

COOPERATIVE SPECTRUM SHARING WITH JOINT RECEIVER DECODING

Songze Li, Urbashi Mitra*

University of Southern California
Ming Hsieh Department of Electrical Engineering
University Park, Los Angeles, CA 90089, USA.

Ashish Pandharipande

Philips Research
High Tech Campus
Eindhoven, 5656 AE, The Netherlands.

ABSTRACT

We consider a spectrum sharing protocol wherein the primary and secondary transmitters cooperatively relay each other's message. Transmission is done in two phases, with each transmitter attempting to decode messages from the other system transmission in a first phase. The second phase transmission consists of the decoded message superposed onto its own message. Priority is given to the primary system transmissions by having the primary message always transmitted over the two phases, while the secondary message is transmitted depending on successful decoding. We consider the scenario where the primary and secondary receivers are co-located, forming a virtual two-antenna receiver. We assess the performance of the system in terms of outage probability and characterize performance corresponding to each state of the Markov chain that governs the proposed transmission protocol. We show that joint decoding offers a 20 dB performance improvement over separate decoding for the primary user and 1.8 dB for the secondary user.

Index Terms— spectrum sharing, cognitive radio, joint decoding, outage performance, virtual MIMO system

1. INTRODUCTION

With growing demand for services relying on wireless communications, there is an increasing need to improve radio spectrum utilization. Cognitive radio with dynamic spectrum sharing is an effective solution to multiple system operation in a single spectrum band, with the provision that QoS degradation is limited. An attractive class of spectrum sharing techniques [1–5] that have emerged recently allow primary and secondary systems to transmit concurrently using cooperative communication principles. Transmission protocols and receiver processing techniques were designed such that the secondary system operation did not adversely impact primary system performance.

Secondary-side cooperation via the secondary transmitter using amplify-and-forward [2] and decode-and-forward [4] techniques has been used to gain spectrum access, while maintaining the performance of the primary system. In [3], a spectrum leasing framework was considered wherein the primary system leased bandwidth for a fraction of time in return for cooperation from a secondary *ad hoc* network using distributed space-time coding. An overlay spectrum sharing scheme with multiple antennas was described in [5] where the primary user leases half of its time slots to the secondary user, while the latter used amplify-and-forward to cooperatively relay primary system messages. Beamforming techniques were considered at

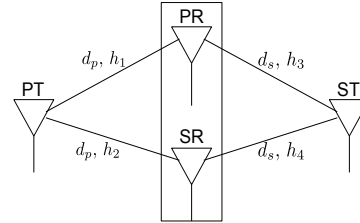


Fig. 1. System topology.

a multi-antenna secondary transmitter to provide secondary access using finite-rate feedback information [6]. In [7], we introduced a transmission protocol employing two-way cooperation between both the primary and secondary users, assuming code-books were shared. In this protocol, in contrast to [4], neither user is idle. In [7], separate decoding was employed by the primary and secondary receivers to retrieve their respective messages.

In this paper, we consider the scenario where joint decoding is possible in a system with co-located primary and secondary receivers. As in [7], the transmission protocol can be characterized as a Markov chain. We obtain the outage probabilities, either in closed form or via tight approximations, for each state of the Markov chain. We assess the gains via joint decoding over the system in [7] with separate decoding. Simulations show substantial performance improvements notably for the primary system in the considered system configuration. Interestingly, we observe good performance even when employing a one-shot decoder (no memory) at the joint receiver.

2. TRANSMISSION PROTOCOL

Consider the scenario that the primary and secondary receivers are co-located as would occur when two providers offer service to a single device, one of which owns the spectrum access rights. The system topology is depicted in Fig. 1 which consists of four nodes: primary transmitter (PT), primary receiver (PR), secondary transmitter (ST) and secondary receiver (SR). We assume Rayleigh fading, which results in the channel power gain being described by an exponential random variable, $\gamma_i = |h_i|^2$, $i \in \{1, 2, 3, 4\}$ with parameter $\lambda_i = d_i^\nu$, $i = 1, 2$ and $\lambda_i = d_i^\nu$, $i = 3, 4$ where ν is the path loss. h_i are assumed to be statistically independent. The transmission powers at the PT and ST are P_p and P_s , respectively.

As illustrated in Fig. 2, during the transmission period, each time frame is split into two phases. Both PT and ST operate in half-duplex mode communication and each transmitter tries to decode the mes-

*This research has been funded in part by the following: ONR N00014-09-1-0700, NSF CNS-0832186, CCF-0917343, CCF-1117896, CNS-1213128, and AFOSR FA9550-12-1-0215.

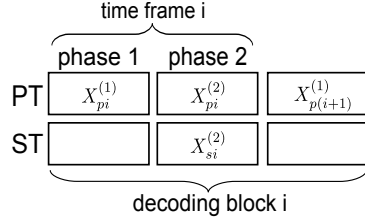


Fig. 2. Transmission scheme.

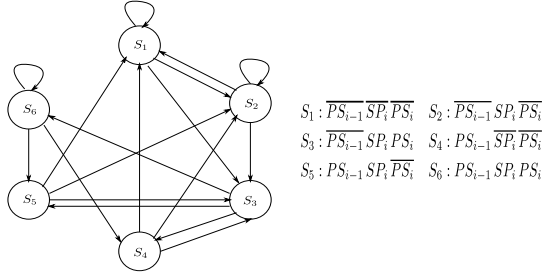


Fig. 3. State transition.

sage from the other transmitter.

In the first transmission phase, the signal transmitted by PT is given by:

$$X_{pi}^{(1)} = \sqrt{\alpha_1 P_p} x_{pi} + \sqrt{(1 - \alpha_1) P_p} x_{s(i-1)}, \quad (1)$$

where $0 < \alpha_1 \leq 1$ is the fraction of power assigned to the primary message. The primary message in the i th frame is denoted by x_{pi} , and $x_{s(i-1)}$ is the secondary message in the previous frame. We assume that $\mathbb{E}\{|x_{pi}|^2\} = \mathbb{E}\{|x_{si}|^2\} = 1$. In our proposed cooperative spectrum sharing protocol, if PT is able to decode the secondary message, it will forward the secondary message along with its own message. Otherwise, PT only transmits its own message and $\alpha_1 = 1$. The ST also feeds back an 1 bit ACK/NACK message to the PT claiming decoding x_{pi} or not.

In the second transmission phase, depending on the primary message decoding outcome in the first phase, the signals transmitted from the ST and PT can be represented by:

$$X_{si}^{(2)} = \begin{cases} \sqrt{\alpha_2 P_s} x_{pi} + \sqrt{(1 - \alpha_2) P_s} x_{si}, & \text{ST decodes } x_{pi} \\ 0, & \text{else} \end{cases}, \quad (2)$$

where $0 < \alpha_2 \leq 1$ and

$$X_{pi}^{(2)} = \begin{cases} 0, & \text{ST decodes } x_{pi} \\ x_{pi}, & \text{ST fails to decode } x_{pi}. \end{cases} \quad (3)$$

α_2 denotes the fraction of power assigned to x_{pi} at the ST.

Under different channel realizations, the decoding block i in Fig. 2 can be in one of the six states $S_k, k = 1, \dots, 6$ characterized by three events: PS_{i-1} = PT decodes $x_{s(i-1)}$ correctly; SP_i = ST decodes x_{pi} correctly; and PS_i = PT decodes x_{si} correctly. The exact expressions for the state transition matrix (Fig. 3) and stationary distributions are given in [7].

In our scheme, the primary message is *always* transmitted for

two phases, by PT or ST. In contrast, the secondary message can be transmitted for two, one, or no phases. This feature of the secondary transmission is due to the fact that the primary system has a higher priority. A similar overlay sharing approach is considered in [5], where amplify-and-forward relaying is performed at ST and the primary user has no incentive to forward the secondary user.

3. JOINT MESSAGE DECODING

We assume that channel state information is perfectly known at both receivers. The PR and SR form a virtual two-antenna receiver, and thus, this system is equivalent to a two-user MIMO MAC channel with one transmit antenna and two receive antennas. In the i th time frame where the PT tries to send x_{pi} , and the ST tries to send x_{si} , the received signal for state $k, k = 1, 2, \dots, 6, \underline{y}(k)$ is:

$$\underline{y}(k) = \underline{H}_{pi}(k)x_{pi} + \underline{H}_{si}(k)x_{si} + \underline{H}_{p(i+1)}(k)x_{p(i+1)} + \underline{H}_{s(i-1)}(k)x_{s(i-1)} + \underline{w}. \quad (4)$$

Denote $\Lambda_p = \frac{P_p}{\sigma^2}$, and $\Lambda_s = \frac{P_s}{\sigma^2}$. For example, the channel vectors for state 5 are:

$$\begin{aligned} \underline{H}_{s(i-1)}(5) &= (\sqrt{\Lambda_p(1 - \alpha_1)}h_1 \quad \sqrt{\Lambda_p(1 - \alpha_1)}h_2 \quad 0 \quad 0)^T \\ \underline{H}_{pi}(5) &= (\sqrt{\Lambda_p\alpha_1}h_1 \quad \sqrt{\Lambda_p\alpha_1}h_2 \quad \sqrt{\Lambda_s\alpha_2}h_3 \quad \sqrt{\Lambda_s\alpha_2}h_4)^T \\ \underline{H}_{si}(5) &= (0 \quad 0 \quad \sqrt{\Lambda_s(1 - \alpha_2)}h_3 \quad \sqrt{\Lambda_s(1 - \alpha_2)}h_4)^T \end{aligned}$$

The channel coherence time is assumed to be two time slots. Channel gains in phase 1 of next decoding block are $h'_i, \gamma'_i = |h'_i|^2, i \in \{1, \dots, 4\}$. We also assume normalized Gaussian white noise at each time instance and each antenna, $\underline{w} \sim N(0, \mathbf{I})$.

In decoding x_{pi} and x_{si} , the previous secondary message $x_{s(i-1)}$ and next primary message $x_{p(i+1)}$ are treated as interfering signals. Disregarding the system memory to reduce complexity, $x_{s(i-1)}$ is treated as noise as we decode and hopefully cancel $x_{p(i+1)}$. In the numerical results, we will show that the treatment of $x_{s(i-1)}$ as noise does not significantly degrade performance.

To classify the capacity region for state k , the noise is whitened and the effective channel vectors for primary and secondary messages $\tilde{\underline{H}}_{pi}(k)$ and $\tilde{\underline{H}}_{si}(k)$ are obtained. According to Eq. (13) in [8], the rate pair for state $k, (R_p(k), R_s(k))$ satisfies $\forall S \subseteq \{p, s\}$,

$$\sum_{j \in S} R_j(k) \leq \frac{1}{2} \log \left[\det \left(\mathbf{I} + \sum_{j \in S} \tilde{\underline{H}}_{ji}(k) \tilde{\underline{H}}_{ji}(k)^T \right) \right]. \quad (5)$$

In Eq. (5), \mathbf{I} is the identity matrix. $\mathbb{E}\{|x_{pi}|^2\} = \mathbb{E}\{|x_{si}|^2\} = 1$, and primary and secondary systems send i.i.d sequences.

For states 3 and 6, the effective channel vectors depend on whether or not $x_{p(i+1)}$ is decoded correctly. When decoding $x_{p(i+1)}$, signals in the first time slot of $(i + 1)$ th time frame are employed and x_{si} is treated as noise. The equivalent noise is denoted \underline{n}_{int} ,

$$\underline{n}_{int} = \left(\frac{\sqrt{\Lambda_p(1 - \alpha_1)}h'_1}{\sqrt{\Lambda_p(1 - \alpha_1)}h'_2} \right) x_{si} + \underline{w} \quad (6)$$

Notice here x_{si} becomes interference when it is decoded in phase 2 at the PT and forwarded in the next phase.

The covariance matrix of \underline{n}_{int} is

$$Q(\underline{n}_{int}) = \begin{pmatrix} \Lambda_p(1-\alpha_1)\gamma'_1 + 1 & \Lambda_p(1-\alpha_1)\sqrt{\gamma'_1\gamma'_2} \\ \Lambda_p(1-\alpha_1)\sqrt{\gamma'_1\gamma'_2} & \Lambda_p(1-\alpha_1)\gamma'_2 + 1 \end{pmatrix}$$

The channel vector for $x_{p(i+1)}$ is found after whitening:

$$\tilde{H}_{int} = G_{int}^{-1} \begin{pmatrix} \sqrt{\Lambda_p\alpha_1}h'_1 \\ \sqrt{\Lambda_p\alpha_1}h'_2 \end{pmatrix},$$

where $Q(\underline{n}_{int}) = G_{int}G_{int}^T$, and G_{int} derives from the eigendecomposition of $Q(\underline{n}_{int})$.

If the PT transmits at a higher rate than $\frac{1}{2} \log(1 + \tilde{H}_{int}^T \tilde{H}_{int})$, $x_{p(i+1)}$ cannot be decoded and is then treated as noise. The equivalent noise $\tilde{\underline{n}}$ is:

$$\tilde{\underline{n}} = \begin{pmatrix} 0 & 0 & 0 & 0 & h'_1 & h'_2 \end{pmatrix}^T \sqrt{\Lambda_p\alpha_1} x_{p(i+1)} + \underline{w}.$$

The whitening process is performed and the corresponding capacities can be calculated.

For states 4,5 and 6, the secondary message from previous time frame $x_{s(i-1)}$ is treated as noise. For example, the equivalent noise $\tilde{\underline{n}}$ for state 4 is

$$\tilde{\underline{n}} = \begin{pmatrix} h_1 & h_2 & 0 & 0 \end{pmatrix}^T \sqrt{\Lambda_p(1-\alpha_1)} x_{s(i-1)} + \underline{w}$$

The covariance matrix of this colored noise is

$$Q(\tilde{\underline{n}}) = \begin{pmatrix} \Lambda_p(1-\alpha_1)\gamma_1 + 1 & \Lambda_p(1-\alpha_1)\sqrt{\gamma_1\gamma_2} & 0 & 0 \\ \Lambda_p(1-\alpha_1)\sqrt{\gamma_1\gamma_2} & \Lambda_p(1-\alpha_1)\gamma_2 + 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\tilde{H}_{pi}(4) = \sqrt{\Lambda_p}M^{-1}(\sqrt{\alpha_1}h_1 \quad \sqrt{\alpha_1}h_2 \quad h_1 \quad h_2)^T$ after whitening, where $Q(\tilde{\underline{n}}) = MM^T$.

Assuming a fixed power allocation at the PT and ST, we can calculate the individual outage probabilities for both systems at each state based on the knowledge of channel statistics, and compare them with the probabilities from the separate decoding situation. In Fig. 1, the channel power gains $\gamma_i = |h_i|^2$, $i \in \{1, 2, 3, 4\}$ are exponentially distributed with parameters $\lambda_p = d_p^\nu$ and $\lambda_s = d_s^\nu$. We define $\gamma_p = \gamma_1 + \gamma_2$, $\gamma_s = \gamma_3 + \gamma_4$, $\gamma'_p = \gamma'_1 + \gamma'_2$ and $\gamma'_s = \gamma'_3 + \gamma'_4$ with distributions:

$$F_{\gamma_k}(z) = [1 - (1 + \lambda_k z) \exp(-\lambda_k z)] U(z), k \in \{p, s\}, \quad (7)$$

where $U(z)$ is the unit step function.

3.1. Primary Outage Performance

The primary capacity R_{pk} of state k is calculated according to the effective channel vectors. Denote the constant primary and secondary target rates by R_{PT} and R_{ST} , $\rho_1 = 2^{2R_{PT}} - 1$, $\rho_2 = 2^{2R_{ST}} - 1$. The primary outage probability for state k can be found by:

$$P_p(out|S_k) = \Pr[R_{pk} < R_{PT}|S_k]. \quad (8)$$

We sketch the computation of primary outages for states 1, 2 and 3, those for states 4, 5 and 6 are similarly computed. For state 1 we have:

$$P_p(out|S_1) = \Pr\left[\gamma_p < \frac{\rho_1}{2\Lambda_p}\right] = F_{\gamma_p}\left(\frac{\rho_1}{2\Lambda_p}\right). \quad (9)$$

When Λ_p is fixed, a smaller target rate results in a lower outage probability.

States 2 and 3 have the same primary outage:

$$P_p(out|S_k) = \Pr[\Lambda_p\gamma_p + \Lambda_s\alpha_2\gamma_s < \rho_1] \\ = F_{\gamma_p}\left(\frac{\rho_1}{\Lambda_p}\right) - \zeta_p - \chi_p, \quad k = 2, 3 \quad (10)$$

Define $\kappa_p = \frac{\lambda_s\Lambda_p}{\alpha_2\Lambda_s} - \lambda_p$, $\epsilon_p = \exp\left(-\lambda_s\frac{\rho_1}{\Lambda_s\alpha_2}\right)\lambda_p^2$, $\eta_p = \exp\left(\frac{\kappa_p\rho_1}{\Lambda_p}\right)$, then ζ_p and χ_p in (10) are given by

$$\zeta_p = \begin{cases} \epsilon_p \left(\frac{\rho_1}{\Lambda_p}\right)^2, & \kappa_p = 0 \\ \frac{\epsilon_p}{\kappa_p} \left[\frac{\rho_1\eta_p}{\Lambda_p} - \frac{\eta_p - 1}{\kappa_p}\right], & \kappa_p \neq 0, \end{cases} \quad (11)$$

$$\chi_p = \begin{cases} \frac{\epsilon_p\lambda_s\rho_1\left(\frac{\rho_1}{\Lambda_p}\right)^2}{6\Lambda_s\alpha_2}, & \kappa_p = 0 \\ \frac{\epsilon_p\lambda_s}{\kappa_p\Lambda_s\alpha_2} \left[\frac{\rho_1\eta_p + \rho_1 - 2\Lambda_p\frac{\eta_p - 1}{\kappa_p}}{\kappa_p}\right], & \kappa_p \neq 0, \end{cases} \quad (12)$$

To obtain a small outage, we want both ζ_p and χ_p to be large. When $\kappa_p = 0$, ϵ_p dominates in both ζ_p and χ_p : transmitting at lower rate and giving more power to relay x_{pi} (larger α_2) both contribute to increasing ζ_p and χ_p , thus reducing the conditional outage. When $\kappa \neq 0$, we claim, numerically, that the conditional outage probability in (10) is a increasing function of R_{PT} and a decreasing function of α_2 with the other variable fixed. Therefore, smaller target rate and larger α_2 are always preferable for the primary user regardless of κ_p .

3.2. Secondary Outage Performance

It is possible for x_{si} to be contained in the second and the third phase of the decoding block (Fig. 2), therefore whether $x_{s(i-1)}$ is present in the first phase or not has no effect on the secondary performance.

$P_s(out|S_1) = P_s(out|S_4) = 1$ in states 1 and 4 because no secondary message is transmitted. Then for states 2 and 5, we have:

$$P_s(out|S_k) = F_{\gamma_s}\left(\frac{\rho_2}{\Lambda_s(1-\alpha_2)}\right), \quad k = 2, 5 \quad (13)$$

This equation is intuitive, because when the fraction of power for the secondary message is increased (smaller α_2), the secondary outage will drop as the distribution function is non-decreasing.

In states 3 and 6, we try to decode $x_{p(i+1)}$ and cancel it based on the signals received in the third time slot. Denote the event W as $x_{p(i+1)}$ correctly decoded, and \bar{W} for its complement. Then for $k = 3, 6$:

$$\Pr[W] = \Pr[\Lambda_p(\alpha_1 - \rho_1(1-\alpha_1))\gamma'_p > \rho_1] \\ = \begin{cases} 0, & 0 < \alpha_1 < \frac{\rho_1}{1+\rho_1} \\ 1 - F_{\gamma_p}\left(\frac{\rho_1}{\Lambda_p(\alpha_1 - \rho_1(1-\alpha_1))}\right), & \frac{\rho_1}{1+\rho_1} < \alpha_1 < 1 \end{cases} \quad (14)$$

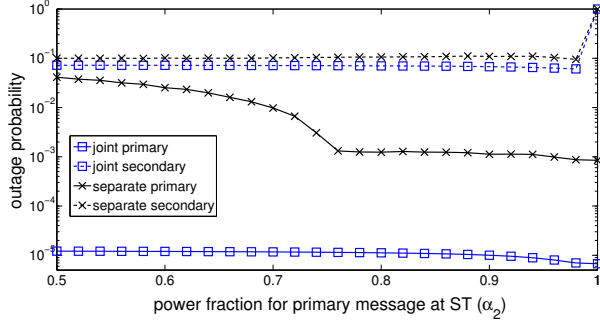


Fig. 4. Outage performances when $d_p=1, d_s=0.2$.

Therefore, giving more power to transmit primary message at the PT (larger α_1) not only yields a higher primary capacity, but also makes it easier to decode and cancel primary interference $x_{p(i+1)}$.

$$P_s(out|S_k, W) = \Pr[\Lambda_p(1 - \alpha_1)\gamma'_p + \Lambda_s(1 - \alpha_2)\gamma_s < \rho_2] \quad (15)$$

Because γ'_p has the same marginal distribution as γ_p , and is statistically independent of γ_s , we use the same steps to evaluate the above probability as we have done for Eq. (10).

$$P_s(out|S_k, \bar{W}) = \Pr\left[\frac{\Lambda_p(1 - \alpha_1)\gamma'_p}{\Lambda_p\alpha_1\gamma'_p + 1} + \Lambda_s(1 - \alpha_2)\gamma_s < \rho_2\right] \\ \approx F_{\gamma_s}\left(\frac{\rho_2 - \frac{1 - \alpha_1}{\alpha_1}}{\Lambda_s(1 - \alpha_2)}\right), \quad (16)$$

by assuming $\Lambda_p \gg 1$, and $\frac{\Lambda_p(1 - \alpha_1)\gamma'_p}{\Lambda_p\alpha_1\gamma'_p + 1} \approx \frac{1 - \alpha_1}{\alpha_1}$.

The probability in (16) is zero when $0 < \alpha_1 < \frac{1}{1 + \rho_1}$. This is ideal for secondary user but highly unlikely in practice, because the primary messages always consume most of the power at the PT. For $\alpha_1 > \frac{1}{1 + \rho_1}$, (16) gives a tight lower bound on desired outage probability.

Overall, for state $k, k = 3, 6$

$$P_s(out|S_k) = P_s(out|S_k, W)\Pr[W] \\ + P_s(out|S_k, \bar{W})(1 - \Pr[W]). \quad (17)$$

4. SIMULATION RESULTS

In this section, we perform simulations for collinear nodes. The path-loss exponent ν is chosen to be 4 as in [4]. The target primary and secondary rates are set to be $R_{ST} = R_{PT} = 1$. The transmit power $\frac{P_p}{\sigma^2} = \frac{P_s}{\sigma^2} = 20$ dB for both PT and ST. In the simulation, $d_p + d_s$ is chosen to be 1.2. According to Eq. (14) in Section 3.2, we fix α_1 at 0.8 which corresponds to the scenario where it is possible for $x_{p(i+1)}$ to be decoded and cancelled. For our system setup, $\frac{\rho_1}{1 + \rho_1} = 0.75$.

From Fig. 4, we find that when $d_p = 1$, as expected, the primary outage probability decreases dramatically from the cases of separate

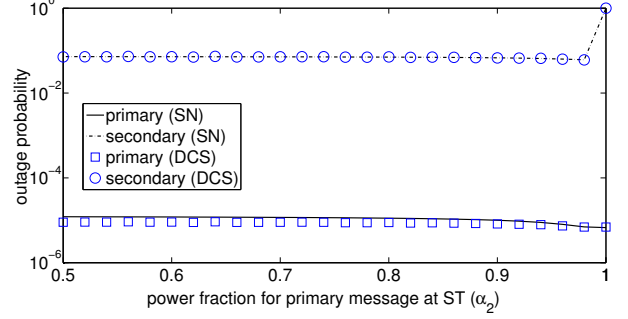


Fig. 5. Performance comparison by treating $x_{s(i-1)}$ as noise and by perfectly cancelling $x_{s(i-1)}$.

decoding in [7] by 20 dB. The secondary outage probability also drops consistently by 1.8 dB over all α_2 larger than 0.5.

We also compare the system performance when treating $x_{s(i-1)}$ as noise (labelled SN) with the case $x_{s(i-1)}$ is perfectly cancelled (labelled DCS) in Fig. 5. It is observed that for both users, these two ways of handling $x_{s(i-1)}$ result in almost identical outage probabilities. For other scenarios with different rate pairs and d_p , the performance improvements by eliminating $x_{s(i-1)}$ are also small, especially for the secondary user. Therefore, we conclude that additional system complexity by trying to decode and cancel $x_{s(i-1)}$ as we have done for $x_{p(i+1)}$ is not attractive due to negligible performance gains.

5. CONCLUSIONS

We extended the spectrum sharing protocol in our earlier work [7] to the scenario of joint receiver decoding. We derived exact and tight approximate expressions for both primary and secondary system outage probabilities corresponding to the different states of the underlying Markov chain. Simulation results showed that joint decoding further reduces outage probabilities as compared to separate decoding [7], resulting in further performance improvement compared to the secondary-side cooperative spectrum sharing protocol of [4]. Furthermore, it was found that good performance is obtained even when memory is not exploited in the joint decoder.

6. REFERENCES

- [1] O. Simeone, J. Gambini, Y. Bar-Ness, and U. Spagnolini, "Cooperation and cognitive radio," in *IEEE International Conference on Communications*, 2007, pp. 6511–6515.
- [2] Y. Han, A. Pandharipande, and S. Ting, "Cooperative spectrum sharing via controlled amplify-and-forward relaying," in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, 2008, pp. 6511–6515.
- [3] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 203–213, 2008.
- [4] Y. Han, A. Pandharipande, and S. Ting, "Cooperative decode-and-forward relaying for secondary spectrum access," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 4945–4950, 2009.

- [5] R. Manna, R.H.Y. Louie, Y. Li, and B. Vucetic, "Cooperative spectrum sharing in cognitive radio networks with multiple antennas," *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5509–5522, 2011.
- [6] K. Huang and R. Zhang, "Cooperative feedback for multi-antenna cognitive radio networks," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 747–758, 2011.
- [7] S. Li, U. Mitra, V. Ratnam, and A. Pandharipande, "Jointly cooperative decode-and-forward relaying for secondary spectrum access," in *46th Annual Conference on Information Sciences and Systems (CISS)*, 2012, March 2012.
- [8] A. Goldsmith, S.A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, 2003.