# ESTIMATION OF NONSTATIONARY HARMONIC SIGNALS AND ITS APPLICATION TO ACTIVE CONTROL OF MRI NOISE

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## ABSTRACT

A new adaptive comb filtering algorithm, capable of tracking the fundamental frequency and amplitudes of different frequency components of a nonstationary harmonic signal embedded in white measurement noise, is proposed. Frequency tracking characteristics of the new scheme are studied analytically, proving (under Gaussian assumptions and optimal tuning) its statistical efficiency for quasi-linear frequency changes. Laboratory tests show that the proposed algorithm can be successfully used for active control of MRI noise.

***Index Terms—*** Adaptive comb filtering, active noise cancelling

## 1. INTRODUCTION

Rotating machinery generates disturbances (noise, vibrations) that usually consist of several sinusoidal components with frequencies, called harmonics, that are integer multiples of the fundamental frequency. When the speed of rotation and/or working load change over time, such signals are subject to both amplitude and frequency changes while preserving their harmonic structure – for this reason they can be called quasi-harmonic. MRI (magnetic resonance imaging) equipment is another well-known source of quasi-harmonic disturbances – due to its high intensity, exceeding 100 dB SPL, MRI noise (generated by vibrating gradient coils) is very annoying both for the patients and for the medical staff.

Quasi-periodic signals with multiple frequency components that are not harmonically related can be estimated/tracked using parallel estimation schemes made up of algorithms known as adaptive notch filters (ANFs) – for more details see e.g. [1]. Since each filter comprising such a bank of ANFs works independently of other filters, the harmonic structure of the analyzed signal – when present – can't be exploited in any way. This results in less efficient tracking, both in terms of accuracy and robustness, compared to estimation based on coordinated frequency search. The algorithms that perform such a coordinated search are called adaptive comb filters (ACFs) [2]–[6].

The contribution of this paper is threefold. First, we propose a novel ACF algorithm and study its tracking properties. Second, we incorporate this new harmonic tracker into SONIC (self-optimizing narrowband interference canceller) – the active noise control algorithm proposed in [7]-[10]. Finally, we demonstrate effectiveness of the new control scheme when applied to reduction of MRI noise.

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## 2. ADAPTIVE COMB FILTER

Consider the problem of extraction or cancellation of a complex-valued nonstationary sinusoidal disturbance $d(t)$ embedded in white measurement noise $v(t)$

$$y(t) = d(t) + v(t) \tag{1}$$

where $y(t)$ denotes the measured signal and $t = \ldots, -1, 0, 1, \ldots$ denotes normalized discrete time. We will assume that

$$d(t) = \sum_{k=1}^{K} d_k(t), \quad d_k(t) = a_k(t)e^{j\phi_k(t)}$$

$$a_k(t) = \beta_k(t)e^{j\nu_k}, \quad \phi_k(t) = \sum_{i=1}^{t} \omega_k(i) \tag{2}$$

where the quantities $\beta_k(t)$, $\omega_k(t)$, and $\nu_k$ (all real-valued) denote respectively the (slowly time varying) amplitude, instantaneous frequency, and initial phase shift of the $k$-th cisoid $d_k(t)$. Furthermore, we will assume that the signal $d(t)$ is quasi-harmonic, i.e., that the frequencies $\omega_k(t)$ are harmonically related

$$\omega_k(t) = m_k\omega_0(t), \;\; k = 1, \ldots, K \tag{3}$$

where $\omega_0(t)$ denotes the fundamental frequency and $m_k$ are integer numbers ($m_k = k$ when all harmonics are present, $m_k = 2k - 1$ when only odd harmonics are present etc.).

### 2.1. Adaptive notch filtering algorithm

A single nonstationary cisoid ($K = 1$) governed by

$$d(t) = a(t)f(t), \;\; f(t) = e^{j\sum_{i=1}^{t}\omega(i)}$$

can be efficiently tracked using the ANF algorithm proposed in [1], presented below in a slightly modified (but equivalent) form

$$\begin{aligned}
\varepsilon(t) &= y(t) - \widehat{d}(t|t-1)\\
\widehat{d}(t+1|t) &= e^{j\widehat{\omega}(t|t-1)}[\widehat{d}(t|t-1) + \mu\varepsilon(t)]\\
g(t) &= \frac{\mathrm{Im}[\varepsilon(t)\widehat{d}^*(t|t-1)]}{|\widehat{d}(t|t-1)|^2}\\
\widehat{\omega}(t|t-1) &= \widehat{\omega}(t-1) + \widehat{\alpha}(t-1)\\
\widehat{\alpha}(t) &= \widehat{\alpha}(t-1) + \gamma_\alpha g(t)\\
\widehat{\omega}(t) &= \widehat{\omega}(t|t-1) + \gamma_\omega g(t)
\end{aligned} \tag{4}$$

where $\widehat{\alpha}(t)$ is an estimate of the rate of change of the instantaneous frequency $\alpha(t) = \omega(t+1) - \omega(t)$, and $\mu > 0$, $\gamma_\omega > 0$, $\gamma_\alpha > 0$, such that $\gamma_\alpha \ll \gamma_\omega \ll \mu$, denote small adaptation gains determining the rate of amplitude adaptation, frequency adaptation and frequency rate adaptation, respectively.

The gradient search strategy, incorporated in (4) for the purpose of tracking $\omega(t)$ and $\alpha(t)$, is based on minimization of the following instantaneous measure of fit $J(t) = |\epsilon(t)|^2/2$, where $\epsilon(t) = y(t) - f(t)\widehat{a}(t-1)$. Note that

$$\begin{aligned}\frac{\partial J(t)}{\partial \omega(t)} &= \mathrm{Re}\left[\epsilon(t)\,\frac{\partial \epsilon^*(t)}{\partial \omega(t)}\right] = -\mathrm{Re}\left[j\epsilon(t) f^*(t)\widehat{a}^*(t-1)\right] \\ &= -\mathrm{Im}\left[\epsilon(t) f^*(t)\widehat{a}^*(t-1)\right]. \qquad (5)\end{aligned}$$

Since $\widehat{d}(t|t-1) = \widehat{f}(t)\widehat{a}(t-1) \cong f(t)\widehat{a}(t-1)$, the term $g(t)$ in (4) can be interpreted as a normalized estimate of the (negative) gradient (5). The normalization term $|\widehat{d}(t|t-1)|^2$, which can be regarded as an estimate of the power of $d(t)$, makes the algorithm scale-invariant.

When the analyzed signal $y(t)$ is real-valued the algorithm (4) can be used after replacing the one-step-ahead prediction error $\varepsilon(t)$ with $\varepsilon_{\mathrm{R}}(t) = \mathrm{Re}[\varepsilon(t)] = y(t) - \mathrm{Re}\big[\widehat{d}(t|t-1)\big]$. Alternatively, one can generate the complex-valued version of the signal $y(t)$ by means of applying the discrete Hilbert transform.

### 2.2. Adaptive comb filtering algorithm

Using the surrogate output technique, introduced in [5], the multiple-frequency ANF algorithm can be obtained by means of combining several single-frequency ANFs. The resulting parallel estimation scheme is driven by the common prediction error $\varepsilon(t) = y(t) - \sum_{k=1}^{K} \widehat{d}_k(t|t-1)$, but the signal components $d_k(t)$ are treated as mutually unrelated quantities and estimated independently of one another. Adaptive comb filter designed in this way has two drawbacks. First, the multiple frequency ANF performs an unconstrained frequency search while the true harmonics vary in a coordinated way. As a consequence, its tracking characteristics are inferior to those offered by algorithms that incorporate the harmonic constraints (3). According to [2] and [3], the accuracy gains achieved by taking into account (3) may be substantial. Second, the multiple-frequency ANFs are usually less robust to incorrect frequency matching than the true ACF algorithms. Even if the initial frequency assignment is correct, the sub-algorithms tracking weak signal components, i.e., those characterized by small values of the signal-to-noise ratio $\mathrm{SNR}_k(t) = |d_k(t)|^2/\sigma_v^2(t)$, may, after some time, lock onto the neighboring, stronger components. The situation becomes even more complicated if the 'strength' of different signal components changes over time.

In order to obtain algorithm that performs coordinated search of the instantaneous fundamental frequency $\omega_0(t)$, one should minimize $J(t)$ for

$$\epsilon(t) = y(t) - \sum_{k=1}^{K} f_k(t)\widehat{a}_k(t-1)$$

where, according to (2) and (3), it holds that $f_k(t) = e^{j\sum_{i=1}^{t}\omega_k(i)} = e^{jm_k\sum_{i=1}^{t}\omega_0(i)}$. Noting that

$$\frac{\partial J(t)}{\partial \omega_0(t)} = -\sum_{k=1}^{K} m_k \mathrm{Im}\left[\epsilon(t) f_k^*(t)\widehat{a}_k^*(t-1)\right],$$

one arrives at the following ACF algorithm which is an extension of the ANF (4) to the multi-harmonic case:

$$\begin{aligned}\varepsilon(t) &= y(t) - \sum_{k=1}^{K}\widehat{d}_k(t|t-1) \\ \widehat{d}_k(t+1|t) &= e^{jm_k\widehat{\omega}_0(t|t-1)}[\widehat{d}_k(t|t-1) + \mu\varepsilon(t)] \\ k &= 1,\ldots,K \\ g(t) &= \frac{\sum_{k=1}^{K} m_k \mathrm{Im}[\varepsilon(t)\widehat{d}_k^*(t|t-1)]}{\sum_{k=1}^{K} m_k^2 |\widehat{d}_k(t|t-1)|^2} \\ \widehat{\omega}_0(t|t-1) &= \widehat{\omega}_0(t-1) + \widehat{\alpha}_0(t-1) \\ \widehat{\alpha}_0(t) &= \widehat{\alpha}_0(t-1) + \gamma_\alpha g(t) \\ \widehat{\omega}_0(t) &= \widehat{\omega}_0(t|t-1) + \gamma_\omega g(t) \\ \widehat{d}(t+1|t) &= \sum_{k=1}^{K}\widehat{d}_k(t+1|t) \qquad (6)\end{aligned}$$

The gradient term in (6) is divided by $\sum_{k=1}^{K} m_k^2|\widehat{d}_k(t|t-1)|^2$, which is an estimate of the so-called effective signal power [2], [3].

### 2.3. Tracking properties of ACF

In order to perform tracking analysis of the ACF algorithm (6), we will assume that

(A1) The measurement noise $\{v(t)\}$ is a zero-mean circular white sequence with variance $\sigma_v^2$.

(A2) $a_k(t) \equiv a_k$, i.e., $d_k(t) = e^{jm_k\omega_0(t)}d_k(t-1), k = 1,\ldots,K,\ \forall t$.

Note that under (A2) the amplitudes of signal harmonics are constant, i.e., the fundamental frequency changes are only source of signal nonstationarity.

Denote by $\Delta\widehat{\omega}(t) = \omega_0(t) - \widehat{\omega}_0(t)$ and $\Delta\widehat{\alpha}(t) = \alpha_0(t) - \widehat{\alpha}_0(t)$ the frequency and frequency rate tracking errors, respectively. Given that the one-step frequency rate changes $\delta(t) = \alpha(t) - \alpha(t-1)$ are uniformly small, the evolution of tracking errors can be analyzed using the approximating linear filter (ALF) technique – the stochastic linearization approach proposed in [11] and [12]. When carrying ALF analysis, one should neglect all terms of order higher than one in $\Delta\widehat{\omega}_k(t)$, $\Delta\widehat{\alpha}_k(t)$, $\delta(t)$ and $v(t)$, including all cross-terms. Applying this approach to analysis of (6), one obtains the following approximate error equations (the derivation follows the lines of the analogous derivation given in [1]; the deterministic averaging technique is used to cope with multiple harmonics)

$$\Delta\widehat{\omega}_0(t) \cong G_1(q^{-1})e_0(t) + G_2(q^{-1})\delta(t) \qquad (7)$$

$$\Delta\widehat{\alpha}_0(t) \cong H_1(q^{-1})e_0(t) + H_2(q^{-1})\delta(t) \qquad (8)$$

where

$$e_0(t) = \sum_{k=1}^{K} m_k e_k(t)\ ,\quad e_k(t) = -\mathrm{Im}\left[d_k^*(t)v(t)/a_0^2\right]$$

and $a_0^2 = \sum_{k=1}^{K} m_k^2 a_k^2$ denotes the effective power of a harmonic signal $d(t)$. Note that in the multiple-frequency case ($K > 1$) it holds that $a_0^2 > \sum_{k=1}^{K} a_k^2$, i.e., the effective power of $d(t)$ is larger than its power. One can show that $\{e_k(t)\}$, $k = 1, \dots, K$ are zero-mean, mutually orthogonal, white noise sequences with variances equal to

$$\mathrm{var}[e_k(t)] = \mathrm{E}[|e_k(t)|^2] = \frac{\sigma_v^2 a_k^2}{2a_0^4} \, .$$

The transfer functions $G_1(q^{-1})$, $G_2(q^{-1})$, $H_1(q^{-1})$ and $H_2(q^{-1})$ are given by

$$\begin{aligned} G_1(q^{-1}) &= (1-q^{-1})[\gamma_\omega + (\gamma_\alpha - \gamma_\omega)q^{-1}]/D(q^{-1}) \\ G_2(q^{-1}) &= q^{-1}[1-\gamma_\omega - (1-\mu)q^{-1}]/D(q^{-1}) \\ H_1(q^{-1}) &= \gamma_\alpha(1-q^{-1})^2/D(q^{-1}) \\ H_2(q^{-1}) &= [1 + (\mu+\gamma_\omega-2)q^{-1} + (1-\mu)q^{-2}]/D(q^{-1}) \end{aligned}$$

where $D(q^{-1}) = 1+d_1q^{-1}+d_2q^{-2}+d_3q^{-3}$, $d_1 = \mu+\gamma_\omega+\gamma_\alpha-3$, $d_2 = 3-2\mu-\gamma_\omega$, $d_3 = \mu-1$. All filters are asymptotically stable if adaptation gains fulfill the following (sufficient) stability conditions: $0 < \mu < 1$, $0 < \gamma_\omega < 1$, $0 < \gamma_\alpha < 1$ and $\mu(\gamma_\omega + \gamma_\alpha) > \gamma_\alpha$.

Since the ALF equations (7)-(8) have identical form as those established earlier for the single-frequency algorithm (5) – only the expression for the variance of white noise $\{e_0(t)\}$ is different – all major conclusions reached in [1] for the ANF algorithm (5) carry on to the ACF algorithm (6). In particular, it can be shown that when the frequency rate $\alpha_0(t)$ drifts according to the random-walk model

(A3) $\{\delta(t)\}$, independent of $\{v(t)\}$, is a zero-mean white sequence with variance $\sigma_\delta^2$,

i.e., when the fundamental frequency $\omega_0(t)$ changes according to the integrated random-walk model (pseudo-linear variation), and when both noise sources are Gaussian

(A4) The sequences $\{v(t)\}$ and $\{\delta(t)\}$ are normally distributed,

the optimally-tuned ACF (6) is a statistically efficient frequency estimation algorithm, i.e., it reaches the Cramér-Rao-type lower tracking bound that limits tracking performance of *any* adaptive comb filter [13].

## 3. ACTIVE CONTROL OF MRI NOISE

One of the potential applications of the proposed adaptive comb filtering scheme is active cancellation of nonstationary harmonic noise. Active cancellation can be achieved by means of generating an acoustic waveform ("antisound") that, in the area/point of interest, has the same shape as the disturbance waveform, but opposite polarity [14], [15]. The problem of narrowband disturbance rejection was considered by many authors under different methodologies, such as filtered-X LMS (FXLMS) compensation [15], [16], internal model principle [17], [18], and phase-locked loop control [19], [20].

Recently a completely new approach, based on adaptive gain scheduling, was introduced in a series of papers [7]-[10]. The new scheme, called SONIC (self-optimizing narrowband interference canceller) offers some unique advantages compared to the existing solutions, such as increased robustness to modeling errors and improved tracking capabilities under nonstationary conditions. SONIC is a feedback ANC designed for the sytem governed by

$$y(t) = H_{\mathrm{s}}(q^{-1})u(t-1) + d(t) + v(t)$$

where $y(t)$ denotes the signal picked by the error microphone, $H_{\mathrm{s}}(q^{-1})$ denotes transfer function of the secondary path, and $u(t)$ denotes the error-dependent canceling signal. Incorporating adaptive comb filter (6) into SONIC, one arrives at the following algorithm

$$z_k(t) = e^{jm_k\widehat{\omega}_0(t|t-1)}\Big[(1-c_\mu)z_k(t-1) - \frac{c_\mu}{\widehat{\mu}_k(t-1)}\, y(t-1)\Big] \tag{9a}$$

$$r_k(t) = \rho r_k(t-1) + |z_k(t)|^2 \tag{9b}$$

$$\widehat{\mu}_k(t) = \widehat{\mu}_k(t-1) - \frac{y(t)z_k^*(t)}{r_k(t)} \tag{9c}$$

$$\widehat{d}_k(t+1|t) = e^{jm_k\widehat{\omega}_0(t|t-1)}[\widehat{d}_k(t|t-1) + \widehat{\mu}_k(t)y(t)] \tag{9d}$$

$$g(t) = \frac{\sum_{k=1}^K m_k \mathrm{Im}[\widehat{\mu}_k(t)y(t)\widehat{d}_k^*(t|t-1)]}{\sum_{k=1}^K m_k^2|\widehat{d}_k(t|t-1)|^2} \tag{9e}$$

$$\widehat{\omega}_0(t|t-1) = \widehat{\omega}_0(t-1) + \widehat{\alpha}_0(t-1) \tag{9f}$$

$$\widehat{\alpha}_0(t) = \widehat{\alpha}_0(t-1) + \gamma_\alpha g(t) \tag{9g}$$

$$\widehat{\omega}_0(t) = \widehat{\omega}_0(t|t-1) + \gamma_\omega g(t) \tag{9h}$$

$$u_k(t) = -\frac{\widehat{d}_k(t+1|t)}{h_{\mathrm{n}}[m_k\widehat{\omega}_0(t)]} \tag{9i}$$

$$k = 1, \dots, K$$

$$u(t) = \mathrm{Re}\left[\sum_{k=1}^K u_k(t)\right] \tag{9j}$$

where $\rho \cong 1$, $0 < \rho < 1$, is the forgetting constant which determines the effective length of the tuning memory, $c_\mu > 0$ is a small constant, and the complex-valued coefficients $h_{\mathrm{n}}[m_k\widehat{\omega}_0(t)] = H_{\mathrm{n}}[e^{-j\widehat{\omega}_k(t)}]$, $k = 1, \dots, K$, denote the nominal (assumed) steady-state gains of the secondary path at the harmonic frequencies $\widehat{\omega}_k(t)$, usually different from the true gains $H_{\mathrm{s}}[e^{-j\widehat{\omega}_k(t)}]$.

The algorithm summarized above consists of three mutually coupled loops. The predictive control loop, governed by (9d), (9i) and (9j), performs the one-step-ahead prediction of the disturbance and works out the cancellation signal. The self-optimization loop (9a)-(9c) is used to adjust the *complex-valued* adaptation gains $\widehat{\mu}_k(t)$ to the rate of nonstationarity of the disturbance. When appropriately tuned, these gains are also capable of compensating modeling errors caused by imprecise knowledge of the secondary path. Finally, the frequency tracking loop (9e)-(9h) performs gradient search of the fundamental frequency and is almost identical with the analogous part of the ACF algorithm (6) – the only (but important) difference is due to the presence of the adaptation gains in the formula used for gradient evaluation. To shed some light on the origin of this modification [derivation of (9) would require much more space] we note that the frequency update mechanism proposed in [10] for the single-frequency ANC, has the form

$$\begin{aligned} \widehat{\omega}(t) &= (1-\gamma_\omega)\widehat{\omega}(t|t-1) + \gamma_\omega \mathrm{Arg}\left[\frac{\widehat{d}(t+1|t)}{\widehat{d}(t|t-1)}\right] \\ &= \widehat{\omega}(t|t-1) + \gamma_\omega \mathrm{Arg}\left[\frac{\widehat{d}(t+1|t)e^{-j\widehat{\omega}(t|t-1)}}{\widehat{d}(t|t-1)}\right] \\ &= \widehat{\omega}(t|t-1) + \gamma_\omega \mathrm{Im}\left\{\log\left[1 + \frac{\widehat{\mu}(t)y(t)}{\widehat{d}(t|t-1)}\right]\right\} \end{aligned} \tag{10}$$

where $\mathrm{Arg}[\cdot]$ denotes the principal argument of a complex number.

For small values of $\widehat{\mu}(t)$, which usually settle down in the closed-loop system, the correction term in (10) is approximately equal to

$$\mathrm{Im}\left[\frac{\widehat{\mu}(t)y(t)}{\widehat{d}(t|t-1)}\right] = \frac{\mathrm{Im}[\widehat{\mu}(t)y(t)\widehat{d}^*(t|t-1)]}{|\widehat{d}(t|t-1)|^2} \tag{11}$$

which can be recognized as the error-compensated gradient term used in (4). When no compensation is needed, i.e., in the absence of modeling errors, $\widehat{\mu}(t)$ is a real-valued quantity which simply scales the gradient term. Note that (9e) can be regarded as a multi-frequency version of (11).

An interesting and practically important application of the proposed scheme is cancellation of MRI noise. MRI scanners are widely used in medical institutions for diagnostic and research purposes. Recently, an open-configuration MR system has been introduced to conduct microwave coagulation therapy with the help of near-realtime MR imaging [21]. The major problem with this technique is, however, due to the high level of acoustic noise (exceeding 100 dB SPL) emitted by gradient coils while the images are taken [22]. Exposure to such an intense noise is very stressful for the surgeons and for the supporting medical staff – it causes fatigue and makes verbal communication difficult. This may result in medical accidents. MRI noise is difficult to cancel as it is nonstationary and consists of a large number ($> 30$) of harmonic components – see Fig. 1a. Another important feature of MRI noise is its discontinuity in the time domain. The scanner performs sequential acquisition of images in a loop. Every time a new acquisition is started, an abrupt change occurs in the noise signal. The problem of active MRI noise reduction was analyzed in many papers – see e.g. [23], [24] and references therein.

The SONIC-based MRI noise cancellation experiment was performed in a lab using commercial off-the-shelf (COTS) equipment. The hardware platform, depicted in Fig. 2, was built on the basis of Texas Instrument's DSK6713 evaluation kit. The DSK board features TMS320C6713 DSP processor clocked at 225 MHz, 512 kB FLASH memory, 16 MB SDRAM and high quality, 24-bit stereo audio codecs (input and output). In addition to the DSK, we used the DBX-RTA-M microphone to measure the system output, two power amplifiers, and custom high-power loudspeakers – one used as a source of the noise (which was recorded, using an optical microphone, in the MR room of the Shiga University of Medical Science, Japan) and another one used to cancel disturbance.

The following settings were adopted: $c_\mu = 0.0002$, $\rho = 0.9999$, $\gamma_\omega = 5 \cdot 10^{-4}$, $\gamma_\alpha = 25 \cdot 10^{-6}$. Additionally, a simple safely feature was added in the form of a bound on the magnitude of estimation gains $\widehat{\mu}_k(t)$, which was set to 0.0015. The total number of cancelled harmonics was equal to $K = 24$. The initial frequency estimate was set to $\widehat{\omega}_0(0) = 0.0556$ rad/Sa (which, under 8 kHz sampling used in the system, corresponds to 70.8 Hz).

Fig. 1b shows the steady-state power spectral density of the signal recorded by the error microphone under the operation of SONIC equipped with the multiple-frequency version of the ANF algorithm (4). The analogous results obtained for the controller (11) are depicted in Fig. 1c. It is clear that the ACF-based controller is much more effective in suppressing signal harmonics than ANF-based controller, even though both algorithms used the same starting values. Even though in terms of sound perception the improvement is significant, it is difficult to quantify this effect since all SNR-type performance measures, dominated by the strongest harmonics, are not trustworthy. Some sort of an agreeable perceptual "measure of relief" should be worked out and used instead. The failure of the ANF-based algorithm can be explained by its poor frequency matching capabilities – after some initial period the algorithm locks on dominant harmonics, leaving the remaining ones unattenuated.

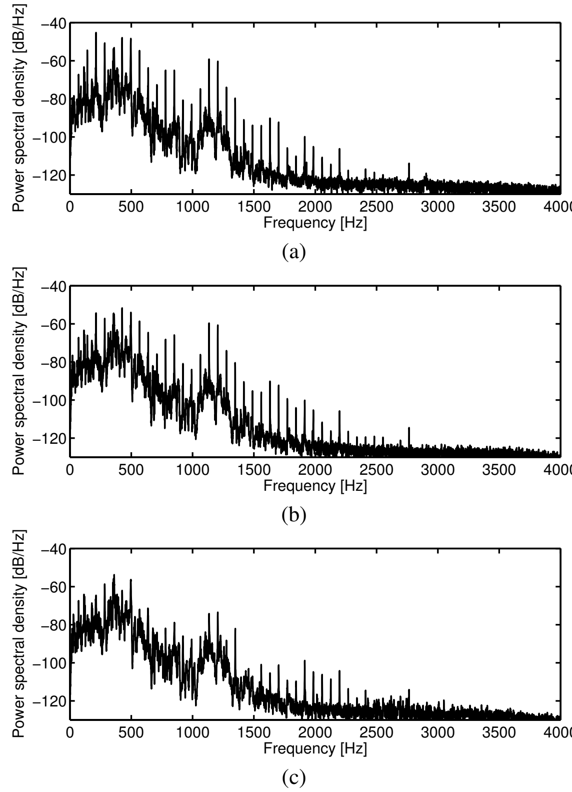


**Fig. 1**. Power spectral density of MRI noise: before cancellation (a), after cancellation using the ANF-based SONIC (b), and after cancellation using the ACF-based SONIC (c).

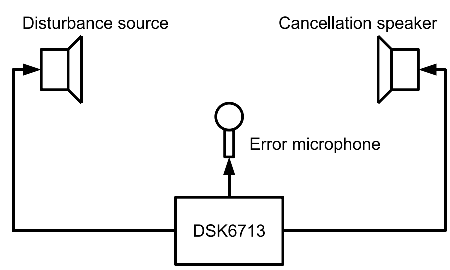


**Fig. 2**. Block diagram of the cancellation system.

## 4. RELATION TO PRIOR WORK

The paper extends the results presented in [1] and [10] to multiharmonic signals. To the best of our knowledge, the proposed adaptive comb filter is the first statistically efficient fundamental frequency tracker – when optimally tuned it reaches (under conditions specified in the paper) the Cramér-Rao-type lower tracking bound that limits tracking behavior of *any* adaptive comb filter. So far the analogous algorithms were only available for nonstationary single-harmonic signals [11], [12] and for stationary multiharmonic signals [2].

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