IMPROVED SPECTRUM-BLIND RECONSTRUCTION OF MULTI-BAND SIGNALS

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ABSTRACT

This paper considers spectrum-blind reconstruction (SBR) of multi-band signals (MBS) which are sampled with multi-coset sampling (MCS) architecture. A new SBR algorithm which we refer to as Khatri-Rao SBR (KR-SBR) is presented. With this new KR-SBR algorithm, the average MCS rate can be reduced by 50% whilst attaining the same performance as that of the existing state-of-art SBR algorithms. Under certain conditions we will also show that the proposed KR-SBR algorithm has the capacity to achieve SBR when the average MCS rate approaches the Landau sampling rate. Simulation results are also presented to demonstrate the advantages of the proposed KR-SBR algorithm.

Index Terms— Multiband sampling, Landau sampling rate, Multicoset sampling, Khatri-Rao product.

1. INTRODUCTION

Spectrum blind reconstruction (SBR) of multi-band signals (MBS) is considered in this paper. MBS is a bandlimited signal whose frequency support resides within several continuous disjoint intervals. The necessity of sampling such signals arise in several applications; a typical example being in cognitive radio where a wide MBS spectrum is required to be sensed in order to determine the spectrum holes [1]. Usually in practice these signals are spread over a wideband, but the occupancy of the information bands within the wideband is sparse. Due to the limitations of the ADC technology and the enormous computation thereafter, sampling MBS spread over a wide spectrum band according to the classical Nyquist sampling may not be feasible as this requires the sampling to be performed at least at twice the entire bandwidth of the wideband spectrum. However, Landau bound [2] shows that the entire wideband spectrum can be perfectly recovered if sampled at the total bandwidth of the information bands, which is far less than the Nyquist rate in practice.

Among several sampling methods that may be found in the literature for sub-Nyquist sampling and reconstruction for such MBS, *multicoset sampling* (MCS) [3] method is a predominant and an important sampling technique considered by many. MCS is essentially a periodic nonuniform sampling technique which can be realized efficiently in practice using a multi-channel architecture. MCS for sampling MBS was first introduced in [4], thereafter several works have been done for spectrum reconstruction based upon this method; most important ones being [4]-[6]. While [4, 5] showed the similarity between the problem of SBR of MBS with MCS architecture and direction of arrival (DOA) estimation, [6] showed the similarity with multiple measurement vector (MMV) - compressive sensing (CS) problem [8], and accordingly applied the corresponding algorithms for SBR.

As it is shown in [4]-[6] that for successful SBR, the algorithms of [4]-[6] requires the number of MCS channels be at least two times the number of information bands (assuming some conditions are satisfied, the details of which are explained later in Section 2), while for non-blind spectrum reconstruction (i.e., the supports are well-known) it is shown in [7] that successful reconstruction is possible when the number of channels approaches the number of information bands. In this paper we propose a new SBR algorithm which we refer as Khatri-Rao SBR (KR-SBR) that has better reconstruction capability than all the existing approaches. The proposed KR-SBR algorithm first converts the MMV-CS problem into a larger space single measurement vector (SMV)-CS problem thereby enhancing the support estimation capability. However some additional conditions on the parameters of the MCS are required to be satisfied in order for successful blind reconstruction of the MBS with this new algorithm. With these conditions being satisfied, this new KR-SBR algorithm will be shown later in Section 3 that it has the capability to successfully reconstruct the spectrum blindly even when the number of MCS channels approaches the number of information bands. In other words with the usage of the KR-SBR algorithm, the average MCS rate can be reduced by 50% to obtain the same performance as that of the existing state-of-art algorithms. Further in Section 4 we will show that when the bandwidth of the information bands approach a uniform bandwidth then blind reconstruction is achievable at Landau sampling rate with the KR-SBR algorithm.

The rest of the paper is organized as follows: in the following section the definition of the MBS followed by a brief description of the MCS, its formulation and existing reconstruction methods are provided. In Section 3 we first provide the formulation of the proposed KR-SBR algorithm, then the necessary conditions on the MCS parameters and this is followed by an outline of the entire algorithm. Section 4 briefly compares the KR-SBR algorithm with the existing algorithms. Simulation results are provided in Section 5, and Section 6 concludes the paper.

2. DEFINITIONS

A brief description of the MBS model and the MCS along with the existing reconstruction methods shall be provided in this section.

2.1. Multi-band signal model

A signal $x(t)$ belongs to the class of MBS denoted by $\mathcal{M}$ if [4, 6]:
i) $x(t)$ is bandlimited to $\mathcal{F} = [0, 1/T]$.
ii) $X(f)$ which is the Fourier transform of $x(t)$ has N information bands that are disjoint i.e., if $\alpha_i = [a_i, b_i], i = \{1, 2, ..., N\}$ denotes the support of the N information bands in $\mathcal{F}$ then $\alpha_j \cap \alpha_k = \emptyset, j \neq k, j, k \in \{1, 2, ..., N\}$.
iii) The bandwidth of the information bands doesn't exceed B i.e., $\lambda(\alpha_i) \leq B$, where $\lambda(k)$ denotes the Lebesgue measure for any $k \subseteq \mathbb{R}$.
In addition to these, we make an important assumption that the *information bands are uncorrelated.*

2.2. Multi-coset sampling

2.2.1. Definition and formulation

MCS refers to a method of selecting certain samples from $x(nT)$, where $x(nT)$ denotes the samples of $x(t)$ that are sampled at uniform Nyquist rate $f_{nyq} = 1/T$. Let $\mathcal{C} = \{c_i\}_{i=1}^{p}$ denote a set of p integers satisfying $0 \leq c_1 < c_2 < ... < c_p \leq L-1$, where L is known as the sub-sampling factor, then for $1 \leq i \leq p$, $x_{c_i}(n)$ may be expressed as [4, 6]

$$x_{c_i}(n) = \begin{cases} x(nT), & n = mL + c_i, m \in \mathbb{Z} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The set $C = \{c_i\}_{i=1}^{p}$ is referred to as the sampling pattern. In practice MCS may be realized by a p-channel system with appropriate time shifts in each channel corresponding to $c_1T, c_2T, ..., c_pT$ followed by an ADC whose sampling clocks are synchronized and samples at a rate $1/LT = f_{nyq}/L$.

The discrete time Fourier transform of $x_{c_i}(n)$, $X_{c_i}(f)$ is given by

$$X_{c_i}(f) = \sum_{n \in \mathbb{Z}} x_{c_i}(n) e^{-j2\pi f n T}. \tag{2}$$

If $X(f)$ denotes the Fourier transform of $x(t)$, then by considering f in the interval $\mathcal{F}_0 = [0, 1/LT]$, the above expression may be simplified to [4]

$$X_{c_i}(f) = \frac{e^{-j2\pi f c_i T}}{LT} \sum_{l=0}^{L-1} e^{-j\frac{2\pi}{L} l c_i} X(f + \frac{l}{LT}) \tag{3}$$

Now for $1 \leq i \leq p$ and $\forall f \in \mathcal{F}_0$, the above equation may be expressed in the matrix form as

$$\mathbf{y}(f) = \mathbf{A}\mathbf{x}(f) \tag{4}$$

where $\mathbf{y}(f) = [e^{j2\pi f c_1 T} X_{c_1}(f), ..., e^{j2\pi f c_p T} X_{c_p}(f)]^T$, $\mathbf{x}(f) = [X(f), X(f + \frac{1}{LT}), ..., X(f + \frac{L-1}{LT})]^T$ and $\mathbf{A}$ is a $p \times L$ matrix with

$$[\mathbf{A}]_{i,l} = \frac{1}{LT} e^{-j\frac{2\pi}{L} l c_i} \tag{5}$$

$l = 0, 1, ..., L-1, i = 1, 2..., p.$

Usually in practice $p < L$ and it may be observed that matrix $\mathbf{A}$ is a partial discrete Fourier transform matrix of order L consisting of p rows corresponding to $\mathcal{C}$. Also notice that the ADC in each channel samples at a rate of $1/LT$. Hence the average rate for the p-channel MCS is $f_{mcs} = p/LT$ and since usually in practice $p < L$, $f_{mcs} < f_{nyq}$.

2.2.2. Reconstruction: Existing approaches

The aim is to reconstruct the unknown signal $x(t)$ from the known MCS samples $x_{c_i}(n)$. Observe that eq.(4) relates the known $X_{c_i}(f)$ to the unknown $\mathbf{x}(f)$. The l^{th} row of $\mathbf{x}(f)$, $\mathbf{x}_l(f) = X(f + \frac{l}{LT}), \forall f \in \mathcal{F}_0$ contains the spectrum in the region $[\frac{l-1}{LT}, \frac{l}{LT})$. Hence by the knowledge of vector $\mathbf{x}(f)$, $X(f), \forall f \in \mathcal{F}$ may be determined and from which $x(t)$ can be estimated. The MCS parameters p, $\mathcal{C}$ and L are chosen based on the MBS parameters (N, B, T). All the existing approaches [4, 5, 6] for SBR (i.e., the support of the bands, $\{\alpha_i\}_{i=1}^{N}$, are not known and are to be estimated) chooses the MCS parameters $L \leq 1/BT$ and $p \geq 2N$ assuming $\mathcal{C}$ to be universal. The set $\mathcal{C}$ is called universal if $\sigma(\mathbf{A}) = p$, where $\sigma(.)$ denotes the Kruskal rank (see [6, Definition 2]) of the matrix. In practice, $p < L$ and the system of equations (4) is an underdetermined system. In order to estimate the support set $\mathcal{S}_{\mathcal{F}_0} = \bigcup_{f \in \mathcal{F}_0} I(\mathbf{x}(f))$, where for any vector $\mathbf{v}$, $I(\mathbf{v}) = \{k | \mathbf{v}_k \neq 0\}$, existing algorithms borrow techniques from DOA estimation and from CS. While [5] showed the similarity between this problem and the DOA estimation problem and proposed to use methods such as MUSIC, [6] showed the similarity with the multiple measurement vector (MMV) problem in CS[8] and standard CS algorithms were used. Upon the knowledge of $\mathcal{S}_{\mathcal{F}_0}$, a submatrix of $\mathbf{A}$, $\mathbf{A}_{\mathcal{S}_{\mathcal{F}_0}}$ was formed that consisted of the columns of $\mathbf{A}$ corresponding to $\mathcal{S}_{\mathcal{F}_0}$ and a least squares solution to eq.(4) provided $\mathbf{x}_{\mathcal{S}_{\mathcal{F}_0}}(f)$ from which $\mathbf{x}(f)$ was estimated.

For N bands, when $L \leq 1/BT$, $|\mathcal{S}_{\mathcal{F}_0}| \leq 2N$ [1], and for every $f \in \mathcal{F}_0$, if $\mathcal{S}_f = I(\mathbf{x}(f))$ then $|\mathcal{S}_f| \leq N$. Thus in order to estimate the supports uniquely for every $f \in \mathcal{F}_0$, it is well known from the results of the CS [8] that we require at least two times the number of measurements. Assuming $\mathcal{C}$ to be universal and $L \leq 1/BT$, in order to blindly reconstruct N bands, the SBR algorithm of [5] requires at least [2] $p = 2N+1$, the SBR algorithm of [6] requires $p = 2N$.

In the following section we propose a new algorithm for SBR that can reconstruct perfectly N bands and requires only $p = N$, thereby reducing the average MCS rate by a factor of two compared to the existing approaches.

3. PROPOSED SPECTRUM-BLIND RECONSTRUCTION

The new formulation first converts the MMV problem (eq.(4)) into a larger space single measurement vector (SMV) problem. CS algorithms are then applied upon this SMV formulation to estimate the supports. Before we describe the method, it is essential to define the following set that is necessary for the description and also for the characterization of the proposed method.

Definition 1 *Difference modulo set:* We define the following difference set $\mathcal{C}_d = \{(c_i - c_j) \mod L | 1 \leq i, j \leq p, (c_i, c_j) \in \mathcal{C}\}$. The set $\mathcal{C}_d$ is allowed to have duplications. Further, we derive another set $\mathcal{C}_{dd}$ from $\mathcal{C}_d$ where the duplications are removed. Since $|\mathcal{C}| = p$, $|\mathcal{C}_d| = p^2$ and $|\mathcal{C}_{dd}| = p_{dd}, p < p_{dd} \leq p^2 - p + 1$.

3.1. Formulation into the SMV problem

Similar to [4, 6] we first form the following covariance matrix

$$\mathbf{Q} = \int_{f \in \mathcal{T}} Y(f) Y^H(f) df = \mathbf{A}\mathbf{Z}_0\mathbf{A}^H \tag{6}$$

where $\mathbf{Z}_0 = \int_{f \in \mathcal{T}} \mathbf{x}(f)\mathbf{x}^H(f) df$. $\mathbf{Q}$ and $\mathbf{Z}_0$ are $p \times p$ and $L \times L$ Hermitian matrices respectively. Because of the assumption of the information bands being uncorrelated (refer to Section 2.1), $\mathbf{Z}_0$ is a diagonal matrix with $[\mathbf{Z}_0]_{i,i} = \int_{f \in \mathcal{T}} \mathbf{x}_i(f)\mathbf{x}_i^H(f) df$. From the Kronecker product property of vectorization [9] the matrix $\mathbf{Q}$ may be expressed in the vector form as

$$\mathbf{q}^d = vec(\mathbf{Q}) = ((\mathbf{A}^H)^T \otimes \mathbf{A}) vec(\mathbf{Z}_0) = (\mathbf{A}^* \otimes \mathbf{A}) vec(\mathbf{Z}_0) \tag{7}$$

where $vec(.)$ denotes the vector of a matrix, $\otimes$ denotes the Kronecker product and $\mathbf{A}^*$ denotes the conjugate matrix of $\mathbf{A}$. Since $\mathbf{Z}_0$ is a diagonal matrix, the above equation may further by simplified by using the well-known property of the Kronecker product for diagonal matrix (see [10, Property 1]) and may be expressed as

$$\mathbf{q}^d = (\mathbf{A}^* \odot \mathbf{A}) diag(\mathbf{Z}_0) = \mathbf{A}^d \mathbf{z} \tag{8}$$

[1] When an information band straddles $k/LT, 1 \leq k \leq L-1$, then support will double, hence for N bands $|\mathcal{S}_{\mathcal{F}_0}| \leq 2N$.

[2] If we impose restriction on the information band location i.e., if it resides within $(\frac{k}{LT}, \frac{k+1}{LT})$ for any k, $0 \leq k \leq L-1$, then the method of [5] requires only $p = N+1$. However, in this paper we consider perfect blind setting and do not impose any restriction on the band location.

where $\mathbf{A}^d = \mathbf{A}^* \odot \mathbf{A}$, $\mathbf{z} = diag(\mathbf{Z}_0)$ and $\odot$ denotes the KR product. Since our algorithm is based on the KR product, we refer to our algorithm as KR-SBR algorithm. From eq.(8) it may be noticed that similar to $\mathbf{A}$, $\mathbf{A}^d$ is also a partial discrete Fourier transform matrix of order L and while $\mathbf{A}$ consists of p rows corresponding to the set $\mathcal{C}$, $\mathbf{A}^d$ consists of p^2 rows corresponding to the set $\mathcal{C}_d$. Since $\mathcal{C}_d$ contain duplicates that are redundant, these duplicates may be removed by choosing only those rows corresponding to the set $\mathcal{C}_{dd}$ and the above equation may be expressed as

$$\mathbf{q}^{dd} = \mathbf{A}^{dd}\mathbf{z} \tag{9}$$

$\mathbf{q}^{dd}$ is a column vector of size p_{dd} and $\mathbf{A}^{dd}$ is a partial Fourier transform matrix that consists of rows corresponding to $\mathcal{C}_{dd}$ and is of size $p_{dd} \times L$.

From the MMV eq.(4) and SMV eq.(9), it is important to notice that $||\mathbf{x}(f)_{\forall f \in \mathcal{T}}||_0 = ||\mathbf{z}||_0$ and the support set $\mathcal{S}_\mathcal{T} = \bigcup_{\forall f \in \mathcal{T}} I(\mathbf{x}(f)) = I(\mathbf{z})$. Hence the support estimation from both the above new SMV formulation (9) and the MMV formulation (4) are identical. Also, notice while the number of measurements (number of rows) of eq.(4) is p, the number of measurements of eq.(9) is $p_{dd} > p$. Since the number of measurements are virtually increased, the SMV eq.(9) has better support estimation capacity than eq.(4). As pointed out in the previous section that we require the number of measurements which is at least twice the number of supports to uniquely determine the supports. In this new formulation we can carefully choose $\mathcal{C}$, thereby $\mathcal{C}_{dd}$, in order to meet this criteria. More details of this shall be described in the following section. It is important to observe that while the existing approaches increase the measurements by increasing p, here we virtually increase the measurements by carefully choosing the MCS parameter. Hence with the proposed method, we require a lower MCS rate compared to the existing approaches for the same performance.

3.2. MCS parameter selection

Given the multiband spectrum parameters (N, B, T), the following theorem provides the necessary conditions in order to estimate the supports uniquely with the above described new formulation.

Theorem 3.1 *For any $x(t) \in \mathcal{M}$, if*
1) $L \leq 1/BT$
2) $p \geq N$
3) $\sigma(\mathbf{A}) \geq N$
4) $\sigma(\mathbf{A}^{dd}) \geq 4N$
then for every $f \in \mathcal{F}_0$, solution of eq.(4) is a unique N-sparse solution [3].

$\mathbf{A}^{dd}$ is a function of $\mathcal{C}_{dd}$ and L. In practice choosing $\mathcal{C}$ in order to satisfy the above theorem (conditions 3) and 4)) is a combinatorial problem. However, the following theorem provides a simpler method to choose MCS parameters that satisfies the conditions of the above theorem.

Theorem 3.2 *Assuming $L \leq 1/BT$, if*
1) $p > 4$ and $p \geq N$
2) L is a prime number
3) $C^N \subseteq C$, C^N is chosen from the Golomb ruler with $c_N \leq \lfloor L/2 \rfloor$ or from a sparse ruler with $2N \leq c_N \leq \lfloor L/2 \rfloor$,
then $\sigma(\mathbf{A}^{dd}) \geq 4N$ and $\sigma(\mathbf{A}) \geq N$

[3]Due to lack of space we only state the theorems here, the proof shall be provided in the journal version of this paper.

where $\mathcal{C}^N = \{c_i\}_{i=1}^N$ denote the N integers with $c_1 < c_2 < ... < c_N$. The above theorem provides only one of the simpler and a practical option for choosing the parameters, exploration of other choices is left to the user and is not within the scope of this paper. Based on the knowledge of the supports, eq.(4) may be used to estimate the spectrum. However some additional steps are necessary while estimating the spectrum in cases when $|\mathcal{S}_{\mathcal{F}_0}| \geq N$. The following section provides the details and the algorithm.

3.3. KR-SBR Algorithm

As pointed out earlier in the Section 2.2.2 that when $L \leq 1/BT$, $\forall f \in \mathcal{F}_0$, $|\mathcal{S}_{\mathcal{F}_0}| \leq 2N$ and for every $f \in \mathcal{F}_0$, $|\mathcal{S}_f| \leq N$. Now choosing $\mathcal{C}$ that satisfies Theorem 3.1 and setting $\mathcal{T} = \mathcal{F}_0$, by solving eq.(9) we obtain the unique support set $\mathcal{S}_{\mathcal{F}_0}$ such that $|\mathcal{S}_{\mathcal{F}_0}| = I(\mathbf{z}) \leq 2N$. Although $\mathcal{S}_{\mathcal{F}_0}$ may be unique, eq.(4) cannot be used in all cases for estimating the spectrum. Eg: if $p = N$, and when $|\mathcal{S}_{\mathcal{F}_0}| > N$ the system of equations (4) becomes underdetermined and cannot be used for spectrum estimation. In such cases we have to use the partition technique in order to estimate the spectrum. In other words, since for every $f \in \mathcal{F}_0$, $|\mathcal{S}_f| \leq N$, there exists $M = 2N + 1$ consecutive intervals $0 = \gamma_1 < \gamma_2 < ...\gamma_{M+1} = 1/LT$ that partitions $\mathcal{F}_0$, $\mathcal{T}_m = [\gamma_m, \gamma_{m+1}), 1 \leq m \leq M$, such that the support of the partition set denoted by $\mathcal{S}_{\mathcal{T}_m}$ will satisfy $|\mathcal{S}_{\mathcal{T}_m}| \leq N$, for every $1 \leq m \leq M$ [7]. In this paper since a blind setting is assumed, unlike [7] the partition set $\{\mathcal{T}_m\}$ is not known a priori. In order to estimate $\{\mathcal{T}_m\}$, we use the bisection algorithm described in [6, Algorithm SBR2].

The following algorithm KR-SBR outlines the details of the reconstruction. First the algorithm considers the entire interval $\mathcal{F}_0$ and checks whether $|\mathcal{S}_{\mathcal{F}_0}| \leq N$. If $|\mathcal{S}_{\mathcal{F}_0}| > N$ then the algorithm will bisect the interval and continue the process till we obtain the interval set $\{\mathcal{T}_m\}$ such that $|\mathcal{S}_{\mathcal{T}_m}| \leq N$. The output of the algorithm, assuming $M+1$ intervals, shall be the partition set $(a, b) = \{(a_m, b_m)\}_{m=0}^M$ support set for each partition $\mathcal{S}_\mathcal{T} = \{\mathcal{S}_{\mathcal{T}_m}\}_{m=0}^M$, and the number of supports in each partition, $N_\mathcal{T} = \{N_{\mathcal{T}_m}\}_{m=0}^M$.

Algorithm 1 : KR-SBR

Input: : $\mathcal{T}$,$Y(f)\forall f \in \mathcal{T}$, **Initialize:** $\mathcal{T} = \mathcal{F}_0$,**Assume:** $\sigma(\mathbf{A}^{dd}) \geq 4N$
Output: : $\mathcal{S}_\mathcal{T}, (a, b), N_\mathcal{T}$

```
if λ(T) < ε then
    return ( S_T = {}, a = inf(T), b = sup(T), N_T = 0)
end if
Compute the matrix Q (eq.(6) and form vector q^dd (eq.(9))
Solve q^dd = A^dd z (eq.(9)) for the sparsest solution z̄
if I(z̄) ≤ N then
    S_T = I(z̄)
    return ( S_T, a = inf(T), b = sup(T), N_T = |S_T|)
end if
if I(z̄) > N then
    Split T into two equal intervals (T_1, T_2)
    (S_T1, a_T1, b_T1, N_T1) = Algorithm1(T_1, Y(f)∀f ∈ T_1)
    (S_T2, a_T2, b_T2, N_T2) = Algorithm1(T_2, Y(f)∀f ∈ T_2)
    S_T = {S_T1, S_T2}
    a = {a_T1, a_T2}
    b = {b_T1, b_T2}
    N_T = {N_T1, N_T2}
end if
return ( S_T, (a, b), N_T)
```

From the output of the above algorithm, appropriate submatrices $\{\mathbf{A}_{\mathcal{S}_{\mathcal{T}_m}}\}_{m=0}^M$ for the intervals may be formed and $\{\mathbf{x}_{\mathcal{S}_{\mathcal{T}_m}}\}_{m=0}^M$ can

Table 1. Minimum average MCS rate necessary for successful blind reconstruction of MBS consisting of N information bands of uniform bandwidth B.

Landau rate (lower bound)	NB
KR-SBR	NB
[6, Algorithm SBR2]	$2NB$
[5]	$(2N+1)B$

be estimated using eq.(4) from which the required $\mathbf{x}(f)$ can be computed.

4. COMPARISONS AND DISCUSSIONS

Table 1 compares the average MCS rate required for different SBR algorithms in order to successfully reconstruct MBS consisting of N information bands each of uniform bandwidth B. Also, for the sake of comparison, the Landau rate which is the minimum sampling rate necessary is also provided. From the table it may be observed that with the KR-SBR algorithm described in this paper, the minimum average MCS rate reduces by a factor of two compared to the state-of-art technique and also approaches the Landau sampling rate.

The reason for this improvement may be better appreciated by comparing Theorem 3.1 and [6, Theorem 3]. As pointed out earlier in Section 3.1 that while the reconstruction of [6] is based on the MMV model (eq.(4)) where only p MMV measurements are available, here the reconstruction is based on the modified SMV model (eq.(9)) where the number of measurements are virtually increased. Hence an additional constraint compared to [6, Theorem 3] in Theorem 3.1 (condition 4) is imposed on $\mathbf{A}^{dd}$. This additional constraint guarantees unique support estimation of all N bands even with $p=N$ and thereby achieve perfect reconstruction, while [6] requires atleast $p=2N$ for unique support estimation in order to achieve perfect reconstruction of N bands.

5. SIMULATION RESULTS

In order to experimentally compare the performance of the algorithms, we considered a MBS $\mathcal{M}$ with $\mathcal{F}=[0,10GHz]$, $N=6$ and uniform bandwidth of $B=243MHz$. 1000 monte-carlo simulations was carried out for this class wherein for each MBS signal the disjoint supports were randomly located within $\mathcal{F}$ and the values for $X(f)$ was generated from an uncorrelated Gaussian distribution. The Landau sampling rate for this class is $f_{LR}=6\times 243MHz=1458MHz$.

For the above MBS class, we chose $L=1/BT=10e9/243e6\approx 41$ which is also a prime number and we chose the set $\mathcal{C}^N=\{1,2,8,13,14,18\}$ which satisfies condition (2) and (3) of Theorem 3.2. The value of p was varied from 4 to 15 and we ran the KR-SBR algorithm to reconstruct the MBS spectrum. The set $\mathcal{C}$ was constructed by adding or removing elements to $\mathcal{C}^N$ corresponding to the value of p. The performance was compared with the SBR method of [4] where we chose the MUSIC algorithm, which we refer as MUSIC-SBR. Fig. 1 shows the probability of success (i.e., the number of MBS signal that was perfectly reconstructed out of these 1000 MBS signals) with varying p for both KR-SBR and MUSIC-SBR algorithms. SBR of [6] was not chosen since it is well known that it breaks for $p<2N$ (12 in this case). From the figure it may be observed that in the region $N+1\leq p\leq 2N$ (in this case $7\leq p\leq 12$), the MUSIC-SBR is able to reconstruct some of the multi-band signals i.e., for those signals when the band supports reside within an interval as pointed out in footnote (2). However for $p\geq 2N+1=13$,

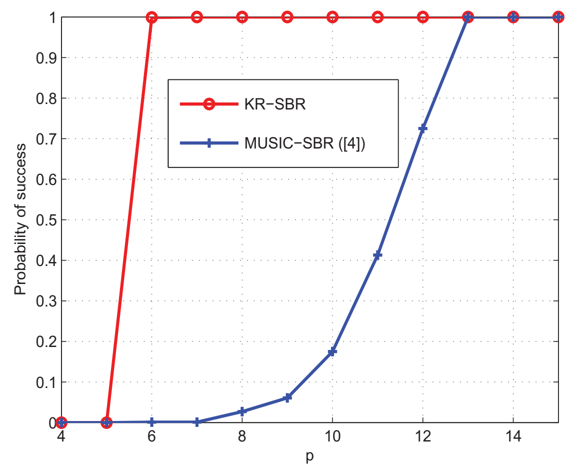

Fig. 1. Comparison of the reconstruction performance between the proposed algorithm 1 and the reconstruction method of [5]

it is able to reconstruct the entire class successfully. On the other hand it may be clearly noticed that the proposed KR-SBR algorithm is able to reconstruct the entire MBS class successfully for all values of $p\geq N=6$. It is important to notice that for $p=6$, the average MCS rate $f_{mcs}\approx f_{LR}$, thus experimentally showing the capability of the KR-SBR algorithm to blindly reconstruct the MBS spectrum at Landau sampling rate.

6. CONCLUSIONS

SBR of multi-band signals which are sampled using the MCS architecture was considered in this paper. A new SBR algorithm, KR-SBR was presented. The proposed algorithm first converted the MMV system problem into a larger dimensional SMV problem, while maintaining the same sparsity, thereby enhancing the support estimation capability. Appropriate conditions on the MCS parameters that are necessary for successful blind reconstruction with this new KR-SBR algorithm were provided. With these conditions being satisfied, we showed that for $L\leq 1/BT$, the proposed algorithm is capable of blindly reconstructing N bands even with $p=N$, while the best state-of-art SBR algorithms require atleast $p=2N$. Further we also showed that when all the information bands approach uniform bandwidth then SBR is achievable even at Landau sampling rate using KR-SBR. Simulation results were also presented to experimentally corroborate the theoretical claims of the proposed algorithm.

7. RELATION TO PRIOR WORK

The SBR of MBS sampled with MCS was considered earlier in [4]-[6]. The work in [4, 5] uses DOA methods and the work in [6] uses methods from CS to estimate the supports, and thereafter estimate the spectrum. The method presented here differs from the above wherein the method here first transforms the problem to a larger Khatri-Rao space and then applies CS upon this transformed problem to estimate the supports. The technique of applying the transformation for SBR before using CS for estimation of supports provides several advantages over the existing techniques as outlined in this paper.

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