COMPLEMENTARY ENVELOPE ESTIMATION FOR FREQUENCY-MODULATED RANDOM SIGNALS

Pascal Clark[1], *Ivars Kirsteins*[2], *and Les Atlas*[3]

[1]Johns Hopkins University Human Language Technology Center of Excellence, Baltimore, MD USA
[2]Naval Undersea Warfare Center, Newport, RI USA
[3]Department of Electrical Engineering, University of Washington, Seattle, WA USA

ABSTRACT

The Hilbert envelope is a well-known indicator for amplitude modulation in subband time series. However, frequency modulation estimators typically impose temporal smoothness constraints that limit their general use for stochastic signals. We introduce the complementary envelope as a direct indicator of stochastic phase coherence and frequency modulation. Rooted in the second-order statistics of randomly-phased sinusoids, the complementary envelope is distinct from the Hilbert envelope, and can be estimated easily and separately. We propose a new complementary envelope estimator based on modified multitaper spectral analysis, and use it to reveal previously unseen FM-like behavior in ship propeller noise.

Index Terms— Modulation, complementary statistics, impropriety, multitaper, time-frequency

1. INTRODUCTION

Many real-world signals can be characterized by amplitude- and frequency-modulation (AM-FM) in subbands. Estimation of these components is called demodulation, and has proven useful in psychoacoustics [1], automatic speech recognition [2], and vocoding for cochlear implants [3].

Much has been written on AM-FM estimation, particularly as an ill-posed problem requiring constraints that are axiomatic [4, 5], operational [6, 7], or bandlimited [8, 9]. However, demodulation is rarely discussed in terms of sufficient statistics. Without a complete statistical characterization of the signal, conventional AM-FM decompositions often perform poorly in realistic conditions including random noise, channel distortion, and interfering sources.

The problem is the absence of a direct estimator for phase coherence in random signals. In contrast, the well-known Hilbert envelope provides an unambiguous, fundamental statistic for subband amplitude. Modern demodulation methods are essentially parametric estimators of the Hilbert envelope, as in frequency domain linear prediction [2] and bandwidth-constrained demodulation [8, 9].

Much of this work was funded by Office of Naval Research grant N00014-11-1-0204, while P. Clark was at the University of Washington.

Without a comparable phase reference, there is an arbitrariness to sinusoidal AM-FM models. For example, assumptions of slowly varying time-frequency trajectories [7, 9, 10] can represent clean speech with high fidelity, but do not generalize to other signals or situations involving non-stationary noise. Alternatively, the Hilbert envelope residual has numerous well-documented artifacts, such as nonphysical discontinuities [5] and bandwidth expansion [6, 7].

In this paper, we introduce the complementary envelope as a direct estimator for subband phase coherence. This new envelope reveals FM-like behavior in stochastic signals, even for non-smooth, bursty modulations. More importantly, the complementary envelope is a second-order estimator with deep roots in recently developed, complex statistics of analytic signals [11]. This means that, unlike residual Hilbert phase, the complementary envelope is applicable to optimal complex subband estimation by means of widely linear [12] operators.

We begin in Section 2 by deriving the two envelopes of a Gaussian random process. Then in Section 3, we connect the complementary envelope to FM-like variations. We derive a practical estimator based on multitaper theory in Section 4, and use it to reveal frequency modulation in ship propeller noise in Section 5. Finally, we conclude in Section 6.

2. THE TWO ENVELOPES OF A GAUSSIAN PROCESS

Let $x[n]$ be a real-valued zero-mean Gaussian random process. The instantaneous variance is

$$V[n] = E\{x^2[n]\} \tag{1}$$

where E denotes the expected, or average, value.

Next, we show that $V[n]$ consists of two "envelopes," one being the squared Hilbert envelope and the other being our new contribution. Let $x_a[n]$ be the analytic signal defined by

$$x_a[n] = \frac{1}{\pi} \int_{0^+}^{\pi} X(\omega) e^{j\omega n} d\omega \tag{2}$$

978-1-4799-0356-6/13/$31.00

where $X(\omega)$ is the complex Fourier transform[1] of $x[n]$, and is itself a random process. It is easy to show that

$$x[n] = \text{Re}\{x_a[n]\} = 1/2(x_a[n] + x_a^*[n]) \tag{3}$$

using * to denote complex conjugation, and therefore

$$2V[n] = E\{|x_a[n]|^2\} + \text{Re}\{E\{x_a^2[n]\}\}. \tag{4}$$

We refer to the two terms in (4) as the "envelopes" of $x[n]$. Let us denote $P[n] = E\{|x_a[n]|^2\}$. This is the instantaneous power envelope, or squared Hilbert envelope, of the subband signal. We also introduce the new, complementary envelope $C[n] = E\{x_a^2[n]\}$.

These two envelopes have distinct spectral characteristics. When $x[n]$ is bandpass, for example a subband signal, the envelopes separate into low-frequency and high-frequency regions. The magnitude-square in (4) corresponds to frequency-domain autocorrelation,

$$P(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} E\{X(u)X^*(\omega + u)\} du \tag{5}$$

which is necessarily lowpass. In contrast, the complex-square term induces frequency-domain *autoconvolution*,

$$C(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} E\{X(u)X(\omega - u)\} du \tag{6}$$

and results in symmetric double-frequency sidebands. Figure 1 illustrates both envelopes and their relation to $x[n]$ in the frequency domain. The symmetries shown in this figure are deliberate. The Hilbert envelope is necessarily Hermitian, or conjugate, symmetric in the frequency domain, whereas there is no symmetry constraint on the complementary envelope spectrum.

Equation (4) is important because squaring and lowpass filtering is a common method of envelope estimation. In the remainder of this paper, our intent is to prove that the sideband terms, previously treated as "unwanted" terms, uniquely capture the phase coherence of $x[n]$.

3. FREQUENCY-MODULATION IN THE COMPLEMENTARY ENVELOPE

In this section, we show how the formalism of subband envelopes leads to useful interpretations of second-order statistics in terms of power and phase coherence, and amplitude and frequency modulation.

[1]Such a process is said to be "harmonizable" [13], and the integral is technically of the Riemann-Stieltjes variety. Here, we use simpler Riemann notation under the mild assumption that $x[n]$ is time-limited to some long interval N, which defines a chosen frequency resolution of interest $2\pi/N$.

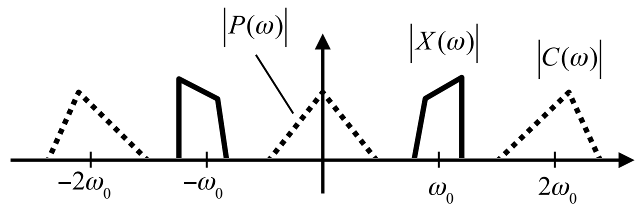

Fig. 1. Frequency-domain sketch showing the basedband power envelope spectrum $P(\omega)$, sideband complementary envelope spectrum $C(\omega)$, and original bandpass signal spectrum $X(\omega)$.

3.1. Connection to the Impropriety Literature

Schreier and Scharf [11] discuss the "complementary covariance" $E\{x_a[n_1]x_a[n_2]\}$ and its conventional counterpart, the Hermitian covariance $E\{x_a[n_1]x_a^*[n_2]\}$ in detail. Here, we restrict our focus to instantaneous variances, or envelopes, as defined in Section 2. In other words, Figure 1 is a one-dimensional reduction from the bivariate spectral characterization of Schreier and Scharf.

If $C[n]$ is nonzero, then $x_a[n]$ falls in the category of improper [14] or second-order noncircular [15] random processes. Otherwise, it is called proper or circular. The distinction becomes clear in the subband formalism of Figure 1, where propriety implies a strictly "slowly-varying" bandpass process due to vanishing complementary sidebands.

The operational consequences of impropriety are well known. Picinbono and Chevalier [12] showed that least-squares estimators are linear only for proper processes, and "widely linear" otherwise. Furthermore, widely linear and similarly "augmented" forms [16] provide a second-order statistical foundation for optimal, coherent estimation of random processes [17, 18].

Widely linear estimation is a possible application for the complementary envelope, but we will not go into detail here. Instead, our present goal is to determine the physical properties of the complementary envelope. In the next subsection, we derive a plausible signal model that leads to meaningful envelopes with AM-like and FM-like properties.

3.2. Polarized Modulation Signal Model

Suppose $x[n]$ is bandpass. For any midband frequency ω_0, there exist real, lowpass Gaussian processes $a[n]$ and $b[n]$ with zero mean, for which

$$x[n] = a[n]\cos(\omega_0 n) - b[n]\sin(\omega_0 n). \tag{7}$$

This is called Rice's representation [19]. Applying Bedrosian's theorem [20], we obtain

$$x_a[n] = (a[n] + jb[n])\, e^{j\omega_0 n}. \tag{8}$$

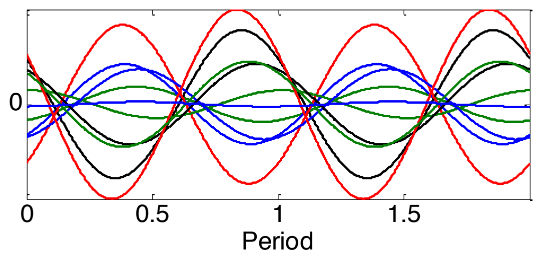

Fig. 2. Example ensemble of a polarized random process with average phase of $\pi/2$ radians.

The power and complementary envelopes are hence related to the quadratic forms

$$\begin{aligned} |x_a[n]|^2 &= a^2[n] + b^2[n] \\ x_a^2[n] &= \left(a^2[n] - b^2[n] + j2a[n]b[n]\right) e^{j2\omega_0 n}. \end{aligned} \quad (9)$$

The point of this derivation is to show that the power envelope $P[n] = E\{|x_a[n]|^2\}$ is the average power of $a[n]$ and $b[n]$, with no indication of relative power difference or correlative interaction. With regard to the cosine and sine components in (7), power difference and cross-correlation encode a form of statistical phasing.

To better understand random phasing, let us consider the elemental case of a sinusoid with scalar coefficients,

$$s[n] = a\cos(\omega_0 n) - b\sin(\omega_0 n) \quad (10)$$

where a and b are jointly Gaussian distributed with zero mean. The phasor given by

$$z = a + jb = (a' + jb')e^{j\theta} \quad (11)$$

is complex-Gaussian with elliptical parameters in the complex plane shown by Ollila [21]. We define θ as the angle of orientation of the main axis of the ellipse, such that a' and b' as mutually uncorrelated. The non-normalized eccentricity is then simply $\varepsilon = E\{a'^2\} - E\{b'^2\}$.

The eccentricity is fundamentally related to local phase coherence of sinusoids in Rice's representation. For $\varepsilon = 0$, the ellipse degenerates to a circle in the complex plane and the phase of z is uniformly distributed in $[0, 2\pi]$. For $\varepsilon > 0$, the phase distribution is no longer uniform and has a preferred angular orientation, the average angle θ. This is equivalent to the improper case $E\{z^2\} \neq 0$ for which $s[n]$ is "polarized" [22] with non-uniform random phase. Figure 2 shows example realizations of $s[n]$ with nonzero eccentricity and average phase $\theta = \pi/2$.

The point of these illustrations is to demonstrate how sinusoidal phase manifests in a Gaussian random signal. Returning to (7), the envelopes $a[n]$ and $b[n]$ can be seen as a time-varying elliptical distribution in the complex plane. The magnitude of $C[n]$ is proportional to the time-varying eccentricity, while variations in $\angle C[n] = \theta[n]$ signify phase or frequency modulations within the subband.

4. MULTITAPER ENVELOPE ESTIMATOR

In practice, the expected value $E\{\cdot\}$ is unobtainable. Estimation of $C[n]$ and $P[n]$ is particularly problematic because impropriety implies that $x[n]$ is nonstationary [11] and hence familiar ergodic theorems do not apply. In the following, we propose a practical estimator based on a new complementary extension of Thomson's classic multitaper technique [23].

Let us consider a real-valued, broadband Gaussian signal $y[n]$. We assume a sum-of-products synthesis

$$y[n] = \sum_i \left(a_i[n] + jb_i[n]\right) e^{j\omega_i n} \quad (12)$$

which combines multiple frequency bands of the form (7).The goal of this section is to estimate the envelopes of any ith band under this model.

Following Thomson's eigenspectral argument [23], it is possible to compute multiple, orthogonal samples of the complementary envelope at the expense of some time-frequency resolution. Defining the estimation bandwidth as $0 < W < 1/2$ and the analysis window length as N samples, we denote $v_k[n]$ as the set of discrete prolate spheroidal sequences (DPSS) with time-frequency product of $2NW$. There are $K = 2NW - 1$ such tapers, for $k = 0, ..., K-1$ and $n = N/2, ..., N/2$. The tapers are mutually orthogonal and maximally concentrated in the frequency range $[-W, W]$.

From the DPSS tapers, we construct K parallel spectral analyzers for the ith band, of the form

$$X_k[n, \omega_i) = \sum_r v_k[n-r] y[r] e^{-j\omega_i r}. \quad (13)$$

We note that these are basebanded subbands, frequency-shifted to center on zero Hertz. As a result, the complementary envelope estimate is similarly basebanded or lowpass. The basebanded, complementary envelope estimate is the alternating average

$$\tilde{C}[n, \omega_i) = \frac{1}{K} \sum_{k=0}^{K-1} (-1)^k X_k^2[n, \omega_i). \quad (14)$$

Sign alternation comes from the alternating even/odd symmetry in the DPSS tapers. The squared spectrum $V_k^2(\omega)$ is positive for k even and negative for k odd, so sign alternation ensures a coherent sum over k. No such alternation is required for the power envelope estimate, obtained by

$$\tilde{H}[n, \omega_i) = \frac{1}{K} \sum_{k=0}^{K-1} |X_k[n, \omega_i)|^2 \quad (15)$$

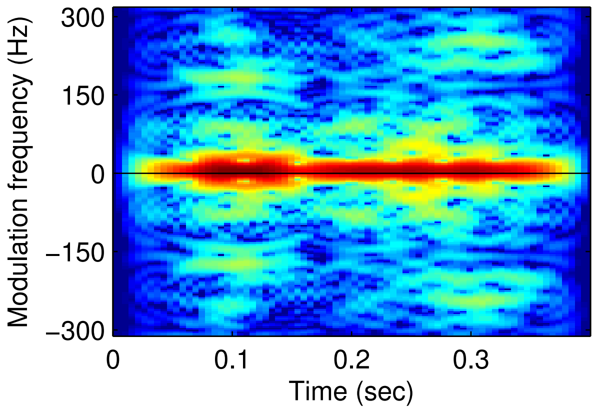

Fig. 3. Spectrogram of the multitaper Hilbert power envelope for propeller cavitation noise, analyzed with a 75-millisecond window and plotted with 30-dB colormap.

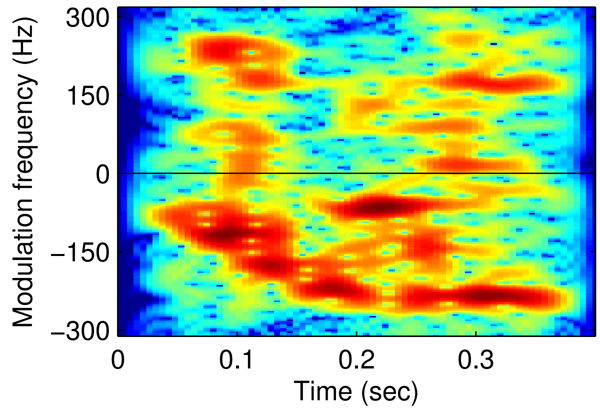

Fig. 4. Spectrogram of a multitaper complementary power envelope for propeller cavitation noise, analyzed with a 75-millisecond window and plotted with 30-dB colormap. Frequencies are plotted relative to 900 Hz.

which is Thomson's multitaper spectrum at discrete frequencies ω_i.

Like any estimator, (14) and (15) operate on assumptions that constrain the observable properties of $y[n]$. We require the following assumptions: 1) the components $a_i[n]$ and $b_i[n]$ each appear nearly stationary within a window of length N, and 2) the power spectra of $a_i[n]$ and $b_i[n]$ are each relatively flat within the bandwidth W of the DPSS analysis tapers. This is emphatically *not* the same as quasi-stationarity in $y[n]$, but rather a requirement on the elliptical parameters of highly nonstationary sinusoids in $y[n]$.

5. ANALYSIS OF PROPELLER CAVITATION NOISE

To see the complementary envelope in practice, we now give an example with underwater propeller noise, or cavitation. This is an important signal because the noise from a ship's propeller is characteristic of the type of ship and its speed. Understanding cavitation noise is vital for tracking and identifying vessels of interest in possibly crowded shipping lanes.

Cavitation noise is also a challenging example because the signal is highly stochastic. Cavitation is a chaotic process of collapsing water vapor bubbles that are modulated by the churning of the propeller blades. Conventional amplitude demodulation extracts only the power envelope [24] under the assumption that the subband phase is statistically indeterminate, or $E\{a^2[n]\} = E\{b^2[n]\}$ in (7).

Multitaper analysis suggests otherwise. We used merchant ship noise obtained from [25], sampled at 16 kHz. We chose the following analysis parameters: $N = 512$ samples, with $K = 8$ for a bandwidth of 280 Hz. Figures 3 and 4 display estimated power and complementary envelopes, respectively. In Figure 4, the complementary envelope reveals clear frequency modulations in the form of narrowband movements in the spectrogram. For comparison, the power envelope estimate is lowpass with forced symmetry in the frequency axis of the spectrogram.

In this example, the complementary envelope reveals a rich time-frequency structure otherwise unseen in the Hilbert envelope. This information could possibly precede the design of least-squares optimal filters and classification features.

6. CONCLUSION

We have shown that the complementary envelope is distinct from the Hilbert, or power, envelope, and equally unambiguous as a second-order statistic of a nonstationary Gaussian process. Our main theoretical result connects the complementary envelope to time-varying elliptical parameters of randomly-phased sinusoids. Consequently, the complementary envelope contains FM and other second-order statistics which are lost in the forced spectral symmetry of the Hilbert envelope. To estimate the complementary envelope, we proposed a new multitaper-based spectral analysis. We used the estimator to reveal FM-like modulations in propeller noise, which opens the possibility of analyzing highly stochastic signals in a coherent least-squares sense. A possible direction for future work is to relate multitaper estimation to the sufficient statistics of a signal.

We would like to acknowledge enlightening discussions with P. Schreier and L. Scharf, and the helpful comments of the anonymous reviewers.

7. REFERENCES

[1] R. Drullman, J.M. Festen, and R. Plomp, "Effect of temporal envelope smearing on speech reception," *J.*

Acoust. Soc. Am., vol. 95, no. 2, pp. 1053–1064, 1994.

[2] S. Ganapathy, S. Thomas, and H. Hermansky, "Temporal envelope compensation for robust phoneme recognition using modulation spectrum," *The Journal of the Acoustical Society of America*, vol. 128, no. 6, pp. 3769–3780, 2010.

[3] R. V. Shannon, F.-G. Zeng, V. Kamath, J. Wygonski, and M. Ekelid, "Speech recognition with primarily temporal cues," *Science*, vol. 270, no. 5234, pp. 303–304, 1995.

[4] P.J. Loughlin and B. Tacer, "On the amplitude- and frequency-modulation decomposition of signals," *J. Acoust. Soc. Am.*, vol. 100, no. 3, pp. 1594–1601, 1996.

[5] L. Cohen, P. Loughlin, and D. Vakman, "On an ambiguity in the definition of the amplitude and phase of a signal," *Elsevier Signal Process.*, vol. 79, pp. 301–307, June 1999.

[6] O. Ghitza, "On the upper cutoff frequency of the auditory critical-band envelope detectors in the context of speech perception," *J. Acoust. Soc. Am.*, vol. 110, no. 3, pp. 1628–1640, 2001.

[7] P. Clark and L. Atlas, "Time-frequency coherent modulation filtering of nonstationary signals," *IEEE Trans. Signal Process.*, vol. 57, no. 11, pp. 4323–4332, Nov. 2009.

[8] G. Sell and M. Slaney, "Solving demodulation as an optimization problem," *IEEE Trans. Audio, Speech, and Language Processing*, vol. 18, no. 8, pp. 2051 –2066, Nov. 2010.

[9] R.E. Turner and M. Sahani, "Probabilistic amplitude and frequency demodulation," in *Advances in Neural Information Processing Systems 24*, J. Shawe-Taylor, R.S. Zemel, P. Bartlett, F.C.N. Pereira, and K.Q. Weinberger, Eds. 2011.

[10] R. McAulay and T. Quatieri, "Speech analysis/synthesis based on a sinusoidal representation," *IEEE Trans. Acoust. Speech, Signal Process.*, vol. 34, no. 4, pp. 744 – 754, Aug. 1986.

[11] P.J. Schreier and L.L. Scharf, "Stochastic time-frequency analysis using the analytic signal: Why the complementary distribution matters," *IEEE Trans. Signal Processing*, vol. 51, no. 12, pp. 3071–3079, Dec. 2003.

[12] B. Picinbono and P. Chevalier, "Widely linear estimation with complex data," *IEEE Trans. Signal Processing*, vol. 43, no. 8, pp. 2030 –2033, Aug. 1995.

[13] M. Loève, *Probability Theory*, Princeton, N.J., Van Nostrand, 3rd edition, 1963.

[14] F.D. Neeser and J.L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. Information Theory*, vol. 39, no. 4, pp. 1293–1302, July 1993.

[15] B. Picinbono and P. Bondon, "Second-order statistics of complex signals," *IEEE Trans. Signal Processing*, vol. 45, no. 2, pp. 411–420, Feb. 1997.

[16] P. J. Schreier and L. L. Scharf, "Second-order analysis of improper complex random vectors and processes," *IEEE Trans. Signal Processing*, vol. 51, no. 3, pp. 714 – 725, 2003.

[17] P.J. Schreier, L.L. Scharf, and C.T. Mullis, "Detection and estimation of improper complex random signals," *IEEE Trans. Information Theory*, vol. 51, no. 1, pp. 306–312, Jan. 2005.

[18] W. M. Brown and R. B. Crane, "Conjugate linear filtering," *Proc. IEEE Trans. Information Theory*, vol. 15, no. 4, pp. 462 – 465, July 1969.

[19] S.O. Rice, "Mathematical analysis of random noise," *The Bell System Technical Journal*, vol. 23-24, 1944.

[20] E. Bedrosian, "A product theorem for hilbert transforms," *Proc. of the IEEE*, vol. 51, no. 5, pp. 868 – 869, May 1963.

[21] E. Ollila, "On the circularity of a complex random variable," *IEEE Signal Processing Letters*, vol. 15, pp. 841 –844, 2008.

[22] P.J. Schreier, "Polarization ellipse analysis of nonstationary random signals," *IEEE Trans. Signal Processing*, vol. 56, no. 9, pp. 4330 –4339, Sept. 2008.

[23] D.J. Thomson, "Spectrum estimation and harmonic analysis," *Proc. of the IEEE*, vol. 70, no. 9, pp. 1055 – 1096, Sept. 1982.

[24] J.G. Lourens and J.A. du Preez, "Passive sonar ML estimator for ship propeller speed," *IEEE Journal of Oceanic Engineering*, vol. 23, no. 4, pp. 448 –453, Oct. 1998.

[25] University of Connecticut, "Bioacoustics laboratory research page," http://www.bioacoustics.uconn.edu/research.html, Sept. 2009.