IIR PHASOR BANKS: CAUSAL, DELAY-FREE, NUMERICALLY ROBUST, CUSTOMIZABLE, UNIFORM-DFT-LIKE PERFECT RECONSTRUCTION FILTER BANKS

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ABSTRACT

We describe a method for generating causal, stable, numerically robust, IIR, customizable (application specific), perfect reconstruction (PR) filter-banks with zero (or adjustable) reconstruction delay. The ideas presented in this paper extend the concept of *Dynamic Phasors* as well as the concept of the so-called *Phasor Bank Transform*, which constitute specific subclasses of filter-banks. We provide an explicit compact characterization of the optimally-robust reconstruction stage of this transform, which can be used with any (customized) choice of its analysis stage. Since the Phasor Bank transform allows causal (real-time) delay-free perfect reconstruction of any input waveform, it is perfectly suited for (closed-loop) control applications. Although the motivating application area of Phasor Banks is Power Systems, such as tracking of time-variant harmonic content under both steady-state (nearly-periodic) and transient operating conditions, the technique presented here can be used in a variety of closed loop applications.

***Index Terms*—** Causal Perfect Reconstruction Filter-Bank, Numerical Robustness, Phasor-Banks, Nyquist Filter, Dynamic Phasors

1. INTRODUCTION

Dynamic Phasors are finding increasing use in a variety of applications and, in particular, in the analysis and control of electric energy components and systems [1-5]. Some of the applications of the Dynamic Phasor framework include power systems (analysis of faults and protection) as well as power electronics (analysis of resonant and high-power converters, control of active filters). The classical Dynamic Phasor representation is based on the so-called Short-Time Fourier Transform (STFT) [6, 7, 9]. Its discrete version can be viewed as an M-channel Perfect Reconstruction (PR) filter-bank (Fig. 1).

In this paper we describe a family of customizable Dynamic Phasors known as Phasor Banks (see Sec. 2) that generalizes the concept of Fourier Phasors, thereby allowing greater design flexibility as well as fine-tuning in order to match the needs of a broad variety of applications. Dynamic Phasors are frequently used in applications that require dynamic processing, namely processing carried out online in real time, using digital signal processing hardware/software, one sample at a time. Consequently, all processing must be causal (and stable). To facilitate digital processing we shall use a *discrete-time* problem formulation.

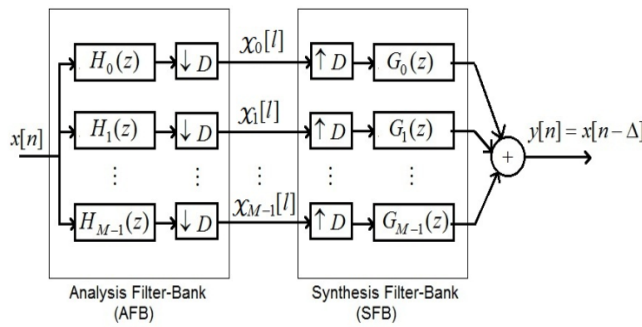

Fig. 1. Filter-bank interpretation of Dynamic Phasors

In contrast to filter banks designed for coding applications, which employ down-sampling to achieve the smallest possible data rate subject to the constraint of perfect reconstruction, we opt to avoid down-sampling altogether. The use of non-decimated filter banks allows much greater flexibility in implementing the reconstruction step, and as a result also in the selection of the analysis filter bank [4, 5]. The resulting single-rate processing scheme also allows for a closer relation between our discrete-time generalized dynamic phasors and the continuous-time systems from which our signals of interest are acquired.

In summary, Phasor Banks are causal, uniform DFT-like, fully-oversampled, perfect reconstruction filter banks, whose reconstruction delay can be selected at will within the integer interval $[0, M-1]$ (Sec. 2). In addition, their analysis stage is entirely unconstrained, while their synthesis stage can be designed to optimize numerical robustness (Sec. 3). In this paper we characterize the design of a numerically robust IIR synthesis stage (Sec. 4), thereby extending our previous results about the design of FIR synthesis prototypes (see [4, 8]). We use an example to demonstrate the advantage of such IIR designs over the FIR case.

2. PHASOR BANK FUNDAMENTALS

Phasor Banks provide a generalization of the well-known discrete Gabor transform [6, 7, 9], which can be (equivalently) viewed as a perfect reconstruction M-channel uniform DFT filter bank. We define a Phasor Bank as a filter bank that satisfies the following conditions:

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(i) Causal & stable:

Phasor Banks are targeted for applications that require dynamic processing, namely processing carried out online in real time, using digital signal processing hardware/software, one sample at a time. As a consequence, all processing must be causal and stable.

(ii) No Decimation:

In contrast to filter banks designed for coding applications, which employ down-sampling to achieve the smallest possible data rate subject to the constraint of perfect reconstruction, we opt to avoid down-sampling altogether, so that $D = 1$ in Fig. 1. The resulting single-rate processing scheme allows for a closer relation between our discrete-time generalized Dynamic Phasors and the continuous-time systems from which our signals of interest are acquired, as well as greater flexibility in the selection of the Phasor Bank synthesis stage [4]. This type of filter-bank is known in the literature as *fully oversampled* [10].

(iii) Uniform DFT-like structure:

Both analysis and synthesis filters have a uniform DFT-like structure, that is, they can be expressed in terms of a *prototype analysis filter* $H(z)$ (PAF) and a *prototype synthesis filter* $G(z)$ (PSF) respectively, viz.

$$H_k(z) = H\left(e^{-jk\omega_0}z\right) \quad (1a)$$

$$G_k(z) = e^{-jk\omega_0\Delta}G\left(e^{-jk\omega_0}z\right) \quad (1b)$$

where $0 \le k \le M-1$, M represents the number of channels, $\omega_0 \triangleq \frac{2\pi}{M}$, and Δ is the controllable reconstruction delay (Fig. 1). When $\Delta = 0$ the expression (1a,b) coincides with the definition of a uniform DFT filter bank.

(iv) Unconstrained Analysis Filter:

The *prototype analysis filter* $H(z)$ of the Phasor Bank is unconstrained and thus can be designed to meet the specifications of a particular application of interest. For example, when used for dynamic frequency analysis in electric power systems, the analysis stage should be highly frequency selective. This reduces the leakage between frequency bands and allows unambiguous association of each Dynamic Phasor $\mathcal{X}_k(\cdot)$ with a *single* harmonic.

(v) Controllable Delay Perfect Reconstruction (PR):

Perfect reconstruction indicates that $y[n] = x[n-\Delta]$, where Δ is the reconstruction delay (Fig. 1). Control applications usually require zero reconstruction delay, which cannot be achieved, for instance, with paraunitary QMF filter banks. The reconstruction delay Δ of our Phasor Banks is a design parameter, with any desired (integer) value in the range $[0, M-1]$, and is entirely independent of the analysis stage of the phasor bank. The perfect reconstruction requirement $y[n] = x[n-\Delta]$ for a phasor-bank with given causal $H(z), G(z)$ reduces to [4].

$$f[n] = \begin{cases} \frac{1}{M} & n = \Delta \\ 0 & n = \Delta + lM,\ l \ge 1 \end{cases} \quad (2)$$

where $F(z) \triangleq H(z)G(z)$, and $f[n]$ represents the impulse response of $F(z)$ [4]. Thus, $F(z)$ is a delayed version of a Nyquist filter [9]. Since $H(z)$ is unconstrained, $G(z)$ is responsible for satisfying (2). Because we do not down-sample the phasor-domain coefficients, every $H(z)$ has (infinitely) many matching choices of $G(z)$ that satisfy the PR condition (2). This added freedom of choice allows us to customize the selection of $G(z)$ with respect to other metrics of performance, such as reconstruction numerical robustness (see Sec. 3).

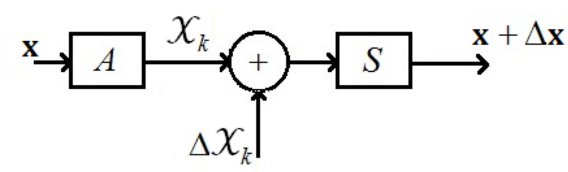

Fig. 2. Simplified perturbation model for a PR filter-bank

3. MEASURES OF NUMERICAL ROBUSTNESS

Numerical robustness (a.k.a "stability of reconstruction") refers to the effects of perturbations/errors in $\mathcal{X}_k[n]$ on the reconstructed signal $y[n]$ [11], (Fig. 2). Using vector-matrix notation, let $\boldsymbol{A}$, $\boldsymbol{S}$ represent the input-output signal space mappings induced by the analysis (AFB) and synthesis (SFB) filters, respectively (Fig. 2). Thus, we have

$$\eta_{rob}(\boldsymbol{A},\boldsymbol{S}) = \frac{\|\boldsymbol{\Delta}\mathbf{x}\|/\|\mathbf{x}\|}{\|\boldsymbol{\Delta z}\|/\|\boldsymbol{z}\|} \le \|\boldsymbol{A}\| \cdot \|\boldsymbol{S}\| \quad (3)$$

which is true for every norm, where $\boldsymbol{z}$ represents the collection of all $\mathcal{X}_k$'s (Fig. 2). Including the requirement of perfect reconstruction (i.e., $\boldsymbol{S} \cdot \boldsymbol{A} = \boldsymbol{I}$) we conclude that $\|\boldsymbol{A}\| \cdot \|\boldsymbol{S}\| \ge 1$, so that numerical robustness of filter banks is synonymous with reducing the product $\|\boldsymbol{A}\| \cdot \|\boldsymbol{S}\|$. The minimum value of this product ($\|\boldsymbol{A}\| \cdot \|\boldsymbol{S}\| = 1$) is achieved by using: (i) the canonical left inverse ($\boldsymbol{S} = \boldsymbol{A}^+$), and (ii) making $\boldsymbol{A}$ unitary [9]. This optimal solution is not causal, and attempting to make it causal results in significant reconstruction delay. We shall focus here on the Euclidean vector norm, so that

$$\eta_{rob}(\boldsymbol{A},\boldsymbol{S}) = \frac{\frac{\|\Delta\mathbf{x}\|_2}{\|\mathbf{x}\|_2}}{\sqrt{\frac{\sum_{k=0}^{M-1}\|\Delta\mathcal{X}_k\|_2^2}{\sum_{k=0}^{M-1}\|\mathcal{X}_k\|_2^2}}} \quad (4)$$

Notice that the same robustness index values are obtained when we replace the Euclidean norm in (4) by the "RMS - norm" $\|\mathbf{x}\|_{RMS} \triangleq \lim_{N\to\infty}\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2}$, which is more appropriate for persistent (finite power) signals. This allows us to model the perturbations $\Delta\mathcal{X}_k(\cdot)$ as independent white noise processes, all with the same variance, and the input signal $x[n]$ as a stationary ergodic process.

Using these assumptions along with causality, stability and the property (1), the robustness index (for $N \to \infty$) is minimized by selecting $G(z)$ as the solution of the linearly constrained quadratic optimization problem:

$$\min_{\boldsymbol{g}} \frac{1}{2}\|\boldsymbol{g}\|_2^2 \quad (5a)$$

subject to

$$\boldsymbol{\mathcal{H}}\boldsymbol{g} = \left(\frac{1}{M}\right)\boldsymbol{e}_0 \quad (5b)$$

where

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} h_\Delta & \cdots & h_0 & 0 & \cdots & \cdots & \cdots & \cdots \\ h_{\Delta+M} & \cdots & \cdots & \cdots & h_0 & 0 & \cdots & \cdots \\ h_{\Delta+2M} & \cdots & \cdots & \cdots & h_M & \cdots & h_0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \quad (6a)$$

$$\boldsymbol{e}_0 \triangleq [\,1\;0\,...\,0\,]^T \qquad (6b)$$

The optimally robust solution of (5) is given by

$$\begin{bmatrix} g[0] \\ g[1] \\ \vdots \end{bmatrix}_{opt} = \left(\frac{1}{M}\right)\boldsymbol{\mathcal{H}}^{\dagger}\boldsymbol{e}_0 \qquad (7)$$

where $\boldsymbol{\mathcal{H}}^{\dagger}$ denotes the generalized (Moore-Penrose) inverse of this infinite matrix. The resulting optimal impulse response $\boldsymbol{g}_{opt}$ has infinite length, since $\boldsymbol{\mathcal{H}}^{\dagger}$ and $\boldsymbol{e}_0$ have infinite dimensions. The optimal IIR solution can be approximated, to any desired level of accuracy, by truncating $\boldsymbol{\mathcal{H}}$, $\boldsymbol{e}_0$ to finite size, which results in an FIR approximation for $\boldsymbol{g}_{opt}$.

4. OPTIMALLY ROBUST IIR FILTER

In view of the need to truncate the expression (7) to finite dimensions, we consider in this section optimized choices of the prototype synthesis filter $G(z)$ that are FIR of a prescribed length. In particular, when the prototype analysis filter $H(z)$ satisfies a mild regularity condition, the optimal FIR $G(z)$ can be expressed, without approximation, in terms of *finite dimensional matrices*.

Theorem 4.1: (Optimal FIR $G(z)$)
Consider a given IIR prototype analysis filter $H(z) = b(z)/a(z)$ with $deg\{b(z)\} \leq deg\{a(z)\} \leq M-1$, so that $a(z) \triangleq \sum_{i=0}^{M-1} a[i]z^{-i}$ and $b(z) \triangleq \sum_{i=0}^{M-1} b[i]z^{-i}$. Let $\{p_i\,; 1 \leq i \leq M-1\}$ denote the poles of this filter (i.e., the roots of $a(z)$), and assume that

$$p_i^M \neq p_j^M \quad \text{for all} \quad i \neq j \qquad (8)$$

Then, the *optimally-robust* FIR prototype synthesis filter $G_{opt}(z)$ of a prescribed length L_g is given by $G_{opt}(z) = \gamma_{opt}(z)a(z)$ where $\gamma_{opt}(z)$ is an FIR filter of length $L_\gamma \triangleq L_g - L_a + 1$ whose impulse response is

$$\boldsymbol{\Gamma}_{opt} = \frac{1}{M}(\boldsymbol{D}^T\boldsymbol{D})^{-\frac{1}{2}}\left(\boldsymbol{B}(\boldsymbol{D}^T\boldsymbol{D})^{-\frac{T}{2}}\right)^{\dagger}\boldsymbol{e}_0 \qquad (9a)$$

or, equivalently,

$$\boldsymbol{\Gamma}_{opt} = \frac{1}{M}(\boldsymbol{D}^T\boldsymbol{D})^{-1}\boldsymbol{B}^T[\,\boldsymbol{B}(\boldsymbol{D}^T\boldsymbol{D})^{-1}\boldsymbol{B}^T\,]^{-1}\boldsymbol{e}_0 \qquad (9b)$$

with $\boldsymbol{e}_0 \triangleq [1\;0\,...\,0]^T$, and

$$\boldsymbol{D} \triangleq \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots \\ a_1 & 1 & 0 & \cdots & \cdots \\ \vdots & a_1 & 1 & 0 & \cdots \\ a_{L_a-1} & \vdots & \ddots & \cdots & \cdots \\ 0 & a_{L_a-1} & \ddots & \cdots & \cdots \\ \vdots & 0 & \ddots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \qquad (9c)$$

$$\boldsymbol{B} \triangleq \begin{bmatrix} b_\Delta & \cdots & b_0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & b_{M-1} & \cdots & b_0 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \qquad (9d)$$

Both $\boldsymbol{B}$ and $\boldsymbol{D}$ are *finite dimensional*, viz., $[\boldsymbol{B}] = K \times L_\gamma$, $[\boldsymbol{D}] = L_g \times L_\gamma$, where $K \triangleq 1 + \text{floor}\left(\frac{L_\gamma + L_b - \Delta - 2}{M}\right)$. We use the notation L_g to denote the length of the impulse response of the FIR filter $G(z)$, i.e., $L_g \triangleq deg\{\,G(z)\,\} + 1$ and similarly for L_γ, L_a, L_b. Also, $(\boldsymbol{D}^T\boldsymbol{D})^{1/2}$ is the lower triangular Cholesky factor of the symmetric positive-definite matrix $\boldsymbol{D}^T\boldsymbol{D}$ ■

Theorem 4.1 guarantees that $F(z) = H(z)G(z)$ is FIR, so that the perfect reconstruction condition (2) gives rise to a finite set of linear equations whose optimal solution is given by (9a-d). Typically, $H(z)$ is the product of a classic filter technique (such as Butterworth, Chebyshev, etc.) in which condition (8) is rarely violated. The following theorem shows that the PR condition can sometimes be satisfied with a specially structured IIR $F(z)$.

Theorem 4.2: (Optimal IIR $G(z)$)
Consider a given IIR prototype analysis filter $H(z) = \frac{b(z)}{a(z)}$ where the poles of $a(z)$ satisfy (8), and a prototype synthesis filter of the form $G(z) = \frac{c(z)}{p_0(z)}$. A necessary condition for $F(z) \triangleq H(z)G(z)$ to satisfy the PR condition (2) is that $c(z) = \gamma(z)a(z)$ for some polynomial $\gamma(z)$, and that all roots of $p_0(z)$ violate (8), namely $\forall p_i\; \exists p_j, j \neq i$, such that $p_i^M = p_j^M$. With this condition satisfied, there are multiple choices of $\gamma(z)$ that result in Perfect Reconstruction (PR). ■

One simple choice of $p_0(z)$ consists of a collection of complex pole pairs $\{\,(p_j,\; p_j^*)\,; 0 \leq j \leq L-1\,\}$ such that $p_j = r_j e^{j\frac{\pi}{M}}$. Thus $p_0(z) \triangleq \prod_{j=0}^{L-1}\left\{1 - 2r_j \cos\left(\frac{\pi}{M}\right)z^{-1} + r_j^2 z^{-2}\right\}$. The resulting optimization problem is now non-linear since we are optimizing for both $\gamma(z)$ and the special pole magnitudes r_j. The optimally-robust IIR prototype synthesis filter with $\gamma(z)$ of a given length L_γ and $p_0(z)$ as in Theorem-4.2, with prescribed pole magnitudes $\boldsymbol{r} \triangleq \{r_0, \dots, r_{L-1}\}$ is given by $G_{opt}(z) = \frac{\gamma_{opt}(z)a(z)}{p_0(z)}$ so that $F(z) \triangleq H(z)G(z) = \frac{b(z)\gamma_{opt}(z)}{p_0(z)}$, as required by Theorem-4.2. The optimal choice of $\gamma_{opt}(z)$ coefficients depends on $\boldsymbol{r}$ and is given by

$$\boldsymbol{\Gamma}_{opt}(\boldsymbol{r}) = \frac{1}{M}(\boldsymbol{D}_1^T\boldsymbol{D}_1)^{-\frac{1}{2}}\left(\boldsymbol{B}_1(\boldsymbol{D}_1^T\boldsymbol{D}_1)^{-\frac{T}{2}}\right)^{\dagger}\boldsymbol{e}_0 \qquad (10)$$

where $\boldsymbol{D}_1 := \boldsymbol{D}_1(\boldsymbol{r}) = \boldsymbol{P}_0^{-1}\boldsymbol{D}$, $\boldsymbol{B}_1 := \boldsymbol{B}_1(\boldsymbol{r}) = \boldsymbol{B}\boldsymbol{P}_0^{-1}$ and $\boldsymbol{P}_0 := \boldsymbol{P}_0(\boldsymbol{r})$ is the lower-triangular Toeplitz matrix associated with $p_0(z)$. Also, $(\boldsymbol{D}_1^T\boldsymbol{D}_1)^{1/2}$ is the lower triangular Cholesky factor of the symmetric positive-definite matrix $\boldsymbol{D}_1^T\boldsymbol{D}_1$. Since $\boldsymbol{P}_0$ is a square matrix of infinite size, this approach requires truncation as opposed to the method based on (9).

Special cases:
(i) The optimized Windowed-FIR Phasor Bank [4] is a special case of Theorem-4.1. It is obtained by setting $a(z) = 1$ so that $G_{opt}(z) = \gamma_{opt}(z)$ and $\boldsymbol{D} = \boldsymbol{I}$ is an identity matrix. Since the rows of $\boldsymbol{B}$ are mutually orthogonal, $\boldsymbol{B}\boldsymbol{B}^T$ is a diagonal matrix.
(ii) The optimized *Constrained FIR* (CFIR) solution (9) can be obtained from the *Constrained IIR* (CIIR) solution (10) by setting $p_0(z) = 1$ and $\boldsymbol{P}_0 = \boldsymbol{I}$, that is, there are no special poles of $G(z)$ that violate (8).

5. NUMERICAL EXAMPLE

The results of using the three different methods discussed in this paper (7), (9), (10) are summarized by means of the following example.

Consider an $M = 11$ channel Phasor Bank with $\Delta = 0$ reconstruction delay. Let the prototype analysis filter be low-pass IIR of the form $H(z) = \frac{b(z)}{a(z)}$ constructed via the MATLAB® command cheby2(6 , 40, 1/M) so that $L_a = L_b = 7$. For the Unconstrained approach we use truncation, so that $[\boldsymbol{\mathcal{H}}] = 37 \times 18$, with $L_g = 18$. For the CFIR approach we use $L_\gamma = L_g - L_a + 1 = 12$ and $K = 2$, so that $[\boldsymbol{B}] = 2 \times 12$, $[\boldsymbol{D}] = 18 \times 12$. The two methods achieve the same level of numerical robustness, as seen in Table I. However, the computational cost of evaluating (9) is smaller than that of (7), and the resulting solution is more accurate, in the sense of achieving a smaller PR error. To quantify the effects of truncation and finite precision on the PR condition we evaluate the gap between the achieved $f[n] = h[n] * g[n]$ and its nominal value, viz.,

$$\text{PR error} \triangleq \left\| \underbrace{h[n] * g[n]}_{f[n]} - \frac{1}{M}\delta[n - \Delta] \right\|_2$$

Table I : Robustness indices for the CFIR & truncated approaches

	$\|g\|_2$	PR error
Unconstrained	4.7374	2.78e-13
CFIR	4.7374	6.66e-16

For the CIIR approach (10) we use a special single pole pair $re^{\pm j\frac{\pi}{M}}$, where r is used as a design parameter, and $L_{\gamma,CIIR} = L_\gamma - 2 = 10$ so that the total number of multipliers remains the same as in the CFIR case. Fig. 3 shows that a suitable choice of r can result in a 15% improvement in the robustness index when we allow a special pole pair in the design of $G(z)$ (indicated by the solid black dot). Finally Fig. 4 shows the effect on the PR error when varying the special pole angle. We observe that although Theorem-4.2 requires that the angle difference π/M is exact, in practice, a small variation around that value (due to finite wordlength) will still result in a reliable PR-Error value.

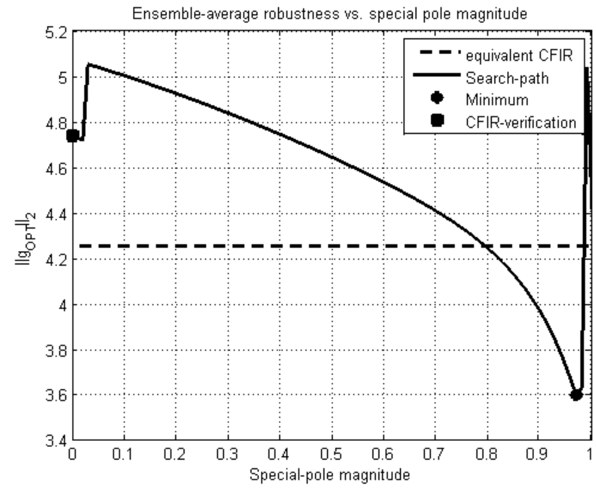

Fig. 3. Robustness index vs. special pole magnitude $\boldsymbol{r}$

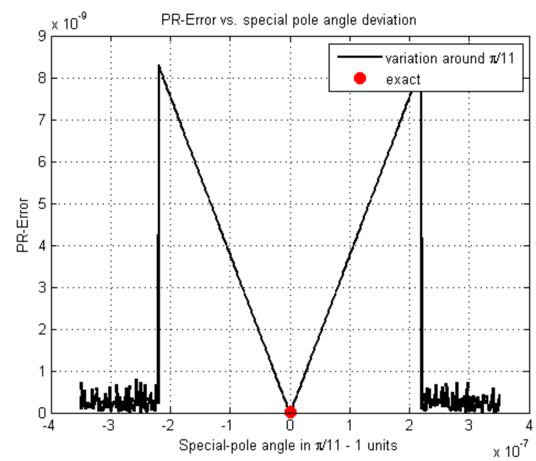

Fig. 4. PR-Error index vs. special pole angle variation

6. RELATION TO PRIOR WORK AND CONCLUDING REMARKS

The design of uniform DFT filter banks has historically focused on critically (or nearly critically) sampled representations, with strong preference for orthogonal mappings (such as the DFT) which exhibit high numerical robustness [6, 9, 10]. This approach results invariably in non-causal synthesis prototypes (obtained via the non-causal canonical dual), so that significant reconstruction delay has to be introduced in order to recover causality. In contrast, our approach includes causality and zero reconstruction delay as strict constraints. We also allow for small ($\Delta \leq M - 1$), controllable reconstruction delay when such a delay is required in applications, such as [12]. In contrast, previous approaches for designing uniform-DFT-like filter banks, such as [13], result in reconstruction delays that are relatively large ($\Delta \geq M$), and completely exclude the possibility of delay-free reconstruction.

The concept of Phasor Banks, as introduced in [4], allows for unconstrained customization of the causal analysis prototype filter $H(z)$. The choice of the causal synthesis prototype filter $G(z)$ must satisfy the perfect reconstruction condition, with reconstruction delay selected at will in the range $0 \leq \Delta \leq M - 1$, and can be optimized in terms of reconstruction numerical robustness. A complete characterization of the optimal FIR $G(z)$ of length M (= windowed FIR) was provided in [4]. A complete characterization of the optimal FIR $G(z)$ of length $L_g \geq M$ was introduced in [8]. In this paper we presented a complete characterization of the optimal IIR $G(z)$ of arbitrary order. We demonstrated via a simple numerical example that such "constrained-IIR" Phasor Banks can achieve the same performance and the same level of numerical robustness as the "constrained-FIR" designs of [8], but at lower implementation cost (= fewer multipliers).

A useful by-product of our analysis of IIR Phasor Banks is a compact characterization of the poles of a (shifted) causal Nyquist filter, which should be contrasted with the previously published, and significantly more complicated, characterization in [14].

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