{\substack{p=0\\(even)}}^{D} \sum_{k=0}^{K} h^{(p)}_k x_{n-k}|x_{n-k}|^p. \quad (4)$$

The ideal predistortion function would guarantee the minimization of the error between the received symbols r_n (1) and the intended transmitted symbols a_n. Such a symbol level approach is not feasible since the transmitter is not privy to the received symbols. Instead, we consider,

$$x_n = \underset{x_n, 0<|x_n|^2<P_x}{\arg\min} \{|H_{nl}(X_{n,K}) - a_n|^2\}. \quad (5)$$

The power constraint in (5) arises from the power constraint served by the transmitter. It also helps to avoid infeasible solutions arising due to the finite degree channel model approximation.

Clearly the minima of (5) is obtained when $H_{nl}(X_{n,K}) = a_n$ if the resulting solution satisfies $0 < |x_n|^2 < P_x$. Towards solving $H_{nl}(X_{n,K}) = a_n$ using (4), we obtain,

$$a_n = \sum_{\substack{p=0\\(even)}}^{D} \sum_{k=0}^{K} h^{(p)}_k x_{n-k}|x_{n-k}|^p. \quad (6)$$

Equation (6) is non-linear with memory in the complex variable x_n that can be transformed in two distinct equations in real variables: a polynomial equation for the amplitude of x_n and a linear equation for the phase. These two equations will be derived in the following.

3.1. Amplitude of the Predistorted Symbol

Equation (6) can be rewritten as:

$$\tilde{a}_n = \sum_{\substack{p=0\\(even)}}^{D} h^{(p)}_0 x_n|x_n|^p \quad (7)$$

$$\tilde{a}_n = a_n - \sum_{\substack{p=0\\(even)}}^{D} \sum_{k=1}^{K} h^{(p)}_k x_{n-k}|x_{n-k}|^p \quad (8)$$

Applying the magnitude operator on both sides of (7), we obtain a $(D+1)$th degree real polynomial equation in $|x_n|^2$:

$$\sum_{\substack{p_1=0\\(even)}}^{D} \sum_{\substack{p_2=0\\(even)}}^{D} h^{(p_1)}_0 [h^{(p_2)}_0]^* |x_n|^{2+p_1+p_2} = |\tilde{a}_n|^2. \quad (9)$$

Assuming $\tilde{a}_n$ to be known, we can find the optimal $|x_n|^2$ as the *positive* solution of (9). In order to obtain a solution we need to find the roots of a real polynomial of degree $(D+1)$. Closed form polynomial solutions are derived up to the third degree and numerical evaluation is applied for higher degrees. If no valid solutions to (9) exist or the resulting solution does not satisfy $0 < |x_n|^2 < P_x$, we redefine the amplitude $|x_n|^2$ as a solution of (9)

$$|x_n|^2 = \underset{0<|x_n|^2<P_x}{\arg\min} \{(f(|x_n|^2) - |\tilde{a}_n|^2)^2\} \quad (10)$$

$$f(|x_n|^2) = \sum_{\substack{p_1=0\\(even)}}^{D} \sum_{\substack{p_2=0\\(even)}}^{D} h^{(p_1)}_0 [h^{(p_2)}_0]^* |x_n|^{2+p_1+p_2}. \quad (11)$$

The problem defined in (10) can be solved finding the local maximum of the polynomial function $f(|x_n|^2)$ under the condition $0 < |x_n|^2 < P_x$ using first and second order derivatives. Alternatively, a purely numerical approach would require a search for the minimum of $(f(|x_n|^2) - |\tilde{a}_n|^2)^2$ in the closed interval $0 < |x_n|^2 < P_x$.

3.2. Phase of the Predistorted Symbol

Once we obtain a valid solution for (9), we can derive the phase of x_n by using the phase relations of (7) as,

$$\angle x_n = \angle \tilde{a}_n - \angle \sum_{\substack{p=0 \\ (even)}}^{D} h_0^{(p)} |x_n|^p. \quad (12)$$

The above process generates a predistorted symbol solving (12) and (9) [or (10)]. This requires information about $\tilde{a}_n$, which in turn, depends on previous predistorted symbols. As a result, x_n needs to be computed for each n and the complexity of such a process is very high. We now consider a reduced complexity approach that allows for *off-line* calculation of x_n and use it as a Look Up Table (LUT).

4. REDUCED COMPLEXITY IMPLEMENTATION

The information about the previous symbols is the cause of increased complexity. Towards implementing the process as a low complexity LUT, we choose to approximate x_{n-k} by their centroids in (8). Centroids of the predistorted symbols are defined as the solution of

$$\mathbb{E}[\tilde{a}_n | a_n] = \sum_{\substack{p=0 \\ (even)}}^{D} h_0^{(p)} \bar{x}_n |\bar{x}_n|^p, \quad (13)$$

where the averaging is performed over the previous transmitted symbols ($\{x_{n-k}\}$ or equivalently $\{a_{n-k}\}$) and $\bar{x}_n$ is defined as the centroid of x_n. For obtaining $\mathbb{E}[\tilde{a}_n|a_n]$, we take recourse to the numerically observed fact that $\mathbb{E}[\sum_{\substack{p=0 \\ (even)}}^{D} \sum_{k=1}^{K} h_k^{(p)} \; x_{n-k}|x_{n-k}|^p | a_n] \approx 0$. Using this and (8) leads to,

$$\mathbb{E}[\tilde{a}_n | a_n] \approx a_n \quad (14)$$

This approximation allows us to define centroids as a solution of an auxiliary $(D+1)$th degree equation in (15) that can be solved as described in Section 3

$$a_n = \sum_{\substack{p=0 \\ (even)}}^{D} h_0^{(p)} \bar{x}_n |\bar{x}_n|^p. \quad (15)$$

Solving (15) allows to map, *off-line*, each constellation symbol with the corresponding centroid. For a finite channel memory K, knowing $\{a_{n-k}\}$, we compute $\{\bar{x}_{n-k}\}$ and use these centroids to approximate $\tilde{a}_n$ as

$$\tilde{a}_n \approx a_n - \sum_{\substack{p=0 \\ (even)}}^{D} \sum_{k=1}^{K} h_k^{(p)} \bar{x}_{n-k} |\bar{x}_{n-k}|^p. \quad (16)$$

The value of $\tilde{a}_n$ evaluated in (16) can be used in (7) and the resulting equation solved to get an approximation of x_n. Notice that in (16) we obtained an approximation of $\tilde{a}_n$ as an implicit function of $[a_{n-K}, \dots, a_{n-1}]$ using only the estimated centroids of the predistorted symbols and a_n. Since the centroid and channel computations are off-line, hence x_n can be obtained off-line entirely and a LUT generated. Such a LUT maps $[a_{n-K}, \dots, a_n]$ to $[\tilde{x}_n]$ and has a dimension of M^{K+1}.

5. NUMERICAL RESULTS

In this section we compare the predistortion techniques designed in Sections 3 and 4 against standard memory polynomial predistortion [5]. To this end we simulated the channel of Fig 1 and Table 1 details the simulation parameters. The Saleh model is a memoryless non-linearity with AM/AM

Table 1. Simulation parameters

Parameter	Value
HPA model	Saleh Model [1]
TX/RX filters	Square Root Raised Cosine, roll-off=0.25
Modulation	32APSK
Coding	LDPC $3/4$

AM/PM characteristics: $A(r) = \frac{\alpha_1 r}{1+\alpha_2 r^2}$, $\Phi(r) = \frac{\beta_1 r^2}{1+\beta_2 r^2}$ with parameters [$\alpha_1 = 1$, $\alpha_2 = 0.25$, $\beta_1 = 0.26$, $\beta_2 = 0.25$]. The predistortion technique based on real-time roots computation (refer to Section 3) has been implemented assuming the channel model in (4), with a memory depth $K = 1$ and polynomial degree $D + 1 = 5$. For the same channel characteristics, we also implemented the reduced complexity predistortion method described in Section 4 generating a LUT with $M^{K+1} = 32^2$ entries addressed with $(K+1)\log_2(M) = 10$ bits. In either case the channel estimation is based on 15000 training symbols and the linear LS minimization [13]. For the matter of comparison, we devised a memory polynomial predistorter as in [5]. This memory polynomial predistorter function has a memory depth of $K = 1$, polynomial degree of $D + 1 = 5$ and is estimated using the indirect learning method [14] with 15000 training symbols.

As metric for HPA power efficiency we use the OBO (Out Back Off) as $OBO = 10 \log \frac{P_{out}}{P_{out}^{SAT}}$ where P_{out} and P_{out}^{SAT} are the output and saturated powers of the HPA, respectively. The OBO defines the working point of the HPA and controls the level of non-linear effects as well as the overall signal power level. Non linear interferences are stronger close to the saturation region (OBO$\approx$ 0 dB) while they tend to disappear moving to the linear region (OBO$\rightarrow \infty$). However, the overall signal power decreases when OBO increases, resulting in a degradation of the effective SNR for a fixed level of noise power at the receiver.

Performance can be evaluated in absence of noise by means of the Normalized Mean Square Error (NMSE) defined as $\mathbb{E}[|r_n - a_n|^2/|r_n|^2]$. Fig 2 shows how the NMSE

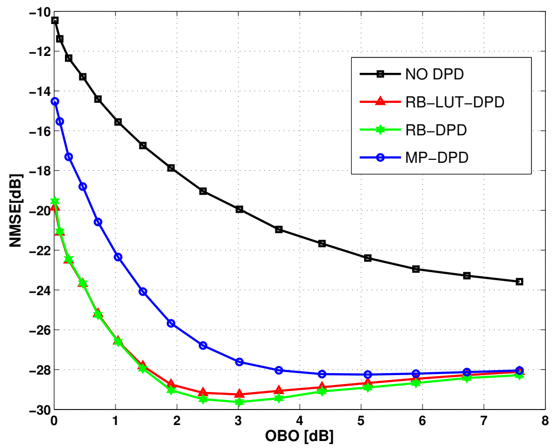

Fig. 2. NMSE vs OBO (Noiseless)

varies with respect to the OBO. We can notice a dramatic reduction in the interference level (here measured as NMSE) for the new techniques (legends RB-DPD and RB-LUT-DPD for techniques described in Section 3 and 4, respectively) compared to the standard memory polynomial predistortion (MP-DPD). Moreover, the performance loss between the real time roots computation (RB-DPD) and its complexity reduced version (RB-LUT-DPD) is almost negligible. The slight increase in NMSE for the predistortion techniques at high OBO can be attributed to the channel mismatch.

Having demonstrated a significant NMSE gain in the non-linear region for the noiseless case, we evaluate the BER trend (see Fig 3) with the amplifier operating very close to saturation (OBO = 1 dB). Fig. 3 provides a measure of the Es/N_0

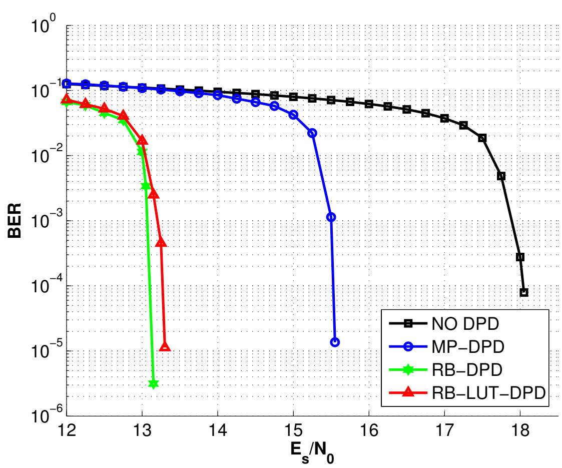

Fig. 3. BER performance of predistortion techniques for OBO=1dB

gain of the new method over the standard memory polynomial technique and the negligible loss in performance due to approximations is also illustrated. In order to investigate BER behavior with respect to the OBO, we set a fixed noise level at the receiver of $N_o = 15dB + E_s^{(SAT)}$ where $E_s{}^{(SAT)}$ is the average signal energy received when the amplifier is in saturation (OBO = 0 dB).

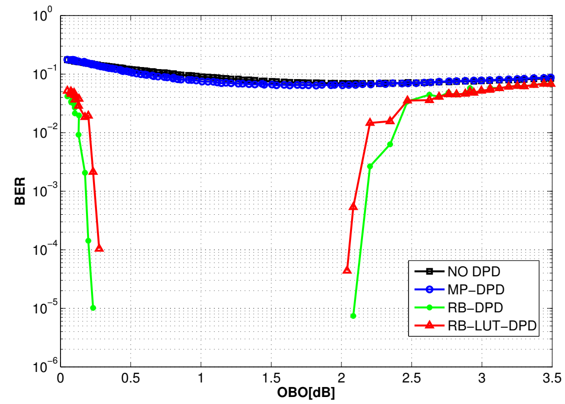

Fig. 4. Impact of OBO on BER of different predistortion techniques

Fig. 4 illustrates the variations in BER due to OBO. Close to the saturation region, the BER is influenced by the strong non-linear interferences, while moving toward the linear region of the amplifier, the BER rises again due to the reduction in the received SNR. For the chosen settings, it can be seen that the devised techniques provide a range of OBO in which the BER is negligible. This is due to the enhanced mitigation offered by the proposed techniques that allow for the optimal performance of LDPC. On the other hand, for the MP-DPD, an increase in Es/N_0 is needed to obtain improved BER.

6. CONCLUSION AND FUTURE WORK

A novel transmitter based technique for the mitigation of the impairments generated by a non-linear channel was designed. Exploiting the transmission of finite constellation symbols, this method provided significant gain over the most commonly applied predistortion techniques. A reduced complexity implementation yielding a LUT was also provided. Such a LUT based technique is a promising candidate for incorporation in next generation terrestrial as well as satellite systems towards improving power and spectral efficiencies. Future research will target the complexity reduction of the LUT as well as the possible extension to the multicarrier scenario.

7. REFERENCES

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