EVEN MIRROR FOURIER NONLINEAR FILTERS

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ABSTRACT

In this paper, a novel sub-class of linear-in-the-parameters (LIP) nonlinear filters, formed by the so-called even mirror Fourier nonlinear (EMFN) filters, is presented. These filters are universal approximators for causal, time invariant, finite-memory, continuous nonlinear systems as the well-known Volterra filters. However, in contrast to Volterra filters, their basis functions are mutually orthogonal for white uniform input signals. Therefore, in adaptive applications, gradient descent algorithms with fast convergence speed and efficient nonlinear system identification algorithms can be devised. Preliminary results, showing the potentialities of EMFN filters in comparison with other LIP nonlinear filters, are presented and commented.

Index Terms— Nonlinear system identification, linear-in-the-parameters nonlinear filters, universal approximators, orthogonality property.

1. INTRODUCTION

Linear-in-the-parameters (LIP) nonlinear filters with finite or infinite memory constitute a relevant class of models for nonlinear systems. In fact, most of the commonly used finite-memory nonlinear models belong to this class. Among them, the most popular are, perhaps, the truncated Volterra filters [1] that are still actively studied and used in applications [2]−[9]. Other elements of the class are either particular cases of Volterra filters, as the Hammerstein filters [1], [10]−[12], or are strictly related to them, as memory and generalized memory polynomial filters [13], [14]. The finite-memory class also includes filters based on functional expansions of the input samples, as functional link artificial neural networks (FLANN) [15] and radial basis function networks [16]. A review in a unified framework of finite-memory LIP nonlinear filters can be found in [17]. More recently, infinite-memory LIP nonlinear filters have been introduced [18]−[22] and used, in particular, for active noise control.

In this paper, we introduce a novel sub-class of finite-memory LIP nonlinear filters, the so-called even mirror Fourier nonlinear (EMFN) filters, that are based on trigonometric functional expansions, as the FLANN filters, and share with Volterra filters the property of being universal approximators for causal, time invariant, finite-memory, continuous nonlinear systems. The FLANN filter, originally introduced in the field of neural networks as an effective alternative to the widely-used multilayer artificial neural network, has been recently exploited in the field of signal processing due to its reduced computational complexity. FLANN filters have been used in nonlinear channel equalization [23], nonlinear active noise control [24], [25], and nonlinear acoustic echo cancellation [8], [26]. However, as pointed out in [27], the performance of a FLANN filter may be negatively affected in some applications, since it does not include cross-terms, i. e. products of samples with different time shifts. To overcome this difficulty, a generalized FLANN (GFLANN) filter has been proposed in [28], by adding appropriate cross-terms to the conventional FLANN filter. Nevertheless, it can be noted that the basis functions of FLANN and GFLANN filters do not satisfy the conditions of the well-known Stone-Weierstrass approximation theorem [29], and thus may not fulfill the requirements for universal approximation. The EMFN filters presented in this paper permit to overcome these limitations. Such filters are introduced resorting to an N-dimensional representation of a given nonlinear continuous function $f[x(n), x(n-1), \dots, x(n-N+1)]$ of the N most recent input samples. In our derivations, we make use of the generalized Fourier series which permits the representation of a continuous function by means of a generic set of orthogonal basis functions [30]. Accordingly, it is shown in the paper that the basis functions of EMFN filters are mutually orthogonal in $\mathbb{R}_1^N$ for white uniform input signals. Moreover, they satisfy the requirements of the Stone-Weierstrass theorem, and thus are universal approximators, i. e. linear combinations of basis functions can arbitrarily well approximate a continuous nonlinear function of N input samples $f[x(n), x(n-1), \dots, x(n-N+1)]$, as the well-known Volterra filters. However, in contrast to Volterra filters, the orthogonality property allows the derivation of gradient descent algorithms with fast convergence speed. Moreover, efficient identification algorithms for nonlinear systems with performance often better than those of Volterra filters, especially in presence of strong nonlinearities, can be devised.

The paper is organized as follows. In Section 2, general considerations on the approach used for introducing a novel class of nonlinear filters on the basis of the Stone-Weierstrass approximation theorem are outlined. In Section 3, EMFN filters are introduced and their properties are described. Some simulation results, showing the potentialities of EMFN filters in comparison with other LIP filters, are presented in Section 4. Conclusions follow in Section 5.

Throughout the paper the following notation is used. Sets are represented with curly brackets, intervals with square brackets, while the following convention for brackets: $\{[(\cdots\{[()]\}\cdots)]\}$ is used elsewhere.

2. GENERAL CONSIDERATIONS

Assuming for simplicity the system to be causal, the input-output relationship of a time-invariant, finite-memory, continuous nonlinear system can be expressed by a nonlinear function f of the N most recent input samples, i. e.

$$y(n) = f[x(n), x(n-1), \dots, x(n-N+1)]. \tag{1}$$

In (1), the input signal $x(n)$ is assumed to take values in the range $[-1, +1]$,

$$x(n) \in \mathbb{R}_1 = \{x \in \mathbb{R}, \text{ with } |x| \leq 1\}, \tag{2}$$

$y(n) \in \mathbb{R}$ is the output signal and N is the system memory. The representation in (1) is useful to efficiently implement the finite-memory nonlinear filters, suitably exploiting their shifting property [17]. From the analysis point of view, equation (1) can be interpreted as a multidimensional function in the $\mathbb{R}_1^N$ space, where each dimension corresponds to a delayed input sample. This representation has been already exploited, for example, to represent truncated Volterra filters where the nonlinearity is mapped to multidimensional kernels that appear linearly in the input-output relationship [1]. Therefore, it is possible to represent the nonlinear function $f[x(n), x(n-1), \ldots, x(n-N+1)]$ with a series of basis functions f_i, as in the following equation

$$f[x(n), x(n-1), \ldots, x(n-N+1)] =$$

$$\sum_{i=1}^{+\infty} c_i f_i[x(n), x(n-1), \ldots, x(n-N-1)], \qquad (3)$$

where $c_i \in \mathbb{R}$, and f_i is a continuous function from $\mathbb{R}_1^N$ to $\mathbb{R}$, for all i. Every choice of the set of basis functions f_i defines a different kind of nonlinear filters, which can be used to approximate the nonlinear systems in (1). In particular, we are interested in nonlinear filters that are able to arbitrarily well approximate every time-invariant, finite-memory, continuous nonlinear system. To this purpose, we resort to the well known Stone-Weierstrass theorem [29]:

> "Let $\mathcal{A}$ be an algebra of real continuous functions on a compact set K. If $\mathcal{A}$ separates points on K and if $\mathcal{A}$ vanishes at no point of K, then the uniform closure $\mathcal{B}$ of $\mathcal{A}$ consists of all real continuous functions on K".

Indeed, according to the Stone-Weierstrass theorem any algebra of real continuous functions on the compact $\mathbb{R}_1^N$ which separates points and vanishes at no point is able to arbitrarily well approximate the continuous function f in (1). A family $\mathcal{A}$ of real functions is said to be an algebra if $\mathcal{A}$ is closed under addition, multiplication, and scalar multiplication, i. e., if (i) $f + g \in \mathcal{A}$, (ii) $f \cdot g \in \mathcal{A}$, and (iii) $cf \in \mathcal{A}$, for all $f \in \mathcal{A}$, $g \in \mathcal{A}$ and for all real constants c.

A set of basis functions that satisfies the requirements of the Stone-Weierstrass theorem is the polynomial set

$$\{1, x(n), x(n-1), \ldots, x(n-N+1), x^2(n), \ldots, x^2(n-N+1),$$
$$x(n)x(n-1), \ldots, x(n-N+2)x(n-N+1), \ldots,$$
$$x(n)x(n-N+1), x^3(n), \ldots\}.$$

The linear combination of these basis functions defines the well-known truncated Volterra filters.

A class of basis functions that does not satisfy the requirements of the Stone-Weierstrass theorem is that of FLANN filters

$$\{1, x(n), x(n-1), \ldots, x(n-N+1),$$
$$\cos[\pi x(n)], \ldots, \cos[\pi x(n-N+1)],$$
$$\sin[\pi x(n)], \ldots, \sin[\pi x(n-N+1)],$$
$$\cos[2\pi x(n)], \ldots, \cos[2\pi x(n-N+1)],$$
$$\sin[2\pi x(n)], \ldots, \sin[2\pi x(n-N+1)], \ldots, \}$$

This class is not complete under multiplication and FLANN filters cannot approximate well system having cross-terms, i. e. product of samples with different time shifts. For instance, they cannot approximate with arbitrary accuracy the system $y(n) = x(n) \cdot x(n-1)$ because none of the FLANN basis functions has this cross product in its Volterra series expansion.

In this paper we are interested in developing a class of nonlinear filters based on sine and cosine functions of the input signal, similar to those of FLANN filters. Differently from FLANN filters, we want our class to satisfy all the requirements of the Stone-Weierstrass theorem to be able to arbitrarily well approximate any continuous, finite memory, causal nonlinear system.

3. EVEN MIRROR FOURIER NONLINEAR FILTERS

For presentation's simplicity, we first consider the case of an 1-dimensional function in (1), where $f(x)$ is a continuous function from the unit interval $[-1, 1]$ to $\mathbb{R}$. In order to expand the function $f(x)$ using sine and cosine functions, it is possible to consider the periodic repetition of $f(x)$ with period 2 and the Fourier series expansion of $f(x)$. It is well-known from the Fourier theory that the series converges to the function $f(x)$ on all the points of the interval $[-1, +1]$, apart from the borders ± 1 where discontinuities caused by the periodic repetition of $f(x)$ can be present. Moreover, as still known from the Fourier theory, these discontinuities originate relevant high-order terms in the Fourier expansion of f. In the 1-dimensional case, apart from the linear term, FLANN filters coincide with a truncation of the Fourier series expansion of f, and thus are affected by these jump discontinuities at ± 1. From the theory of Discrete Cosine Transform we know that a simple expedient to avoid this drawback is that of considering a mirror image periodic repetition of f. Thus, here we extend $f(x)$ on the entire real axis $\mathbb{R}$ by considering its periodic even mirror repetition, so that

$$f(1 + x) = f(1 - x). \qquad (4)$$

and

$$f(x + 4) = f(x). \qquad (5)$$

Since $f(x)$ is periodic of period 4, we can consider its Fourier series expansion using the basis functions

$$\{1, \cos(\frac{\pi}{2}x), \sin(\frac{\pi}{2}x), \cos(\pi x), \sin(\pi x), \cos(\frac{3\pi}{2}x), \sin(\frac{3\pi}{2}x),$$
$$\cos(2\pi x), \sin(2\pi x), \cos(\frac{5\pi}{2}x), \sin(\frac{5\pi}{2}x), \ldots\}. \qquad (6)$$

The basis functions

$$\{\cos(\frac{\pi}{2}x), \sin(\pi x), \cos(\frac{3\pi}{2}x), \sin(2\pi x), \cos(\frac{5\pi}{2}x), \ldots\}$$

are not even mirror, i. e. they do not satisfy the condition in (4), and consequently they do not contribute to the even mirror periodic expansion of the function $f(x)$. It can be easily verified that the remaining basis functions

$$\{1, \sin(\frac{\pi}{2}x), \cos(\pi x), \sin(\frac{3\pi}{2}x), \cos(2\pi x) \sin(\frac{5\pi}{2}x), \ldots\} \quad (7)$$

satisfy the conditions (4) and (5), and thus can be used to approximate the 1-dimensional even mirror periodic extension of $f(x)$. It is convenient now to attribute the following orders to the basis functions: 1 is the basis function of order 0, $\sin(\frac{\pi}{2}x)$ is the basis function of order 1, $\cos(\pi x)$ is the basis function of order 2, ..., $\cos(k\pi x)$ is the basis function of order $2k$ and $\sin(\frac{(2k+1)\pi}{2}x)$ is the basis function of order $2k + 1$.

Let us now interpret the continuous nonlinear function $f[x(n), x(n-1), \ldots, (n-N+1)]$ as a multidimensional function in the $\mathbb{R}_1^N$ space, where each dimension corresponds to a delayed input sample. It is then possible to give account of the even mirror nonlinear basis functions in the N-dimensional case, passing from $\mathbb{R}_1$ to $\mathbb{R}_1^N$. To this purpose, we first consider the 1-dimensional basis functions in (7) for $x = x(n), x(n-1), \ldots, x(n-N+1)$:

$$1, \sin[\frac{\pi}{2}x(n)], \cos[\pi x(n)], \sin[\frac{3\pi}{2}x(n)], \ldots$$

$$1, \sin[\frac{\pi}{2}x(n-1)], \cos[\pi x(n-1)], \sin[\frac{3\pi}{2}x(n-1)], \ldots$$

$$\vdots$$

$$1, \sin[\frac{\pi}{2}x(n-N+1)], \cos[\pi x(n-N+1)], \sin[\frac{3\pi}{2}x(n-N+1)] \ldots$$

Then, to guarantee completeness of the algebra under multiplication, we multiply the terms having different variables in any possible manner, taking care of avoiding repetitions. It is easy to verify that this family of real functions and their linear combinations constitutes an algebra on the compact $[-1, 1]$ that satisfies all the requirements of the Stone-Weierstrass theorem. Indeed, the set of functions is closed under addition, multiplication and scalar multiplication. The algebra vanishes at no point due to the presence of the function of order 0, which is equal to 1. Moreover, it separates points, since two separate points must have at least one different coordinate $x(n-k)$ and $\sin[\frac{\pi}{2}x(n-k)]$ separates these points. As a consequence, the nonlinear filters exploiting these basis functions are able to arbitrarily well approximate any time-invariant, finite-memory, continuous nonlinear system.

More specifically, let us define the order of an N-dimensional basis function as the sum of the orders of the constituent 1-dimensional basis functions. For example, $\cos[2\pi x(n)] \cdot \cos[\pi x(n-1)] \cdot \sin[\frac{\pi}{2}x(n-2)]$ has order $4+2+1=7$. Avoiding repetitions, we thus obtain the following basis functions:
The basis function of order 0 is the constant 1.
The basis functions of order 1 are the N 1-dimensional basis functions of the same order:

$$\sin[\frac{\pi}{2}x(n)], \sin[\frac{\pi}{2}x(n-1)], \ldots, \sin[\frac{\pi}{2}x(n-N+1)].$$

The basis functions of order 2 are the N 1-dimensional basis functions of the same order and the basis functions originated by the product of two 1-dimensional basis functions of order 1. Avoiding repetitions, the basis functions are:

$$\cos[\pi x(n)], \cos[\pi x(n-1)], \ldots, \cos[\pi x(n-N+1)],$$
$$\sin[\frac{\pi}{2}x(n)] \cdot \sin[\frac{\pi}{2}x(n-1)], \ldots,$$
$$\sin[\frac{\pi}{2}x(n-N+2)] \cdot \sin[\frac{\pi}{2}x(n-N+1)],$$
$$\sin[\frac{\pi}{2}x(n)] \cdot \sin[\frac{\pi}{2}x(n-2)], \ldots,$$
$$\sin[\frac{\pi}{2}x(n-N+3)] \cdot \sin[\frac{\pi}{2}x(n-N+1)],$$
$$\vdots$$
$$\sin[\frac{\pi}{2}x(n)] \cdot \sin[\frac{\pi}{2}x(n-N+1)].$$

Thus, we have $N \cdot (N+1)/2$ basis functions of order 2.
Similarly, the basis functions of order 3 are the N 1-dimensional basis functions of the same order, the basis functions originated by the product between an 1-dimensional basis function of order 2 and an 1-dimensional basis function of order 1, and the basis functions originated by the product of three 1-dimensional basis functions of order 1. This constructive rule can be iterated for any order P.

It is worth noting that the basis functions of order P can also be obtained by multiplying in every possible way the basis functions of order $P-1$ by those of order 1 and deleting repetitions. In this case it is necessary to apply the following substitution rule for products between factors having the same time index:

$$\cos(m\pi x)\sin(\frac{\pi}{2}x) \longrightarrow \sin(\frac{2m+1}{2}\pi x),$$
$$\sin(\frac{2m+1}{2}\pi x)\sin(\frac{\pi}{2}x) \longrightarrow \cos[(m+1)\pi x].$$

In fact, due to the prosthaphaeresis formulas, $\cos(m\pi x)\sin(\frac{\pi}{2}x)$ is a linear combination of the basis function $\sin(\frac{2m+1}{2}\pi x)$ and of a basis function of order $2m-1$, $\sin(\frac{2m-1}{2}\pi x)$. A similar justification applies to the second replacement rule.

It clearly appears that the multiplicative rule for generating the basis functions of order P from those of order $P-1$ is the same rule applied for Volterra filters. In our case, the linear combination of all the even mirror basis functions of the same order P defines an EMFN filter of uniform order P. The number of its terms is

$$\binom{N+P-1}{P}, \tag{8}$$

where N is the memory length. Clearly, this number is the same of the polynomial basis functions of a Volterra filter with the same memory of N samples. The linear combination of all the basis functions with order ranging from 0 to P and memory length of N samples defines an EMFN filter of nonuniform order P. The number of its terms is

$$\binom{N+P}{N}. \tag{9}$$

It is now possible to show that the set of basis functions of EMFN filters is orthogonal in $\mathbb{R}_1^N$. In fact, taking two different basis functions f_i and f_j, the orthogonality condition is written as

$$\int_{-1}^{+1} \cdots \int_{-1}^{+1} f_i[x(n), x(n-1), \ldots, x(n-N+1)] \cdot$$
$$f_j[x(n), x(n-1), \ldots, x(n-N+1)] \cdot \mathrm{d}x(n) \cdots \mathrm{d}x(n-N+1) = 0 \tag{10}$$

It is easy to verify that the basis functions of EMFN filters satisfy (10) since for any integer m

$$\int_{-1}^{+1} \sin(\frac{2m+1}{2}\pi x)\mathrm{d}x = \int_{-1}^{+1} \cos(m\pi x)\mathrm{d}x = 0. \tag{11}$$

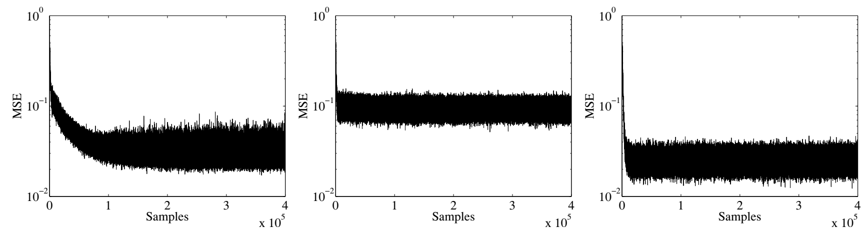
As a direct consequence of this orthogonality property, the expansion of $f[x(n), \ldots, x(n-N+1)]$ with the proposed basis functions is a generalized Fourier series expansion [30]. Moreover, the following condition also holds for a white uniform distribution of the input signal samples

$$\int_{-1}^{+1} \cdots \int_{-1}^{+1} f_i[x(n), \ldots, x(n-N+1)] \cdot f_j[x(n), \ldots, x(n-N+1)] \cdot$$
$$\cdot p[x(n), \ldots, x(n-N+1)] \cdot \mathrm{d}x(n) \cdots \mathrm{d}x(n-N+1) = 0, \tag{12}$$

where $p[x(n), \ldots, x(n-N+1)]$ is the probability density of the N-tuple $[x(n), \ldots, x(n-N+1)]$, equal to the constant $1/2^N$ for a white uniform distribution in $\mathbb{R}_1$. As a consequence, it is possible to devise for EMFN filters simple identification algorithms using input signals with white uniform distributions in the range $[-1, +1]$. Moreover, a fast convergence of the gradient descent adaptation algorithms, used for nonlinear systems identification, is expected in this situation.

Table 1. MSE versus K for Volterra, FLANN, and EMFN filters.

K	0.125	0.25	0.50	0.75	1.00
Volterra	$1.3 \cdot 10^{-1}$	$8.3 \cdot 10^{-2}$	$3.0 \cdot 10^{-2}$	$1.0 \cdot 10^{-2}$	$3.6 \cdot 10^{-3}$
FLANN	$2.3 \cdot 10^{-1}$	$1.7 \cdot 10^{-1}$	$9.4 \cdot 10^{-2}$	$5.0 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$
EMFN	$1.2 \cdot 10^{-1}$	$7.2 \cdot 10^{-2}$	$2.5 \cdot 10^{-2}$	$8.6 \cdot 10^{-3}$	$3.3 \cdot 10^{-3}$



(a) (b) (c)

Fig. 1. Learning curves of (a) Volterra, (b) FLANN, (c) EMFN filters.

4. SIMULATION RESULTS

In this section we provide some simulation results to show the potentialities of EMFN filters in comparison with Volterra and FLANN filters, and we highlight the advantages of the orthogonality property in (12). Here we work in a simulated environment to better control the characteristics of the unknown system to be modeled. We will provide experimental results for the identification of real systems in a companion paper discussing algorithms for the identification of long-memory nonlinear systems.

When dealing with different nonlinear filter structures we must take into account that each structure is better fitted to certain nonlinear conditions, and may work worse in other conditions. When modeling a nonlinear system affected by a small nonlinearity, a Volterra or FLANN filter may be a model better than the EMFN filter, because the latter does not use explicitly a linear term. In contrast, in presence of a strong nonlinearity as, for example, a saturation effect, EMFN filters can offer performance better than Volterra filters and much better than FLANN filters.

In our experiments we consider the identification of a Wiener model obtained by cascading a linear filter with the memoryless nonlinearity

$$y(n) = \tanh[x(n)/K], \tag{13}$$

where K is a positive constant that controls the strength of the nonlinearity. The entries of the impulse response of the linear filter, $\{-1.630, -0.741, -0.111, 0.350, -0.194, 0.002, 0.750\}$, have been selected randomly. This system is modeled with Volterra, FLANN and EMFN filters with memory of 7 samples. Since the nonlinear system is odd, we consider only the odd terms of Volterra and EMFN filters, and the sine terms of FLANN filters. The order of the Volterra and EMFN filters is 3 and the order of the FLANN filter is 12. Thus, the number of coefficients is 91 for all filters. An LMS algorithm with a white uniform input signal in $[-1, +1]$ is used for the identification. Since the three different nonlinear filters converge to different Mean Square Errors (MSE), whatever the choice of the step-size, we compare the performance of the algorithms for the same step-size, equal to 0.001. Figure 1 shows the ensemble averages over 100 realizations of the learning curves of MSE for the three filters in a situation of noticeable nonlinearity ($K = 0.5$). The first characteristic we can observe is the fast convergence speed of EMFN and FLANN filters compared to the Volterra filter. According to the orthogonality property in (12), the autocorrelation matrices of the input data vector of EMFN filters are diagonal (a property never shared by Volterra filters for any input signal [1]). This fact explains the fast convergence of EMFN filters in comparison with Volterra filters. With FLANN filters, we loose the orthogonality of the autocorrelation matrix, but we still have a good conditioning, and thus the convergence speed remain similar to that of EMFN filters. The steady-state modeling properties of the three filters strongly depend on the level of the nonlinearity, i. e. on the value of K in our case. Table 1 provides the average value of MSE estimated on 100 000 samples after convergence (specifically, after a simulation 10^6 samples long) for K ranging between 0.125 and 1. The larger is K, the smaller is the effect of the nonlinearity. For K greater than 1, the system is almost linear, and the Volterra and FLANN filters provide the best MSEs thanks to their linear term. On the other hand, for K between 1 and 0.25, i. e. for a nonlinearity ranging from mild to strong, the EMFN filter provides the best MSEs. For $K = 0.125$, the nonlinearity is very strong and all filters become less efficient in modeling it within the chosen filter orders. Clearly, higher filter orders can give better results, but with higher computational complexity. In conclusion, in the reported nonlinear situation, the EMFN filter is able to offer fast convergence together with small residual MSEs with respect to other LIP nonlinear filters, such as FLANN and Volterra filters, on a wide range of nonlinearity.

5. CONCLUSIONS

In this paper, a novel sub-class of finite-memory LIP nonlinear filters, the EMFN filters, has been introduced. EMFN filters are based on trigonometric functions which are mutually orthogonal for white uniform input signals. It has been shown that their linear combinations are universal approximators, according to the Stone-Weierstrass theorem, for causal, time invariant, finite-memory, continuous nonlinear systems. As a consequence of the orthogonality property, EMFN filters may offer better convergence speed and lower approximation errors than FLANN and Volterra filters, especially in presence of noticeable nonlinearities. Presently, work is in progress to apply EMFN filters to the identification of real-world nonlinear systems.

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