HYBRID SPECTRUM SHARING WITH IMPERFECT SENSING IN FADING CHANNELS

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ABSTRACT

This paper considers the hybrid spectrum sharing paradigm where a cognitive radio system first performs spectrum sensing to identify primary users' (PU) status (idle/busy) and then adapts its transmit power according to sensing outcomes. To maximize ergodic throughput in fading environments, joint sensing and power allocation has to be considered. However, existing studies determine the optimal sensing time based on instantaneous channel state information (CSI) at each time slot, which imposes a stringent requirement in practice. In this paper, we obtained a statistical CSI-based optimal sensing time by exploring the ergodic rates of both overlay and underlay access in Rayleigh fading environments. Simulation results validate the derived analytic expressions, showing that significantly higher maximum throughput can be achieved by hybrid access compared with conventional overlay access.

Index Terms— Cognitive radio, hybrid spectrum sharing, ergodic throughput, spectrum sensing, power allocation

1. INTRODUCTION

Cognitive radio (CR) has been proposed as a new spectrum sharing paradigm that enables a CR system to operate on a licensed spectrum without harmful interference to licensed systems [1, 2]. Licensed systems with higher priorities over spectrum use are called primary user (PU), whereas CR systems with lower priority over spectrum use are called secondary users (SU) [3]. SUs have two basic spectrum access strategies: overlay access and underlay access [4–7].

Overlay access is also termed as opportunistic spectrum access, where SUs perform spectrum sensing to perceive spectrum status and transmit on "spectrum holes" that are temporarily unused by PUs [2]. Overlay access has an inherent drawback in that SUs are prohibited to transmit when the spectrum is occupied by PUs. Unlike overlay access which requires SUs to wait for an idle status to transmit, in underlay access, SUs can coexist with PUs by transmitting with extremely low power, such that the interference to PUs can be constrained [8,9]. Although SUs obtain spectrum access opportunities in the presence of PUs by underlay access, available spectrums are not fully utilized in idle statuses because the transmit power is always restricted by the interference power constraint without [4].

The hybrid spectrum access strategy has been proposed recently to combine the advantages of both overlay and underlay access [4, 5, 7, 10, 11], where hybrid access is shown to be more effective in terms of achievable throughput. To maximize the ergodic throughput of hybrid CR in fading environments, joint sensing and power allocation design has been studied under transmit power and interference constraints [10, 11]. However, the ergodic rates under different transmit powers and PUs' statuses are not given, and the sensing time has to be determined for each channel realization, which imposes in practice a stringent requirement that instantaneous CSIs have to be available before spectrum sensing. Unlike the studies in [4, 10, 11], where SUs adapt their transmit power based on sensing results, SUs in [7] fix the transmit power and adapt their access modes based on the prediction of PUs' traffic. The ergodic rate of hybrid access has been investigated under perfect sensing [4, 7]. However, new issues arise with imperfect sensing scenarios, where the optimal sensing time and the impact of sensing errors have to be considered.

We are thus motivated to study spectrum sensing and power allocation for hybrid CR with imperfect sensing. To be specific, we aim to maximize the ergodic throughput of hybrid CR under the peak transmit power and interference constraints in Rayleigh fading channels. The alternating optimization method is applied to obtain the optimal power allocation and miss detection probability. We derive analytic expressions of ergodic rates under the optimal power allocation in overlay and underlay access scenarios, based on which we determine a unique optimal sensing time to maximize ergodic throughput of hybrid CR.

The relation of the work presented here with prior work is summarized as follows. Similar to the existing works in [4, 7, 10, 11], the proposed work tackles the problem which maximizes the ergodic throughput of hybrid cognitive radio subject to transmit power and interference constraints to PUs. However, unlike the studies in [4,7,11], which assumed perfect sensing, we focus on more practical imperfect sensing scenarios and consider the impacts of sensing errors and sensing time on system throughput. Although existing works in [10, 11] did consider imperfect sensing, ergodic rates were not explored and the sensing time essentially depends on random instantaneous CSIs. Sensing-throughput tradeoff is investigated for overlay access CR in [12], which did not consider the impact of power allocation and channel fading. In this work, we derive ergodic rates for both underlay access and overlay access scenarios, which provides a closed-form expression for the ergodic throughput under perfect sensing. We investigate the impact of sensing errors and develop a statistical CSI-based sensing time scheme, which is more favorable in practice due to the reduced implementation complexity.

2. SYSTEM MODEL

We consider a spectrum sharing system with one primary link and one secondary link sharing the same frequency band in Fig. 1. The primary link comprises a PU transmitter (PT) and a PU receiver (PR). Likewise, the secondary link comprises an SU transmitter (ST) and an SU receiver (SR). The instantaneous channel power gains for the secondary link, the link between ST and PR, the primary link and the link between PT and SR, are denoted respectively by g_{ss}, g_{sp}, g_{pp} and g_{ps} , which are assumed to be exponentially distributed with unit means, following small-scale Rayleigh fading¹.

¹The large-scale channel effect such as lognormal shadowing and pathloss effects are scaled into the noise power and interference constraint.



Fig. 1. System Model

We assume the system is time-slotted with frame period T, which consists of a sensing duration τ and a transmission duration $T_d = T - \tau$. Suppose PUs' status (idle/busy) and channel power gains are stationary and ergodic but remain unchanged over a frame period. At the beginning of each time frame, ST performs spectrum sensing within sensing time τ to determine the PUs' status. Due to imperfect sensing, for a given sensing outcome (idle/busy), the actual PUs' status could be either idle or busy (absence or presence of PUs). If the sensing outcome is idle in the presence of PUs, a miss detection occurs. If the sensing outcome is busy in the absence of PUs, a false alarm occurs. Miss detection imposes interference to PUs, whereas false alarm under-utilizes spectrum opportunity. The probabilities of miss detection and false alarm are denoted by P_F and P_M , respectively. Ideally, P_M and P_F should be small.

Specific expressions of P_F and P_M are related to the detection algorithm to be used. For simplicity, we consider energy detection, which compares the received signal power with a threshold. The false alarm probability P_F and miss detection probability P_M of energy detection in AWGN channel can be expressed respectively [13]

$$P_F = Q \left((1+\gamma)Q^{-1}(1-P_M) + \gamma \sqrt{0.5\tau f_s} \right), \tag{1}$$

$$P_M = 1 - Q\left(\frac{Q^{-1}(P_F) - \gamma \sqrt{0.5\tau f_s}}{(1+\gamma)}\right),$$
(2)

where γ is the signal-to-noise ratio (SNR) at SUs, τ is sensing time, and f_s is the sampling frequency, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$ is the tail probability of the normal distribution. In practice, both the miss detection probability and the false alarm probability have to be no greater than 0.1 [14]. Furthermore, it is required that the target P_M and P_F should be satisfied even in the low SNR γ region, which corresponds to the sensing sensitivity. For instance, in IEEE 802.22 WRAN [14], the sensing receiver sensitivity is -116 dBm for digital TV and -107 dBm for wireless microphone, associated with the SNR $\gamma = -21$ dB and $\gamma = -12$ dB, respectively. It is worth mentioning that in reality, the SNR γ may not always to be low down to the sensing sensitivity region; however, CR systems have to be designed to be able to tackle the worst scenarios. Therefore, the sensing sensitivity-based detection performance in AWGN channel provides a performance lower bound in fading environments [15].

Hybrid transmission mechanism is based on sensing outcomes. If the detection outcome is idle, ST transmits with power P_0 ; otherwise, ST transmits with a lower power P_1 . When ST transmits in the presence of PUs, SR suffers the amount of interference $P_p g_{ps}$ from PT. In reality, the instantaneous transmit power of PT P_p and the channel power gain g_{ps} are difficult to obtain, while the expected value $I_{ps} = \mathbb{E}[P_p g_{ps}]$ is assumed to be perfectly measured by SR and feedback to ST [4]. To avoid harmful interference to PR, the transmit power P_1 has to satisfy interference constraint $P_1g_{sp} \leq \Gamma$, where Γ is the predefined maximum interference power that the PUs can tolerate and is known as "interference temperature" constraint [2, 16].

Let σ_0^2 be the noise power. When sensing outcome is idle, the instantaneous achievable rates in the absence and presence of PUs are $r_{00} = \ln(1 + \frac{P_{0gss}}{\sigma_0^2})$ and $r_{10} = \ln(1 + \frac{P_{0gss}}{\sigma_0^2 + P_{pgps}})$, respectively. On the other hand, when sensing outcome is busy, the instantaneous rates in the absence and presence of PUs are $r_{01} = \ln(1 + \frac{P_{1}g_{ss}}{\sigma_0^2})$ and $r_{11} =$

 $\ln(1 + \frac{p_{1g_{ss}}}{\sigma_0^2 + p_{pg_{ps}}}), \text{ respectively.}$ Let $\mathbf{g} = (g_{ss}, g_{sp}, g_{ps})$. Given the ergodic properties of CSIs \mathbf{g} sensing under a given sensing time τ is expressed as

$$C(\tau, P_0, P_1, P_M) = \mathbb{E}_{\mathbf{g}}[\mathcal{P}(\mathcal{H}_0)(1 - P_F(\tau))\ln(1 + \frac{P_0 g_{ss}}{\sigma_0^2}) + \mathcal{P}(\mathcal{H}_0)P_F(\tau)\ln(1 + \frac{P_0 g_{ss}}{\sigma_0^2 + P_p g_{ps}}) + \mathcal{P}(\mathcal{H}_1)P_M(\tau)\ln(1 + \frac{P_1 g_{ss}}{\sigma_0^2}) + \mathcal{P}(\mathcal{H}_1)(1 - P_M(\tau))\ln(1 + \frac{P_1 g_{ss}}{\sigma_0^2 + P_p g_{ps}})], \quad (3)$$

where $\mathcal{P}(\mathcal{H}_0)$ and $\mathcal{P}(\mathcal{H}_1)$ denote respectively the probability of the occurrence of the idle status and the busy status. Expectation, denoted by \mathbb{E} , is taken over all the channel power gains g, which are assumed to be available to SUs for each realization but independently change over each frame.

Consider the effective transmission time, the ergodic throughput of SUs can be written as

$$r = (1 - \tau/T)C(\tau, P_0, P_1, P_M)$$
(4)

where $1 - \tau/T$ is the normalized transmission time.

3. PROBLEM FORMULATION AND ANALYSIS

3.1. Problem Formulation

The objective is to maximize the ergodic throughput of hybrid CR in (4) through optimal sensing and power allocation.

Given the interference constraint $P_1g_{sp} \leq \Gamma$ and the peak transmit power constraint P_{pk} , we can easily show that to maximize the ergodic throughput, the optimal power allocation should be

$$P_0^* = P_{pk}, P_1^* = \min(P_{pk}, \Gamma/g_{sp}).$$
 (5)

Notably, P_1^* is in general a function of the random CSI g_{sp} , which indicates that even though P_1^* is fixed under a given channel realization, it changes with g_{sp} as time evolves. Under a given g_{sp} , P_1^* increases with the maximum allowable interference Γ . However, the larger Γ , the higher interference is potentially imposed to PUs.

Substituting the optimal power into (3) and by linearity property of expectation, we can formulate the ergodic throughput maximization problem as

$$\begin{aligned} \max_{\tau, P_M} (1 - \frac{\tau}{T}) \{ \mathcal{P}(\mathcal{H}_0)(1 - P_F(\tau)) \mathbb{E}_{\mathbf{g}}[r_{00}] + \mathcal{P}(\mathcal{H}_0) P_F(\tau) \mathbb{E}_{\mathbf{g}}[r_{01}] + \\ \mathcal{P}(\mathcal{H}_1) P_M(\tau) \mathbb{E}_{\mathbf{g}}[r_{01}] + \mathcal{P}(\mathcal{H}_1)(1 - P_M(\tau)) \mathbb{E}_{\mathbf{g}}[r_{11}] \} \\ \text{s.t.} \quad P_M(\tau) \leq \varepsilon, \\ P_F(\tau) = Q \left((1 + \gamma) Q^{-1}(1 - P_M) + \gamma \sqrt{0.5\tau f_s} \right), \\ 0 < \tau < T. \end{aligned}$$

$$(6)$$

where ε is the maximum tolerable miss detection probability to control interference in overlay access, e.g., $\varepsilon = 0.1$ in the IEEE 802.22 specification [14]. Noticeably, the expectation can skip over P_F and P_M in (3) because we consider the sensing sensitivity-based detection performance, which is independent of the instantaneous channel power gains. The problem in (6) is in general a bivariate non-convex optimization problem, and the alternating optimization can be used to handle the problem [17].

We then consider for each sensing time τ how to obtain the optimal miss detection probability. By calculating the derivative of the objective function r with respect to P_M , we can show that

$$\frac{\partial r}{\partial P_M} = \mathbb{E}_{\mathbf{g}}[(1 - \frac{\tau}{T})(\mathcal{P}(\mathcal{H}_0)P'_F(r_{01} - r_{00}) + \mathcal{P}(\mathcal{H}_1)(r_{10} - r_{11}))] > 0,$$

which indicates that the objective function increases with P_M for each τ . The optimal P_M then takes the maximum feasible value ε , i.e., $P_M(\tau) = \varepsilon$, which has to be satisfied for every sensing time τ .

To solve the problem (6), we have to derive the expected rates $\mathbb{E}_{\mathbf{g}}[r_{ij}] = C_{ij}, i, j \in \{0, 1\}$. We will elaborate on this issue in the remaining subsections.

3.2. Ergodic rates in overlay mode

When an idle status is detected, SUs operate in overlay access by transmitting with an optimal power $P_0^* = P_{pk}$. If PUs are absent, the expected rate C_{00} can be derived as

$$C_{00} = \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_0^* g_{ss}}{\sigma_0^2})] = \int_0^\infty \ln(1 + \frac{P_{pk}}{\sigma_0^2} g_{ss}) e^{-g_{ss}} dg_{ss}$$
$$= \exp(\sigma_0^2 / P_{pk}) E_1(\sigma_0^2 / P_{pk}), \tag{7}$$

where $\exp(x)$ denotes exponential function e^x and $E_1(x)$ is the exponential integral function defined as [18].

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt.$$
 (8)

Due to imperfect sensing, if the sensing outcome is idle in the presence of PUs, the expected rate is obtained as

$$C_{10} = \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_0 g_{ss}}{\sigma_0^2 + P_p g_{ps}})] \ge \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_0 g_{ss}}{\sigma_0^2 + \mathbb{E}[P_p g_{ps}]})] = \hat{C}_{10} = \exp(\frac{\sigma_0^2 + I_{ps}}{P_{pk}})E_1(\frac{\sigma_0^2 + I_{ps}}{P_{pk}}).$$
(9)

where deriving exact C_{10} is difficult and the Jensen's inequality is thus applied to obtain a lower bound of C_{10} with $I_{ps} = \mathbb{E}[P_p g_{ps}]$.

3.3. Ergodic rates in underlay mode

When a busy status is detected, SUs can still access the channel in underlay mode by transmitting with power P_1^* under interference constraint. Due to false alarm, SUs would transmit with power P_1^* in the absence of PUs with the ergodic rate

$$C_{01} = \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_{1}^{*}g_{ss}}{\sigma_{0}^{2}})]$$

$$= \int_{0}^{\infty} \int_{0}^{\Gamma/P_{pk}} \ln(1 + \frac{P_{pk}g_{ss}}{\sigma_{0}^{2}})f(g_{sp})dg_{sp}f(g_{ss})dg_{ss} + \underbrace{\int_{0}^{\infty} \int_{\Gamma/P_{pk}}^{\infty} \ln(1 + \frac{\Gamma}{\sigma_{0}^{2}}\frac{g_{ss}}{g_{sp}})f(g_{sp})dg_{sp}f(g_{ss})dg_{ss}}_{L_{01}}$$

$$= \exp(\sigma_{0}^{2}/P_{pk})E_{1}(\sigma_{0}^{2}/P_{pk})(1 - e^{-\Gamma/P_{pk}}) + L_{01}, \quad (10)$$

which is complicated because of the double integration in L_{01} and a simplification will be made. Let $a_{01} = \Gamma/\sigma_0^2$, we have

$$L_{01} = \underbrace{\int_{0}^{\infty} \int_{0}^{\infty} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\infty}} - \underbrace{\int_{0}^{\infty} \int_{0}^{\Gamma/P_{pk}} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{\Gamma/P_{pk}} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\Gamma/P_{pk}} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\Gamma/P_{pk}} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{ss}}{g_{sp}}) f(g_{sp}) dg_{sp} f(g_{ss}) dg_{ss}}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{\infty} \int_{0}^{0} \ln(1 + a_{01} \frac{g_{sp}}{g_{sp}}) f(g_{sp}) dg_{sp}}}_{L_{01}^{\Gamma/P_{pk}}}} - \underbrace{\int_{0}^{0} \ln(1 + a_{01} \frac{g_{sp}}{g_{sp}}) f(g_{sp}) dg_{sp}}} - \underbrace{\int_{0}^{0} \ln$$

Since g_{ss}, g_{sp} both follow Rayleigh distribution, we have the distribution $f_{\frac{g_{ss}}{g_{sex}}}(x) = 1/(1+x)^2$ [8]. We can show that

$$L_{01}^{\infty} = \int_{0}^{\infty} \frac{\ln(1+a_{01}x)}{(1+x)^{2}} dx$$

= $-\frac{\ln(1+a_{01}x)}{(1+x)} \Big|_{0}^{\infty} + \int_{0}^{\infty} \frac{a_{01}}{(1+x)(1+a_{01}x)} dx$
= $\int_{0}^{\infty} \frac{a_{01}}{a_{01}x^{2} + (a_{01}+1)x + 1} dx.$ (12)

From [19], we can obtain that $L_{01}^{\infty} = \frac{a_{01} \ln a_{01}}{a_{01}-1}$ if $a_{01} \neq 1$, and $L_{01}^{\infty} = 1$ if $a_{01} = 1$. Now we consider L_{01}^{0} , which can be simplified as

$$L_{01}^{\Gamma/P_{pk}} = \int_0^{\Gamma/P_{pk}} \exp(\frac{1}{a_{01}g_{sp}}) E_1(\frac{1}{a_{01}g_{sp}}) e^{-g_{sp}} dg_{sp}.$$
 (13)

Finally, we obtain the expected rate of C_{01}

$$C_{01} = L_{01}^{\infty} - L_{01}^{\Gamma/P_{pk}} + \exp(\frac{\sigma_0^2}{P_{pk}})E_1(\frac{\sigma_0^2}{P_{pk}})(1 - e^{-\Gamma/P_{pk}}).$$
(14)

With $I_{ps} = \mathbb{E}[P_p g_{ps}]$, the ergodic rate C_{11} under correct detection of busy status is lower-bounded by \hat{C}_{11} from Jensen's inequality,

$$C_{11} = \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_1^* g_{ss}}{\sigma_0^2 + P_p g_{ps}})] \ge \mathbb{E}_{\mathbf{g}}[\ln(1 + \frac{P_1^* g_{ss}}{\sigma_0^2 + I_{ps}})] = \hat{C}_{11}.$$

Let $a_{11} = \Gamma/(\sigma_0^2 + I_{ps})$. In a similar fashion to derive C_{01} , we obtain

$$\hat{C}_{11} = L_{11}^{\infty} - L_{11}^{\Gamma/P_{pk}} + \exp(\frac{\sigma_0^2 + I_{ps}}{P_{pk}}) E_1(\frac{\sigma_0^2 + I_{ps}}{P_{pk}}) (1 - e^{-\Gamma/P_{pk}}), \quad (15)$$

where,

$$L_{11}^{\infty} = \begin{cases} \frac{a_{11} \ln a_{11}}{a_{11} - 1}, & \text{if } a_{11} \neq 1\\ 1, & \text{if } a_{11} = 1 \end{cases}$$
(16)

$$L_{11}^{\Gamma/P_{pk}} = \int_0^{\Gamma/P_{pk}} \exp(\frac{1}{a_{11}g_{sp}}) E_1(\frac{1}{a_{11}g_{sp}}) e^{-g_{sp}} dg_{sp}.$$
 (17)

The expressions of both C_{01} and \hat{C}_{11} appear to be complicated. To validate the derived results, their numerical values and simulation results (10,000 runs for each result) are plotted in Fig. 2, where $P_{pk} = 20$ dB, $P_p = 10$ dB and $I_{ps} = 10$ dB, are measured with respect to noise power σ_0^2 in the unit of dB.

The simulation results of C_{01} overlap with its numerical values, which verifies the derived expression of C_{01} . Furthermore, the numerical values of \hat{C}_{11} closely approach and lower-bound the simulation results of \hat{C}_{11} , which shows the tightness of the approximation.

The derived ergodic rates provide a way to obtain the closed-form ergodic throughput with perfect sensing ($P_F = 0$ and $P_M = 0$)

$$r = \mathcal{P}(\mathcal{H}_0)C_{00} + \mathcal{P}(\mathcal{H}_1)\hat{C}_{11}.$$
(18)



Fig. 2. Validation of C_{01} and \hat{C}_{11} ($P_{pk} = 20 \text{ dB}$, $P_p = 10 \text{ dB}$)

3.4. Optimum Sensing Time

Substituting the optimal $P_M^*(\tau) = \varepsilon$ and the obtained ergodic rates into (6), we have a simplified univariate problem

$$\max_{0 < \tau < T} (1 - \tau/T) \{ \mathcal{P}(\mathcal{H}_0)(1 - P_F(\tau))C_{00} + \mathcal{P}(\mathcal{H}_0)P_F(\tau)C_{01} + \mathcal{P}(\mathcal{H}_1)\varepsilon\hat{C}_{10} + \mathcal{P}(\mathcal{H}_1)(1 - \varepsilon)\hat{C}_{11} \},$$
(19)

which can be solved efficiently by univariate optimization methods. The obtained optimal sensing time τ^* is related to the distribution of CSIs and is termed as statistical CSI-based sensing time.

Noticeably, although Rayleigh fading channels are considered in this paper, the proposed statistical CSI-based sensing time can also be applied to other channel environments as long as the corresponding ergodic rates C_{ij} , $\forall i, j = \{0, 1\}$ can be obtained.

4. SIMULATION RESULTS

Simulations were conducted to demonstrate the performance of the proposed sensing and power allocation in hybrid CR. Simulation settings are: maximum allowable miss detection probability $\varepsilon = 0.1$, the frame period T = 100 ms, the sampling frequency $f_s = 1$ MHz, the SNR $\gamma = -15$ dB, noise power $\sigma_0^2 = 1$, SUs' peak transmit power $P_{pk} = 20$ dB and PUs's transmit power $P_p = 10$ dB (relative to σ_0^2). All the channel power gains are independently following exponential distribution with unit means. Each simulation result was averaged over 10,000 runs.

Fig. 3 shows the simulated and numerical ergodic throughput of hybrid access versus sensing time for $\mathcal{P}(\mathcal{H}_0) = 0.5$. The simulation results of overlay access is also plotted for comparison.

The numerical results of hybrid access are tightly upperbounded by the simulation results, which indicates that \hat{C}_{10} and \hat{C}_{11} are sufficient approximations for the optimal sensing time design in hybrid CR. For both hybrid access and overlay access, a unique optimal sensing time exists to maximize the ergodic throughput. We also note that the maximum throughput increases significantly by hybrid access compared with that of overlay access, where the gain is achieved by exploiting underlay access in hybrid CR.

Fig. 4 shows the simulation results of the maximum ergodic throughput of hybrid access versus interference constraint Γ under different idle probabilities. The simulation results of instantaneous CSI-based scheme [10] are also plotted for comparison. It is clear that the proposed statistical CSI-based scheme approaches closely to the instantaneous CSI-based scheme [10]. However, the proposed



Fig. 3. Ergodic throughput versus sensing time ($\mathcal{P}(\mathcal{H}_0) = 0.5$)

statistical CSI-based scheme is practically preferable because it does not rely on instantaneous CSIs. With the increase of idle probability $\mathcal{P}(\mathcal{H}_0)$, the maximum throughput increases significantly, this is reasonable since a larger idle probability indicates a higher chance for SUs to transmit with higher power. In addition, the maximum achievable ergodic throughput increases with the interference constraint Γ , which suggests that an effective way to improve throughput of hybrid CR is to relax interference constraint at PUs. However, relaxation of interference constraint may require additional payoff incentives for PUs in practice.



Fig. 4. Maximum ergodic throughput under optimal sensing and power allocation

5. CONCLUSION

In conclusion, this paper investigates optimal sensing and power allocation to maximize ergodic throughput of hybrid CR with imperfect sensing. We derived the expected rates for both overlay and underlay access in Rayleigh fading environments, based on which we obtained a unique optimal sensing time. The proposed sensing time scheme is related to the distribution of CSIs rather than instantaneous CSIs in existing works and is more flexible in practice. Simulation results validate the derived results, showing distinctly higher throughput achieved by hybrid access compared with conventional overlay access.

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