# OPPORTUNISTIC CHANNEL-AWARE SPECTRUM ACCESS FOR COGNITIVE RADIO NETWORKS WITH PERIODIC SENSING

Sheu-Sheu Tan, James Zeidler, and Bhaskar Rao

Department of Electrical and Computer Engineering University of California, San Diego La Jolla, CA 92093, USA

## ABSTRACT

Opportunistic spectrum access in a cognitive radio network has been a challenge due to the dynamic nature of spectrum availability and possible collisions between the primary user (PU) and the secondary user (SU). To maximize the spectrum utilization, we propose a spectrum access strategy where SU's packets are interleaved with periodic sensing to detect PU's return. We formulate the sensing/probing/access process as a maximum rate-of-return problem in the optimal stopping theory framework and show that the optimal channel access strategy is a pure threshold policy. We consider a realistic channel and system model by taking into account channel fading and sensing errors. We jointly optimize the rate threshold and the packet transmission time to maximize the average throughput of SU, while limiting interference to PU.

# 1. INTRODUCTION

Cognitive radio appears as one very viable technology that can optimize the use of available radio frequency spectrum [1]. In this paper, we propose an optimal spectrum access strategy involving transmission interleaved with periodic sensing that leverages sensing, channel-aware scheduling and optimization of transmission time in a joint manner to maximize SU's throughput. One of the key observations on cognitive radio is that the successful transmission of SU depends on PUs' activities. The return of PU would cause the transmission of SU to fail. However, while SU is transmitting, it has no knowledge of the return of PU. We therefore propose periodic sensing while transmission to track PU. First, channel sensing is carried out to explore a spectrum hole for SU's transmission. Second, while a channel is used by SU, periodic sensing is deployed to detect the return of PU. The benefit of periodic sensing is that when PU returns, only the data transmitted since the last successful sensing may be lost - prior transmitted packets are not affected.

In transmission with periodic sensing, there exists a tradeoff between data lost due to PU's return while using long packets, and the time cost of frequent sensing using short packets. If the transmission time is long, i.e., the frequency of periodic sensing is low, the time cost of tracking the return of PU is small but the amount of lost data when PU returns is large. On the contrary, if the transmission time is small and the frequency of periodic sensing is high, the amount of lost data when PU returns is small but at the expense of high cost of tracking PU. Motivated by this, we optimize the transmission time of SU between consecutive sensing phases to maximize the network throughput, which is equivalent to optimizing the frequency of periodic sensing.

We consider a system consisting of multiple channels. We characterize the joint sensing, probing and channel access with optimal transmission duration in a stochastic decisionmaking framework and formulate the decision problem as an optimal stopping problem [2]. When the sensing indicates that a channel is idle, probing is carried out to estimate the channel quality and the highest data rate it can support. Based on this estimate, one can decide either to proceed with transmission on this channel or to give up the opportunity and continue sensing for a potentially better channel. We show that the optimal channel access strategy exhibits a threshold structure, i.e., the channel access decision can be made by comparing the rate to a threshold. Furthermore, we jointly optimize the threshold and the transmission time between consecutive sensing phases to maximize the average throughput.

# 1.1. Related Works

Channel knowledge can be used as one criterion for channel selection to improve spectrum efficiency in wireless networks [2–5]. Zheng *et al.* [2] use optimal stopping theory to develop distributed opportunistic scheduling (DOS) for exploiting multiuser diversity and time diversity in a single channel model for wireless *ad hoc* networks. Chang *et al.* [3] address the optimal channel selection problem in a multichannel system by considering the channel conditions. In our work, besides gaining the benefits of channel knowledge, we consider cognitive radio networks with incumbent PUs and also optimize the transmission time of SU to maximize throughput.

Shu *et al.* [4] show that joint sensing/probing scheme for cognitive radio can achieve significant throughput gains over

This research was supported by the NSF Grant No. CCF-1115645.

conventional mechanisms that use sensing alone. Our channel access scheme is an extension of optimal stopping results in [2], [4] and is more complex due to the variable transmission times, probing of the channels only when they are sensed to be idle and consideration of sensing errors. Additionally, we consider periodic sensing while transmission and further optimize the transmission time, i.e. the frequency of sensing.

There are few works in the literature that explicitly optimize the transmission time or perform periodic sensing while transmission [6], [7], [5]. Pei *et al.* [6] optimize the frame duration to maximize the throughput of the cognitive radio. In contrast, our work considers multichannel system and also takes channel quality into consideration.

Apart from the optimal stopping theory approach to the channel access problem, another popular approach in the literature is based on the Partially Observable Markov Decision Process (POMDP) framework [8–10]. POMDP-based schemes attempt to dynamically track the idle state of various channels and maximize throughput by exploiting the spectrum opportunities. Our scheme explores channels uniformly at random and doesn't dynamically track the idle channels. But it fully utilizes the idle state of the channels it accesses. As we show in the numerical results, our scheme that takes channel quality into consideration, performs periodic sensing while transmission and jointly optimizes the packet duration and the channel quality threshold, outperforms the POMDP scheme in [8].

## 2. CHANNEL AND SYSTEM MODEL

We consider a frequency-selective multi-channel system. We assume each channel experiences flat and slow fading. We further assume that all channels have the same statistics, and are subject to Rayleigh fading. The distribution of rate R is continuous and is given by the Shannon channel capacity  $R = \log(1 + \rho |h|^2)$  nats/s/Hz, where  $\rho$  is the normalized average SNR, and h is the random channel coefficient with a complex Gaussian distribution  $\mathcal{CN}(0, 1)$ .

We assume that each channel has only one designated PU. The L channels are opportunistically available to SU. Although we consider a system where there is only one SU, it can be readily extended to the case when there are multiple SUs and each SU has opportunistic access to a different set of L channels. Each channel's status is modeled as a continuous-time random process that alternates between busy and idle states. We consider a system in which the idle/busy states of different PU channels are homogeneous, independent and identically distributed. We assume that for all PUs, the time durations of the idle and busy states are exponentially distributed with parameters a and b [11].

For selecting a channel, SU uses the scheme of sequential sensing and probing without recall [12]. We consider sensing errors and denote the probability of false alarm as  $P_{\rm fa}$  and miss detection as  $P_{\rm md}$ . A sample realization of the



**Fig. 1**. A sample realization of channel sensing, probing and data transmission with PU returns

sensing/probing for channel selection is depicted in Fig. 1. When SU intends to transmit, it searches for an available channel by randomly choosing channels one at a time and sensing/probing them. If the outcome of the sensing stage is busy, the probing stage is skipped and SU randomly selects another channel for sensing. The time cost for sensing/probing a busy channel is  $\tau_s$ . However, if the sensed channel is idle, SU proceeds with probing to determine the channel quality and the time cost for sensing/probing an idle channel is  $\tau_{\rm s} + \tau_{\rm p}$ . The transmitter compares the achievable data rate to an optimal threshold ( $\lambda^*$ ) pre-designed using the optimal stopping theory. If the data rate is less than the threshold due to the poor channel condition, the SU forgoes its transmission opportunity and continues with sensing/probing another randomly selected channel. However, if the data rate exceeds the threshold, the SU proceeds with the data transmission. The SU will periodically sense the channel after transmitting for time  $T_{\rm s}$ . The transmission of SU stops once it senses the return of a PU. If PU returns during SU's transmission, the current sub-packet being transmitted is destroyed, but the previously transmitted sub-packets are still valid.

# 3. DERIVATION OF CHANNEL ACCESS STRATEGY

We consider the problem of finding an optimal strategy for SU to decide whether or not to transmit on an idle channel based on its quality, so as to maximize the long-term average throughput. We consider a maximum rate-of-return problem in the optimal stopping theory framework [13, 14]. The return is defined as the net gain between the reward achieved and the cost spent. The reward is the rate of the channel probed and the cost is the total time taken to explore the channels so far.

Consider a round of channel searching and transmission by the SU. Let N be the (random) number of channels explored by the SU in this round before it decides to transmit. Accordingly, let  $T_N$  be the total duration of the round, inclusive of the time for which SU transmits data till it detects the return of PU. Let T' be the effective data transmission time in a round that excludes the time spent on exploration and periodic sensing. And let  $R_N$  be the transmission rate in a round. The long term average throughput is therefore

$$x \stackrel{\scriptscriptstyle \triangle}{=} \frac{E[R_N T']}{E[T_N]}.\tag{1}$$

The stopping rule N governs when to stop exploring channels and therefore governs the distributions of  $R_N$  and  $T_N$ .

The total time  $T_N$  of a round consists of the time  $T'_N$  spent in sensing and probing to acquire a good channel and the time  $T_{tr}$  for transmitting SU's packets over this channel. The time  $T_{tr}$  includes both the successfully and unsuccessfully transmitted packets until SU senses PU's return, and the time spent due to periodic sensing between the packets. We have  $E[T_N] = E[T'_N] + E[T_{tr}]$ .

The problem of maximizing the long-term average throughput can be formulated as a maximal-rate-of-return problem [2]. The goal is to find an optimal stopping rule  $N^*$  that maximizes the average rate-of-return x, and the corresponding maximal throughput  $x^*$ :

$$N^* \triangleq \arg\max_N \frac{E[R_N T']}{E[T_N]}, \ x^* \triangleq \sup_N \frac{E[R_N T']}{E[T_N]}, \quad (2)$$

where the maximization is over all stopping rules  $\{N : N \ge 1, E[T_N] < \infty\}$ . Similar to [2], we use optimal stopping theory to solve (2) and have the following proposition.

**Proposition 3.1.** There exists an optimal stopping rule  $N^*$  for the opportunistic spectrum access and is a pure threshold policy given by

$$N^* = \min\left\{n \ge 1 : R_n \ge \lambda^*\right\},\tag{3}$$

where the optimal threshold  $\lambda^*$  is the unique solution for  $\lambda$  in

$$\mathbf{E}[(R-\lambda)^{+}] = \lambda \big( \mathbf{E}[K_{\rm s}]\tau_{\rm s} + \mathbf{E}[K_{\rm p}]\tau_{\rm p} \big) / \mathbf{E}[T_{\rm tr}].$$
(4)

Here, R is a r.v. which refers to the rate whose CDF is  $F_R(r)$ , and  $K_s$  and  $K_p$  are the number of channels sensed and probed respectively to find a channel in which PU is idle for the time  $(\tau_s + \tau_p)$ . Furthermore, the maximum throughput is given by  $x^* = \frac{\lambda^* E[T']}{E[T_{tr}]}$ .

Thus, SU will transmit on a successfully contended channel if the transmission rate found by probing is bigger than a threshold  $\lambda^*$ . Else, it continues to explore other channels.

To further analyze the maximal throughput  $x^*$  and the optimal stopping rule  $N^*$ , i.e., the threshold  $\lambda^*$  in terms of various channel and system parameters, we first calculate the various expectations that were encountered in Proposition 3.1.

**Proposition 3.2.** For any pure threshold policy  $N = \min\{n : R_n \ge \lambda\}$ , the expected times of effective transmission ( $\mathbb{E}[T']$ ), channel access ( $\mathbb{E}[T'_N]$ ), transmission with periodic sensing ( $\mathbb{E}[T_{tr}]$ ), and the rate of transmission ( $\mathbb{E}[R_N]$ ) are given by

$$E[T'] = \frac{T_{s} \cdot e^{-aT_{s}}}{1 - e^{-a(T_{s} + \tau_{s})}(1 - P_{fa})},$$
  

$$E[T'_{N}] = \frac{\tau_{s} + Q'_{I}\tau_{p}}{\left(\frac{b}{a+b}\right)e^{-a(\tau_{s} + \tau_{p})}(1 - P_{fa})(1 - F_{R}(\lambda))},$$
  

$$E[T_{tr}] = \frac{1 - P_{md}e^{-a(T_{s} + \tau_{s})}}{1 - P_{md}}\frac{(T_{s} + \tau_{s})}{1 - e^{-a(T_{s} + \tau_{s})}(1 - P_{fa})}.$$
  

$$E[R_{N}] = \frac{\int_{\lambda}^{\infty} r \, dF_{R}(r)}{1 - F_{R}(\lambda)}.$$

Here,  $Q'_{\rm I} = (\frac{a}{a+b})P_{\rm md} + (\frac{b}{a+b})((1-e^{-a\tau_{\rm s}})P_{\rm md} + e^{-a\tau_{\rm s}}(1-P_{\rm fa}))$  is the probability of finding a channel in which PU is sensed to be idle.

The proof mostly relies on properties of Poisson processes, and exponentially and geometrically distributed random variables.

Using Proposition 3.1 and Equation (1), since  $\lambda^*$  and  $x^*$ satisfy  $x^* = \frac{\lambda^* \mathbf{E}[T']}{\mathbf{E}[T_{\mathrm{tr}}]}$ , i.e.,  $\lambda^* = \frac{\mathbf{E}[T_{\mathrm{tr}}]}{\mathbf{E}[T']}x^* = \frac{\mathbf{E}[T_{\mathrm{tr}}]}{\mathbf{E}[T']}\frac{\mathbf{E}[R_{N^*}]\mathbf{E}[T']}{\mathbf{E}[T_{N^*}]}$ , we see that  $\lambda^*$  is a solution to the fixed-point equation in  $\lambda$ , given by

$$\lambda = \frac{\mathrm{E}[T_{\mathrm{tr}}]}{\mathrm{E}[T']} x = \frac{\mathrm{E}[R_N]}{\frac{\mathrm{E}[T'_N]}{\mathrm{E}[T_{\mathrm{tr}}]} + 1} = \frac{\int_{\lambda}^{\infty} r dF_R(r)}{c_0 - F_R(\lambda)} \stackrel{\triangle}{=} \psi(\lambda).$$
(5)

Here,  $c_0 = 1 + \frac{\mathrm{E}[T'_N](1-F_R(\lambda))}{\mathrm{E}[T_{\mathrm{tr}}]}$  is a constant that does not depend on  $\lambda$  using Proposition 3.2. Similar to [2, Prop. 3.4], we have the following result for finding  $\lambda^*$  when  $T_{\mathrm{s}}$  is given.

**Proposition 3.3.** For a given  $T_s$ , the fixed-point iteration

$$\lambda_{k+1} = \psi(\lambda_k),\tag{6}$$

for  $k \in \{0, 1, 2, ...\}$  and for any nonnegative  $\lambda_0$  converges to the optimum threshold  $\lambda^*$ .

# 4. JOINT OPTIMIZATION OF THRESHOLD AND TRANSMISSION DURATION

We jointly optimize the transmission time  $T_s$  and the threshold  $\lambda$  to maximize the throughput  $x = x(\lambda, T_s)$ . We show that for a given threshold rule  $N = \min\{n \ge 1 : R_n \ge \lambda\}$ , i.e., for a fixed threshold  $\lambda$ , the optimum transmission time  $T_s$  that maximizes throughput can be obtained by taking the derivative with respect to  $T_s$  and equating to zero, i.e., solving for  $T_s$  in  $\frac{\partial}{\partial T_s}x(\lambda, T_s) = 0$ . On simplification, this results in the equation

$$\zeta(T_{\rm s}) \stackrel{\scriptscriptstyle \triangle}{=} c_1 - c_2 e^{-aT_{\rm s}} - ac_1 T_{\rm s} - ac_3 T_{\rm s}^2 = 0, \qquad (7)$$

where

$$\begin{split} c_1 &= \frac{\left(\frac{a+b}{b}\right) e^{a(\tau_{\rm s}+\tau_{\rm p})}(\tau_{\rm s}+Q_{\rm I}'\tau_{\rm p})}{1-P_{\rm fa}} + \frac{\left(1-F_R(\lambda)\right)}{1-P_{\rm md}}\tau_{\rm s},\\ c_2 &= \frac{\left(\frac{a+b}{b}\right) e^{a(\tau_{\rm s}+\tau_{\rm p})}(\tau_{\rm s}+Q_{\rm I}'\tau_{\rm p})}{1-P_{\rm fa}}(1-P_{\rm fa}) e^{-a\tau_{\rm s}} \\ &+ \frac{\left(1-F_R(\lambda)\right)}{1-P_{\rm md}}P_{\rm md} e^{-a\tau_{\rm s}}\tau_{\rm s},\\ c_3 &= \frac{\left(1-F_R(\lambda)\right)}{1-P_{\rm md}}. \end{split}$$

It can be further shown that  $\zeta(T_s) = 0$  has a unique solution  $T_s^*$  for  $T_s > 0$  and  $T_s^* < \frac{1}{a}$ . We use Newton's method with initial value  $\frac{1}{a}$  to obtain  $T_s^*$ . We therefore propose the alternating maximization scheme given by Algorithm 1 for finding  $\lambda^*$  and  $T_s^*$  that jointly maximize the throughput  $x(\lambda, T_s)$ .

**Algorithm 1** Joint maximization of throughput  $x(\lambda, T_s)$ 

- 1: Given: sufficiently small error bounds  $\epsilon_{\lambda}$ ,  $\epsilon_{T_s}$
- 2: Initialize  $\lambda = 1, T_{\rm s} = \frac{1}{a}$
- 3: repeat
- 4:  $\lambda^{\text{old}} = \lambda, T_{\text{s}}^{\text{old}} = T_{\text{s}}$
- 5: **repeat** {Optimize  $\lambda$  for current  $T_s$  by fixed-point iterations}
- 6:  $\lambda = \psi(\lambda)$
- 7: **until**  $|\lambda \psi(\lambda)| \le \epsilon_{\lambda}/2$
- 8: **repeat** {Optimize  $T_s$  for current  $\lambda$  by Newton's method}

9: 
$$T_{s} = \frac{c_{2}e^{-aT_{s}}(aT_{s}+1)-ac_{3}T_{s}^{2}-c_{1}}{a(c_{2}e^{-aT_{s}}-2c_{3}T_{s}-c_{1})} \left(=T_{s}-\frac{\zeta(T_{s})}{\frac{\partial}{\partial T_{s}}\zeta(T_{s})}\right)$$

10: **until** 
$$|T_{\rm s} - \frac{c_2 e^{-a_1 s} (aT_{\rm s}+1) - ac_3 T_{\rm s}^2 - c_1}{a(c_2 e^{-aT_{\rm s}} - 2c_3 T_{\rm s} - c_1)}| \le \epsilon_{T_{\rm s}}/2$$

11: **until**  $|\lambda^{\text{old}} - \lambda| \leq \epsilon_{\lambda}$  and  $|T_{\text{s}}^{\text{old}} - T_{\text{s}}| \leq \epsilon_{T_{\text{s}}}$ 

12: Return  $\lambda$  and  $T_s$  as approximations of  $\lambda^*$  and  $T_s^*$ 

# 5. INTERFERENCE TO PRIMARY USER

If PU returns during SU's transmission, there may be a collision, leading to interference to PU. One way of quantifying this interference is in terms of the fraction of time for which each PU experiences interference in the long term. We calculate the expected collision duration  $T_c$  at the end of each round of transmission. Under ideal, error-free periodic sensing, the expected duration between the return of the PU and the end of SU's current packet can be calculated as  $T_s - \frac{(1-e^{-aT_s})}{a}$ . Misdetections cause an additional expected collision duration of  $T_s \cdot \frac{P_{\rm md}}{1-P_{\rm md}}$ . Thus,

$$E[T_c] = T_s - \frac{(1 - e^{-aT_s})}{a} + T_s \cdot \frac{P_{md}}{1 - P_{md}}.$$
 (8)

Taking into account that the expected duration of a round is  $E[T_N]$ , and the SU is equally likely to transmit on any of the *L* channels, the average fraction of time for which each PU experiences collision is

$$\eta_{\rm c} = \frac{1}{L} \cdot \frac{{\rm E}[T_{\rm c}]}{{\rm E}[T_N]},\tag{9}$$

where  $E[T_c]$  and  $E[T_N] = E[T'_N] + E[T_{tr}]$  are given by Equation (8) and Proposition 3.2 respectively.

We see that  $\eta_c$  is an increasing function of  $T_s$ . Given a bound  $\hat{\eta_c}$  on the interference, we consider the following modification of Algorithm 1 to find  $\lambda$  and  $T_s$  that maximize the throughput, while causing low interference. If the outputs  $\lambda^*$  and  $T_s^*$  from Algorithm 1 are such that the corresponding  $\eta_c \leq \hat{\eta_c}$ , then we use them as the threshold and packet time respectively. If not, starting with  $\lambda^*$  and  $T_s^*$ , we use a modified Algorithm 1 where in the inner loop for optimizing  $T_s$ , each time the  $T_s$  obtained at the end of the loop is such that corresponding  $\eta_c > \hat{\eta_c}$ , we lower  $T_s$  to the solution of  $\hat{\eta_c} = \frac{1}{L} \cdot \frac{E[T_c]}{E[T_n]}$ , obtained by Newton's method.

#### 6. NUMERICAL RESULTS

We present numerical results to evaluate the performance of our proposed scheme. The values of the various parameters used are  $\rho = 10$ ,  $\tau_s = 20$  ms,  $\tau_p = 30$  ms,  $\frac{1}{a} = 500$  ms,  $\frac{1}{b} = 666.67$  ms,  $P_{fa} = 0.1$  and  $P_{md} = 0.05$ . For simplicity, we assume that the optimal transmission time  $T_s^*$  obtained in Algorithm 1 meets the interference requirement  $\eta_c \leq \hat{\eta_c}$ .

In Fig. 2, we compare our scheme with the three other schemes - one without probing, one without periodic sensing and one with POMDP in [8]. For fair comparison, the transmission time  $T'_{s}$  used for the scheme without periodic sensing is the same as expected time of transmission  $E[T_{tr}]$  for the periodic sensing scheme. A threshold based strategy is used for the scheme without periodic sensing, where the threshold that maximizes throughput is derived using optimal stopping theory, similar to the scheme with periodic sensing. For POMDP scheme, we set the rates of all channels to E[R], slot length to  $\tau_{\rm s} + T_{\rm s}$  and the transition probabilities  $p_{\rm busy \rightarrow idle} = b(\tau_{\rm s} + T_{\rm s})$ and  $p_{\text{idle}\rightarrow\text{busy}} = a(\tau_{\text{s}}+T_{\text{s}})$  so that average idle and busy times of PUs are 1/a and 1/b. POMDP-based schemes dynamically track the idle state of various channels in a slotted system and maximize throughput by exploiting the spectrum opportunities. Our scheme fully utilizes the idle state of the channels it accesses by periodic sensing, and together with exploitation of channel quality information and optimization of transmission time, it outperforms the POMDP-based scheme.



Fig. 2. Comparison between our scheme and other schemes: Maximal throughput  $x^*$  versus average SNR  $\rho$ 

## 7. CONCLUSIONS

In summary, we proposed an opportunistic channel access framework where the transmissions are interleaved with periodic sensing. For the proposed scheme, we obtained the optimal threshold and the optimal transmission period that jointly maximize the average throughput. We consider the effect of sensing errors throughout the analysis. Numerical results show that our scheme can offer a much higher throughput than other well-known schemes.

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