PEAK-TO-AVERAGE POWER RATIO AND ILLUMINATION-TO-COMMUNICATION EFFICIENCY CONSIDERATIONS IN VISIBLE LIGHT OFDM SYSTEMS

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ABSTRACT

Visible light communication (VLC) systems can provide illumination and communication simultaneously via light emitting diodes (LEDs). Orthogonal frequency division multiplexing (OFDM) waveforms transmitted in a VLC system will have high peak-to-average power ratios (PAPRs). Since the transmitting LED is dynamic-range limited, peaks of the OFDM waveform can be clipped causing signal degradation. This same phenomenon also occurs in RF communication system, although in RF systems it is straightforward to quantify the performance in terms of RF power conversion efficiency. Results on quantifying VLC performance are scarce. Specifically, because VLC differs from RF communication in system constraint, baseband signal format, and nonlinearity characteristic of the transmitter, it is not obvious how PAPR is related to illumination-to-communication conversion efficiency in VLC. In this paper, we will attempt to quantify the illumination-to-communication conversion efficiency and clarify how PAPR is related to efficiency in VLC systems. We also present a method to improve the efficiency of VLC OFDM systems.

Index Terms— Visible light communication (VLC), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), illumination-to-communication conversion efficiency

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has a well-known disadvantage of high peak-to-average power ratio (PAPR). In the transmitter, to avoid nonlinear distortions, an OFDM signal has to be backed-off to the linear region of a power amplifier (PA), suppressing DC-to-RF power conversion efficiency of the PA. PAPR is an important metric because it impacts directly the power efficiency of a radio frequency (RF) communication system [1, 2]. A number of techniques have been proposed to reduce the PAPR of OFDM [3, 4, 5, 6, 7, 8, 9, 10], in order to improve the power efficiency in RF communication systems and to reduce the sensitivity to nonlinear system components.

Visible light communication (VLC) uses the visible light spectrum to transmit information, and has become a promising candidate to complement conventional RF communication [11, 12, 13]. VLC can provide illumination and communication simultaneously by way of light emitting diodes (LEDs). Therefore, a major constraint in a VLC system is not the electrical power consumption as in RF communication, but the average radiated optical power, which is determined by illumination level. Thus, instead of being concerned with the DC-to-RF power conversion efficiency, we are interested in the illumination-to-communication conversion efficiency. In VLC, simple and low-cost intensity modulation and direct detection (IM/DD) techniques are employed, thus only signal intensity, not phase information, is modulated. IM/DD require the electric signal to be real-valued and unipolar (positive-valued). Recently, OFDM has been considered for VLC due to its ability to boost data rates and efficiently combat inter-symbol-interference (ISI) [14, 15, 16, 17, 18, 19, 20]. The high PAPR problem also exists in visible light OFDM systems because LEDs are dynamic range limited devices [21]. However, since VLC differs from RF communication in system constraint, baseband signal format, and nonlinearity characteristic of front-end devices, it is not obvious how the PAPR is related to the illumination-to-communication conversion efficiency.

The objectives of this paper are three-fold: i) to define the illumination-to-communication conversion efficiency metric, ii) to clarify how PAPR is related to efficiency in a VLC system, iii) to investigate methods to improve the efficiency of visible light OFDM systems.

2. REVIEW OF POWER EFFICIENCY IN RF OFDM SYSTEM

In RF OFDM systems, the time-domain signal $x(t)$ is generated by applying inverse Fourier transform to the frequency-domain signal:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kt/T}, \quad t \in (0, T], \tag{1}$$

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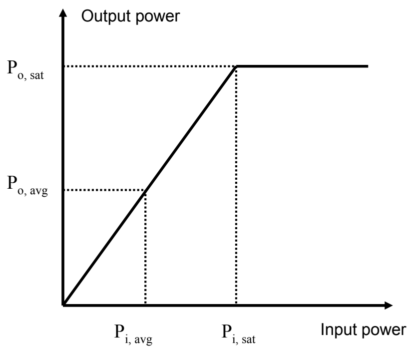

Fig. 1. Input-output relationship of an ideal linear PA.

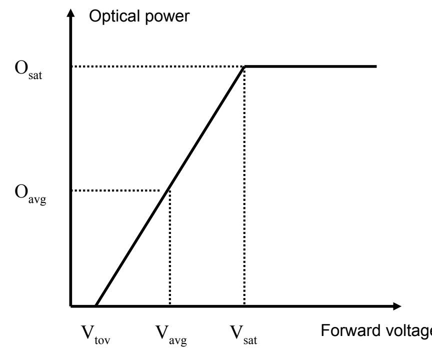

Fig. 2. Ideal linear LED characteristic.

where X_k denotes the frequency-domain signal, $j = \sqrt{-1}$, N is the number of subcarrier, and T is the symbol duration. It is well-known that the OFDM time-domain signal has high peak-to-average power ratio (PAPR) [22], which is defined as

$$\text{PAPR} = \frac{\max\limits_{t\in(0,T]} |x(t)|^2}{E[|x(t)|^2]}, \tag{2}$$

where $E[\cdot]$ stands for the statistical expectation. PAs are peak power limited devices. Fig. 1 shows the input-output characteristic of an ideal linear PA [2]. The DC-to-RF power conversion efficiency is

$$\zeta \triangleq \frac{P_{o,\text{avg}}}{P_{\text{dc}}}, \tag{3}$$

where $P_{o,\text{avg}}$ is the average output power, and P_{dc} is the power drawn from the DC source. We can linearly scale the peak power of the input signal to the saturation level $P_{i,\text{sat}}$ to deliver the maximum power efficiency. For a class A PA, $P_{\text{dc}} = 2P_{o,\text{sat}}$, where $P_{o,\text{sat}}$ denotes the output saturation power [2]. As such, the efficiency of a class A PA is

$$\zeta = \frac{P_{o,\text{avg}}}{P_{\text{dc}}} = \frac{P_{o,\text{avg}}}{2P_{o,\text{sat}}} = \frac{P_{i,\text{avg}}}{2P_{i,\text{sat}}} = \frac{0.5}{\text{PAPR}}. \tag{4}$$

3. EFFICIENCY ANALYSIS IN VISIBLE LIGHT OFDM SYSTEM

3.1. Visible light communication

In visible light communication systems, LED is utilized to simultaneously transmit information and illuminate. Intensity modulation (IM) is employed at the transmitter. The forward signal $y(t)$ drives the LED which in turn converts the magnitude of the input electric signals $y(t)$ into optical intensity. The human eye cannot perceive fast-changing variations of the light intensity, and only respond to the average light intensity. Direct detection (DD) is employed at the receiver. A photodiode (PD) transforms received optical power into the amplitude of an electrical signal. VLC differs from RF in two major ways:

- IM / DD require the baseband signal in VLC to be real-valued, rather than complex-valued as in RF communication.
- Since VLC systems provide illumination and LED is electrical power efficient, electrical power consumption is not a major concern. The operational constraint in VLC is the average optical intensity, which depends on the illumination level. For example, when a room is to be dimly lit, the communication capacity drops accordingly.

In VLC, LEDs are the main source of non-linearity. With predistortion, the input-output characteristic of the LED can be linearized, but only within a limited interval $[V_{\text{tov}}, V_{\text{sat}}]$, where V_{tov} denotes the turn on voltage and V_{sat} denotes the saturation input voltage [21]. Fig. 2 shows the input-output characteristic of an ideal LED. O_{sat} denotes the output optical power corresponding to the input voltage V_{sat}. The illumination level determines the average optical power, which is fixed to a value O_{avg}. Denote by V_{avg} the input average voltage corresponding to O_{avg}. Thus, the input signal $y(t)$ must satisfy

$$E[y(t)] = V_{\text{avg}}. \tag{5}$$

The VLC constraint in (5) differs from the peak power constraint in RF systems. Accordingly the power conversion efficiency is not an appropriate metric for VLC systems. Instead, we define the illumination-to-communication conversion efficiency in VLC

$$\eta \triangleq \frac{D_o}{O_{\text{avg}}} = \frac{D_i}{V_{\text{avg}} - V_{\text{tov}}}, \tag{6}$$

where $D_i = \sqrt{E[y^2(t)] - (E[y(t)])^2}$ denotes the standard deviation of the input electrical signal, and D_o denotes the corresponding standard deviation of the output optical intensity.

To generate real-valued baseband OFDM signal, DC biased optical OFDM (DCO-OFDM) [14] was introduced for

VLC. According to the property of inverse Fourier transform, a real-valued time-domain signal $z(t)$ corresponds to a frequency-domain signal Z_k that is Hermitian symmetric, i.e.,

$$Z_k \quad = \quad Z^*_{N-k}, \quad 1 \le k \le N-1, \tag{7}$$

where $*$ denotes complex conjugate. In DCO-OFDM, the 0th and $N/2$th subcarrier are null (they do not carry data). The time-domain signal $z(t)$ can be obtained as

$$z(t) = \frac{2}{\sqrt{N}} \sum_{k=1}^{N/2-1} \left(\Re(Z_k) \cos\left(\frac{2\pi kt}{T}\right) - \Im(Z_k) \sin\left(\frac{2\pi kt}{T}\right) \right), \quad t \in (0, T], \tag{8}$$

which is real-valued and zero-mean.

3.2. Linear scaling and biasing

The forward signal $y(t)$ is obtained from the OFDM signal $z(t)$ after both a linear scaling and a biasing operation such that

$$y(t) = \alpha z(t) + B, \tag{9}$$

where α and B are both real-valued. The resulting signal, $y(t)$, has a mean value B and a standard deviation $D_i = |\alpha|\sqrt{E[(z(t))^2]}$. To satisfy Eq. (5), we set $B = V_{\text{avg}}$. The standard deviation D_i can be maximized by selecting a scaling factor with the greatest absolute value $|\alpha|$. To ensure $y(t)$ is within the dynamic range of LED, when α is positive, we can obtain an α with the greatest absolute value as

$$\alpha^{(+)} = \min\left\{ \frac{V_{\text{sat}} - V_{\text{avg}}}{\max\limits_{t\in(0,T]} z(t)}, \frac{V_{\text{tov}} - V_{\text{avg}}}{\min\limits_{t\in(0,T]} z(t)} \right\}. \tag{10}$$

When α is negative, we can obtain an α with the greatest absolute value as

$$\alpha^{(-)} = \max\left\{ \frac{V_{\text{sat}} - V_{\text{avg}}}{\min\limits_{t\in(0,T]} z(t)}, \frac{V_{\text{tov}} - V_{\text{avg}}}{\max\limits_{t\in(0,T]} z(t)} \right\}. \tag{11}$$

In summary, an α with the maximum absolute value can be obtained as

$$\alpha = \begin{cases} \alpha^{(+)}, & \text{if} \quad |\alpha^{(+)}| \ge |\alpha^{(-)}| \\ \alpha^{(-)}, & \text{if} \quad |\alpha^{(+)}| < |\alpha^{(-)}| \end{cases} \tag{12}$$

Let us define the positive peak-to-average power ratio of $z(t)$ as

$$\text{pPAPR} \triangleq \frac{\left(\max\limits_{t\in(0,T]} z(t)\right)^2}{E[z^2(t)]}, \tag{13}$$

the negative peak-to-average power ratio of $z(t)$ as

$$\text{nPAPR} \triangleq \frac{\left(\min\limits_{t\in(0,T]} z(t)\right)^2}{E[z^2(t)]}, \tag{14}$$

and the illumination coefficient

$$I_c \triangleq \frac{V_{\text{avg}} - V_{\text{tov}}}{V_{\text{sat}} - V_{\text{tov}}}, \tag{15}$$

where $I_c \in [0, 1]$ represents the illumination level relative to the dynamic range of the LED. Substituting Eqs. (10), (11) and (12) into Eq. (6), we can obtain the illumination-to-communication conversion efficiency

$$\begin{aligned} \eta\{z(t)\} &= \frac{|\alpha|\sqrt{E[z^2(t)]}}{V_{\text{avg}} - V_{\text{tov}}} \\ &= \frac{\sqrt{E[z^2(t)]}}{V_{\text{avg}} - V_{\text{tov}}} \max\left\{\alpha^{(+)}, -\alpha^{(-)}\right\} \\ &= \max\left\{ \min\left\{ \frac{1-I_c}{I_c}\frac{1}{\sqrt{\text{pPAPR}}}, \frac{1}{\sqrt{\text{nPAPR}}} \right\}, \right. \\ &\qquad \left. \min\left\{ \frac{1-I_c}{I_c}\frac{1}{\sqrt{\text{nPAPR}}}, \frac{1}{\sqrt{\text{pPAPR}}} \right\} \right\}. \end{aligned} \tag{16}$$

We can observe that the efficiency depends on three factors:

- Illumination level and LED dynamic range;
- Positive PAPR of OFDM signals;
- Negative PAPR of OFDM signals.

It is worth mentioning that PAPR, pPAPR, and nPAPR obey the following relationship:

$$\text{PAPR} = \max\{\text{pPAPR}, \text{nPAPR}\}. \tag{17}$$

Fig. 3 shows the complementary cumulative distribution function (CCDF) of PAPR, pPAPR, and nPAPR, respectively, from 10000 DCO-OFDM symbols. We chose QPSK modulation, $N = 64$ or $N = 1024$. We can see that pPAPR and nPAPR have similar CCDF curves since DCO-OFDM are symmetric distributed. PAPR is greater than pPAPR and nPAPR because of their relationship as in (17). Fig. 4 is a plot of the mean illumination-to-communication conversion efficiency with varying illumination coefficients and numbers of subcarriers. Observe that the efficiency decreases with increasing illumination level. In Eq. (16), when I_c tends to 0, the efficiency approaches $\max\left\{1/\sqrt{\text{nPAPR}}, 1/\sqrt{\text{pPAPR}}\right\}$. When I_c tends to 1, the efficiency approaches 0. The efficiency becomes worse with more subcarriers.

3.3. Efficiency improvement with selected mapping

In RF communication, DC-to-RF power conversion efficiency can be improved by reducing the PAPR. However, in VLC, the relationship between efficiency and PAPR is not as straightforward as that in RF. We are interested in investigating methods to improve the illumination-to-communication

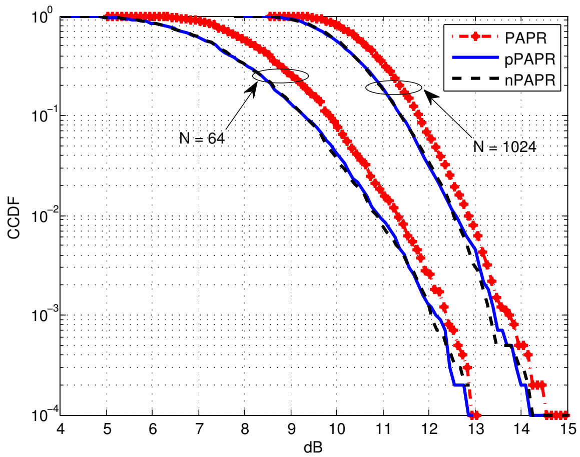

Fig. 3. Complementary cumulative distribution function (CCDF) of PAPR, pPAPR, and nPAPR, respectively, from 10000 DCO-OFDM symbols (QPSK modulation).

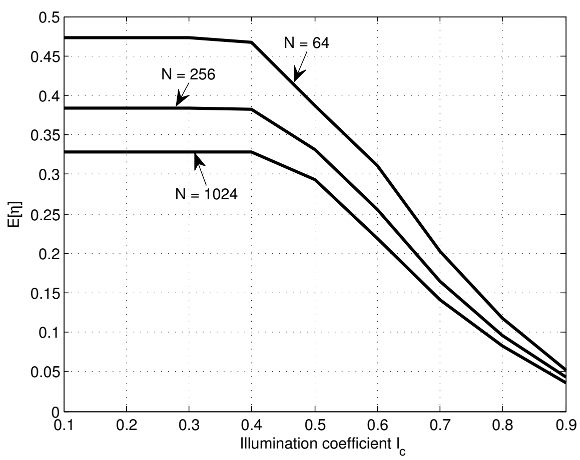

Fig. 4. Illumination-to-communication conversion efficiency with varying illumination coefficients and numbers of subcarriers (10000 DCO-OFDM symbols, QPSK modulation).

conversion efficiency. In this paper, we consider the selected mapping (SLM) method [4].

In SLM, The assumption is that the same phase table $\{\phi_k^{(m)}\}$, $1 \leq k \leq N/2-1$, $1 \leq m \leq M$, where $\phi_k^{(1)} = 0, \forall 1 \leq k \leq N/2-1$, is available to both the transmitter and the receiver. We first rotate the phase of Z_k as

$$Z_k^{(m)} = Z_k e^{j\phi_k^{(m)}}, \; 1 \leq k \leq N/2-1, \tag{18}$$

and the corresponding time-domain signal can be obtained as

$$z^{(m)}(t) = \frac{2}{\sqrt{N}} \sum_{k=1}^{N/2-1} \left(\Re\left(Z_k^{(m)}\right) \cos\left(\frac{2\pi kt}{T}\right) \right. \tag{19}$$

$$\left. - \Im\left(Z_k^{(m)}\right) \sin\left(\frac{2\pi kt}{T}\right) \right). \tag{20}$$

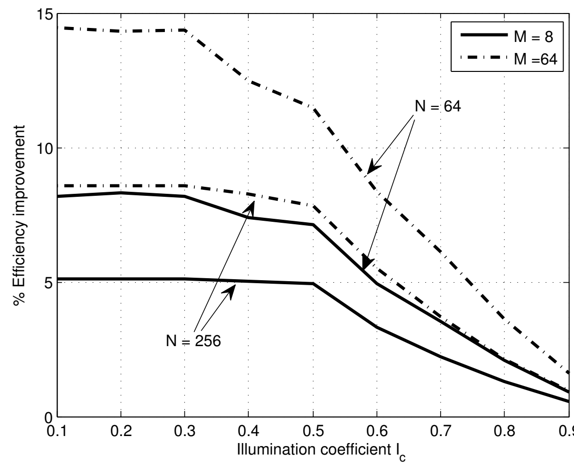

Fig. 5. Illumination-to-communication conversion efficiency improvement with selected mapping method (10000 DCO-OFDM symbols, QPSK modulation).

Instead of selecting the mapped signal $z^{(m)}(t)$ with the lowest PAPR [4], we select the $z^{(m)}(t)$ with the maximum illumination-to-communication conversion efficiency

$$m^* = \underset{1 \leq m \leq M}{\arg\max} \quad \eta\{z^{(m)}(t)\}. \tag{21}$$

As an example, we generated the phase table randomly from the set $\{0, \pm\pi/2, \pi\}$ with equal probability, and $M = 8$ or $M = 64$. We chose QPSK modulation and generated 10000 DCO-OFDM symbols with $N = 64$ or $N = 256$. Fig. 5 shows the illumination-to-communication conversion efficiency improvement with the selected mapping method. We can see that the efficiency can be improved more when N is small and I_c is low.

4. CONCLUSION

In this paper, we investigated the concept of illumination-to-communication conversion efficiency under the average optical power constraint in visible light OFDM systems. We discussed the relationship between efficiency and the positive PAPR, negative PAPR and the illumination coefficient. Finally, we proposed the SLM as a means of improving the illumination-to-communication conversion efficiency.

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