PERFORMANCE ANALYSIS OF SEQUENTIAL DETECTION FOR COLLISION AVOIDANCE IN SENSOR NETWORKS

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ABSTRACT

Many of today's wireless sensor networks operate under the strict requirement that only a single sensor transmits data at a time. One way to guarantee this is to use protocols that detect and prevent package collisions on the MAC layer. These, however, come at the cost of increased transmission delays, reduced throughput and higher energy consumption. We propose a PHY layer approach to collision avoidance that is based on sequential detection and significantly reduces the risk of collisions while simultaneously minimizing the transmission delay. For this approach, a performance analysis is given whose results are shown to closely match numerical simulations.

***Index Terms*—** wireless sensor networks, collision avoidance, sequential analysis, distributed detection

1. INTRODUCTION

The design of efficient and adaptive wireless sensor networks is a topic of increasingly active research [1, 2]. However, while the benefit of multi-user transmission schemes (e.g. spatial multiplexing) is well known in theory, the hardware and power limitations of current low-cost sensor nodes often allow only a single sensor to transmit data at a time [3]. In addition, commonly used scheduling protocols are only partially applicable to such networks. They most often require synchronisation, create overhead data, and have to be adapted if the network topology or the number of sensors changes, hence limiting the flexibility and scalability of the network.

Currently, the most common approach to collision avoidance in wireless sensor networks is to let each sensor sense the spectrum for ongoing transmissions before sending its own message, see e.g. [4] and references therein. While this procedure works reasonably well for randomly timed, sporadic transmissions, its performance quickly degenerates in case of frequent or event triggered transmissions. This effect is due to an increasing probability that several (e.g. closely spaced) sensors start their sensing period at roughly the same time and, in turn, simultaneously start to transmit over the presumably clear channel [5].

To deal with this problem, protocols have been proposed [6] that work as follows: After finishing spectrum sensing, each sensor additionally sends a preamble of random length, then senses the spectrum for a second time and only starts transmitting if the channel is still clear. In this way, only the sensor with the longest preamble will get a clear channel acknowledgement in the end.

The idea presented in this paper is to create this randomisation effect not by adding noise preambles, but by using random sensing times. More precisely, we propose the use of sequential detectors, which not only introduce a random sensing time, but additionally minimise the average number of samples needed to make a decision [7]. In an ideal case, the first sensor to finish the detection phase starts transmitting while all the others are still sensing the spectrum. Due to the adaptive nature of sequential tests, they are then able to "change their minds" and detect the initiated transmission. In this way, not the sensor with the longest, but the one with the shortest sensing delay is served, while simultaneously the probability of collisions is significantly reduced.

The paper is organised as follows: In Section 2 we introduce the system model and problem formulation. The performance analysis, which constitutes the main contribution, is presented in Section 3. Finally, in Section 4, we give an example and compare the analytical results to Monte Carlo simulations.

2. PROBLEM FORMULATION

We consider a worst case scenario, where the transmissions of m sensors are simultaneously triggered by some event. Each sensor performs a statistical test to check whether the channel is clear or not and, in case of a positive result, starts transmitting after a certain transition time $\Delta T \in \mathbb{N}_0$. Note that in the case of fixed sample size tests this scenario necessarily leads to collisions.

In the sequel, we assume that each sensor $i \in \mathcal{M} = \{1, \dots, m\}$ performs a sequential probability ratio tests between the two hypotheses

$$\begin{aligned} \mathcal{H}_0 :&\quad P = P_0 \quad \text{(noise only)} \\ \mathcal{H}_1 :&\quad P = P_1 \quad \text{(signal present)} \end{aligned}$$

where P_0, P_1 denote two probability measures and P_1 is absolutely continuous w.r.t. P_0. Let further S_i^n $(i \in \mathcal{M},\, n \geq 1)$ denote the log-likelihood ratio at the i-th sensor at time n. The stopping and decision rule for each sequential test is given by

$$S_i^n \begin{cases} \geq A, & \text{decide for } \mathcal{H}_1 \\ \in (B, A), & \text{continue testing} \\ \leq B, & \text{decide for } \mathcal{H}_0 \end{cases}$$

where A, B are given constants. The according error probabilities of the first kind (false alarm) and second kind (missed detection)

This work was supported by the LOEWE Priority Program Cocoon (http://www.cocoon.tu-darmstadt.de) supported by the LOEWE research initiative of the state of Hesse/Germany.

are denoted $\alpha_i = \alpha$ and $\beta_i = \beta$, respectively. The (stopped) test statistics S_i^n are recursively defined via

$$S^n \sim S_i^n = \begin{cases} S_i^{n-1}, & S_i^{n-1} \notin (B,A) \\ S_i^{n-1} + X_i^n, & S_i^{n-1} \in (B,A) \end{cases}$$

with $S_i^0 = 0$, X_i^n denoting the log-likelihood increments, and $\sim$ denoting equality in distribution. In what follows, we assume all X_i^n to be i.i.d. under $\mathcal{H}_0$ according a known distribution F_0 with continuous density function f_0. Under $\mathcal{H}_1$ we assume the increments to be still independent between the sensors, but possibly correlated in time.

We further assume that the duration of a transmission is "infinitely long", i.e., that all sequential tests finish before the transmission ends. Again, this is a kind of worst case assumption, since longer transmission times obviously increase the collision probabilities.

3. PERFORMANCE ANALYSIS

The performance analysis given in this section aims to provide exact (or closely approximated) expressions for the collision probabilities. Due to the considered application, we focus on scenarios with a moderate number of sensors and finite error probabilities instead of doing an asymptotic analysis. The latter one may be the subject of future work.

Our analysis relies on knowledge of the distribution of the extended stopping time

$$T \sim T_i = \min\{n \geq 1 : \; S_i^n \leq B\},$$

under P_0, i.e., the probability of the i-th sequential test to correctly decide for $\mathcal{H}_0$ at time T_i. Since we assume the tests to be independent and identical, T_i $(i \in \mathcal{M})$ are i.i.d. and their distribution can be calculated recursively [8] via

$$\begin{aligned} P_0[T = n] &= \int_{(-\infty,B)} q_n(x)dx, \\ q_n(x) &= \int_{(B,A)} q_{n-1}(\omega) f_0(x-\omega) d\omega \end{aligned} \tag{1}$$

using the starting point $q_1 = f_0$. Note that

$$q_n(x) = \frac{\partial}{\partial x} P_0 \left[S_i^n \leq x \, , \, S_i^{1:n-1} \in (B,A) \right],$$

where $S_i^{1:n}$ is short for $S_i^1, \ldots, S_i^n$.

Let us further introduce the ordered stopping times $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(m)}$, which are adapted to the joint sequence $(X_1^n, \ldots, X_m^n)$. Of particular interest for us is $T_{(1)}$, which is the time the first sensor finishes its test with a decision for $\mathcal{H}_0$. This means that a change in probability measure occurs at $T_C = T_{(1)} + \Delta T$ when this sensor starts transmitting. The distribution of $T_{(1)}$, and thereby T_C, is given by [9]

$$P[T_{(1)} \leq n] = \sum_{i=0}^{m-1} \binom{m}{i} (1 - P[T \leq n])^i \, P[T \leq n]^{m-i}.$$

A collision free transmission happens whenever T_C is finite and no other sensor starts transmitting after T_C. The probability of a failed transmission is hence given by

$$\begin{aligned} P(\{\text{"failed transmission"}\}) &= P[T_{(1)} = \infty] + P[T_{(2)} < \infty] \\ &\approx P[T_{(2)} < \infty], \end{aligned}$$

where $P[T_{(1)} = \infty] = \alpha^m$ denotes the probability that all sequential tests erroneously decide for $\mathcal{H}_1$. Throughout the paper we assume (mainly to simplify notation) that this probability is negligibly small.

Due to the change point at T_C, we can further write

$$P[T_{(2)} < \infty] = P_0[T_{(2)} \leq T_C] + P[T_C < T_{(2)} < \infty].$$

Here we can identify two possible sorts of collisions, in the following called Type I and Type II. The first type of collision appears whenever several sensors decide for $\mathcal{H}_0$ within a time slot of length ΔT. The second type of collision appears when a transmission already started, but at least one sensor fails to detect it. We will treat these two types of collisions separately.

3.1. Type I Collisions

The probability of a Type I collision can be calculated straightforwardly by

$$P_0[T_{(2)} \leq T_C] = \sum_{w=0}^{\Delta T} P_0[W_{12} = w]$$

where $W_{12} = T_{(2)} - T_{(1)}$ denotes the spacing between the first and second order statistic. Neglecting the possibility of $T_{(1)} = \infty$, it can further be derived from the results in [9] that the distribution of W_{12} is given by

$$\frac{P[W_{12} = w]}{m} = \sum_{n \in \mathbb{N}} P_0[T = n] \, d_{m-1}(n + w),$$

for $w \geq 1$ and

$$\frac{P[W_{12} = 0]}{m} = \sum_{n \in \mathbb{N}} P_0[T \geq n] \, d_{m-1}(n) - \frac{m-1}{m} d_m(n),$$

where

$$d_m(x) = (P_0[T \geq x])^m - (P_0[T > x])^m.$$

Note that $d_m(x) \leq m \, P_0[T = x]$ can be used to derive Markov type upper bounds on collision probabilities of the first type.

3.2. Type II Collisions

The calculation of the probability of collisions of the second type is a bit more involved. They occur if a single sensor starts transmitting while all other sensors are still sensing or already erroneously decided for $\mathcal{H}_1$. This means that at time T_C some $m' \leq m-1$ sequential test are still running. Let us denote the set of these tests by $\mathcal{M}' \subset \mathcal{M}$. Under some mild independence assumptions, this situation can as well be interpreted as m' tests *starting* at time T_C, under the new measure P_1, and with starting points $S_i^{T_C}$ $(i \in \mathcal{M}')$. Or equivalently: m' tests starting from zero, but with adapted thresholds $A_i' = A - S_i^{T_C}$ and $B_i' = B - S_i^{T_C}$.

Using Wald's approximation [10] (which also holds in case of time correlated signals) we obtain that for these tests

$$\beta_i'\left(S_i^{T_C}\right) \approx 1 - \frac{e^B(e^A e^{-S_i^{T_C}} - 1)}{e^A - e^B} \approx 1 - e^B e^{-S_i^{T_C}}.$$

where β_i' $(i \in \mathcal{M}')$ denotes the probability of an erroneous decision for $\mathcal{H}_0$ as a function of $S_i^{T_C}$.

The approximate probability that all m' tests correctly decide for $\mathcal{H}_1$ is accordingly given by

$$1 - E_0 \left[\prod_{i \in \mathcal{M}'} \left(1 - e^B e^{-S_i^{T_C}} \right) \right], \tag{2}$$

where the expected value is taken with respect to P_0. Evaluating this expression analytically is formidable. We therefore resort to a second approximation step, inspired by the following result of decoupling theory [11, Theorem 2.1.1']:

Given a stopping time T, adapted to a sequence X_n of finite mean i.i.d. random variables, and an independent copy $\tilde{X}_n$ of this sequence, we have

$$E\big[(S_T)^i\big] = E\big[(\tilde{S}_T)^i\big] \quad \text{for} \quad i = 1, 2$$

where $S_T = X_1 + \ldots + X_T$ and $\tilde{S}_T = \tilde{X}_1 + \ldots + \tilde{X}_T$. This means that the distribution of the actual stopping value S_T can, at least up to the first two moments, be approximated by the stopping value of a sequence that is independent of T.

Following this idea, we replace each S_i^n in (2) by an independent copy $\tilde{S}_i^n \sim \tilde{S}^n \sim S^n$, which in turn is also independent of T_i. This allows us to break the dependencies between the stopping time $T_{(1)}$ and the sequences S_i^n.

Obviously, the tests $i \in \mathcal{M}'$ are only those out of the m initial tests, for which $S_i^{1:T_C} \in (B, A)$ holds. Accordingly, we restrict the evaluation of the expected value in (2) to sequences with this behaviour, i.e., we take it with respect to the compound distribution

$$\begin{aligned} \tilde{Q}(x) &= P_0 \left[\tilde{S}^{T_C} \leq x \,|\, \tilde{S}^{1:T_C} \in (B, A) \right] \\ &= \sum_{n \in \mathbb{N}} P_0[\tilde{S}^n < x \,|\, \tilde{S}^{1:n} \in (B, A)] \, P_0[T_C = n] \\ &=: \sum_{n \in \mathbb{N}} \tilde{Q}_n(x) \, P_0[T_C = n]. \end{aligned}$$

Note that the second step in this reformulation requires the sequence and the stopping time to be independent.

Now, since $\tilde{S}^n$ and S^n are identically distributed, it can be easily shown that the density corresponding to $\tilde{Q}_n$ is just a truncated and scaled version of q_n, as defined in (1), i.e.,

$$\tilde{q}_n(x) = \begin{cases} \frac{q(x)}{c_n}, & x \in (B, A) \\ 0, & x \notin (B, A) \end{cases} \tag{3}$$

where

$$c_n = \int_{(B,A)} q_n(x) dx.$$

Combining these results, we get

$$E_0 \left[\prod_{i \in \mathcal{M}'} \left(1 - e^B e^{-S^{T_C}} \right) \right] \approx \left(1 - e^B E_{\tilde{Q}} \left[e^{-\tilde{S}^{T_C}} \right] \right)^{m'} \tag{4}$$

with

$$E_{\tilde{Q}} \left[e^{-\tilde{S}^{T_C}} \right] = \sum_{n \in \mathbb{N}} P_0[T_C = n] \int_{(B,A)} e^{-x} \tilde{q}_n(x) dx \tag{5}$$

and $\tilde{q}_n$ defined in (3) and (1).

Note that to obtain the unconditioned probability of collisions of the second type, (2) has to be weighted by the probability that no collision of the first type happened, i.e., by $P_0[T_{(2)} > T_C]$. This probability has already been calculated in the previous subsection.

At this point, m' is still left to be determined. Since the right hand side of (4) is decreasing in m', we can set $m' = m - 1$ to get a conservative approximation. Alternatively, m' might be seen as a free parameter that can be used to compensate for the independence approximation, that tends to provide rather pessimistic results – see the next section for more details.

4. EXAMPLE AND NUMERICAL RESULTS

In this section we demonstrate the performance analysis by an example, where each sensor has to detect whether the parameter σ of a Rayleigh distribution

$$F(x) = 1 - e^{\frac{x^2}{2\sigma^2}},$$

is given by $\sigma = \sigma_0$ or $\sigma = \sigma_1 > \sigma_0$. This corresponds to an energy detection of complex Gaussian signals at a signal-to-noise ratio of $\text{SNR} = \frac{\sigma_1^2}{\sigma_0^2}$.

To perform the analysis presented in the previous section, knowledge of the functions q_n is of utmost importance. In the case considered here, these functions can be shown to be of the convenient form

$$q_n(x) = \lambda^n e^{-\lambda(x+ns)} \, a_n(x), \tag{6}$$

where $s = \log(\text{SNR})$, $\lambda = \text{SNR}(\text{SNR} - 1)^{-1}$, and a_n are piecewise polynomial functions defined via

$$a_n(x) = \int_{(B, \min\{A, s+x\})} a_{n-1}(\omega) d\omega$$

and $a_1 = \mathbf{1}_{(-s,\infty)}$, where $\mathbf{1}_{\mathcal{A}}$ denotes the indicator function of the set $\mathcal{A}$. Obtaining analytical expressions for a_n is straightforward, but extremely tedious. However, a recursive definition of the polynomial coefficients is possible without solving any integrals numerically. This allows us to calculate explicit expressions for q_n in a highly efficient way.

From the structure of q_n in (6) it further follows that any integral over $q_n(x)$ or $e^{-x} q_n(x)$ can be expressed as a weighted sum of upper incomplete gamma functions Γ of order $1, \ldots, n+1$. These expressions become particularly simple if the thresholds A and B are chosen as multiples of s. In this case, the stopping probabilities are of the form

$$P_0[T = n] = \sum_{i=1}^{n+1} \gamma_i \, \Gamma(i, \lambda s).$$

The coefficients $\gamma_i \in \mathbb{R}$ can be obtained from (6) and a_n. Analogously, the expected value in (5) can be expressed in terms of the ratio of weighted Γ-functions.

An overview of some numerical results is given in Table 1. The simulation results were obtained by averaging over 10^4 Monte Carlo runs. The analytical results were computed using the formulae provided in Section 3. To this end, the distribution of the stopping time T was calculated up to some N for which $P[T > N] \leq 10^{-5}$. The infinite sums were accordingly evaluated over $\{0, \ldots, N\}$ instead of $\mathbb{N}_0$. We further give results for $m' = m-1$ and $m' = \frac{2}{3}(m-1)$. The latter value has been chosen heuristically and, at this point, has no analytical justification. However, for the parameter constellations we considered, it provided results closer to the simulated values, while still tending to slightly over-estimating the collision probabilities.

	Monte Carlo	$m' = m - 1$	$m' = \frac{2}{3}(m-1)$
	$s = 1,\ \ m = 3,\ \ A = 10,\ \ \Delta T = 1$		
p_1	0.1856	0.1848	
p_2	0.1646	0.2389	0.1683
	$s = 1,\ \ m = 6,\ \ A = 10,\ \ \Delta T = 1$		
p_1	0.3189	0.3205	
p_2	0.2641	0.3581	0.2670
	$s = 1,\ \ m = 3,\ \ A = 10,\ \ \Delta T = 2$		
p_1	0.3040	0.2986	
p_2	0.1495	0.2162	0.1528
	$s = 2,\ \ m = 3,\ \ A = 10,\ \ \Delta T = 1$		
p_1	0.6104	0.6084	
p_2	0.0807	0.1781	0.1303
	$s = 1,\ \ m = 3,\ \ A = 5,\ \ \Delta T = 1$		
p_1	0.3025	0.3028	
p_2	0.2171	0.3067	0.2234
	$s = 3,\ \ m = 4,\ \ A = 21,\ \ \Delta T = 1$		
p_1	0.8402	0.8410	
p_2	0.0259	0.0900	0.0679

Table 1. Probabilities of collisions of the first (p_1) and second (p_2) type, for different numbers of users m, thresholds $A = -B$, transition times ΔT, and $s = \log(\text{SNR})$.

Note that the analytic results for the probabilities of collisions of the first type can be considered as nearly exact, since the only inaccuracy involved in their calculation is the negligence of $P_0[T_{(1)} = \infty]$ and $P_0[T > N]$.

All in all, the analytical results are reasonably close to the simulated ones. Considering the rather strong independence approximation used to obtain the probabilities of collisions of the second type, this is quite a pleasant outcome. Especially the results obtained with the adjusted choice of m' match the actual values surprisingly well. The higher the probability of first type collisions, though, the more the analytic results tend to overestimate the probability of second type collisions – see the last row of Table 1. The relation between this effect and the applied approximations might need some further investigation.

Considering the feasibility of the approach to use sequential tests as a means of collision avoidance in sensor networks, the simulations mostly coincide with what one would expect from intuition: The more spread the distribution of T is, the lower is the probability of collisions of the first type. This is in contrast to most use cases of sequential detection, where the random stopping time is considered an unwanted, but unavoidable side effect. Conversely, the concentrated distributions occurring at higher SNR values lead to an increased probability of collisions of the first type. Lowering the thresholds A and B apparently has a similar effect.

Increasing the number of sensors and the transition time ΔT also shows the anticipated effects. The number of users surely is the more critical parameter here since, considering the bound on d_m in Section 3, its influence on the collision probability is stronger than linear. Additional simulations seem to confirm this.

The impact of the SNR, m and ΔT on the probability of collisions of the second type is less evident, but in general follows the same lines. Again, the number of users turns out to be the most critical parameter. Note, though, that obviously a high probability of errors of the first type, automatically leads to a small error probability of the second type.

A more in depth investigation and quantification of the (asymptotic) impact of the parameters on the overall probability of collisions is left for future work.

5. CONCLUSIONS

We have proposed the application of sequential detectors to reduce the probability of packet collisions in wireless sensor networks, while maintaining low delay times and power consumption. Exact and approximated expressions for the probabilities of two different types of collisions have been derived and, using the example of energy detection, have been shown to closely match simulation results. Favourable and unfavourable constellations in terms of SNR and the number of users have been briefly discussed as well.

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