

# ONLINE CONTROL FOR ENERGY STORAGE MANAGEMENT WITH RENEWABLE ENERGY INTEGRATION

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## ABSTRACT

In this work, we study the problem of energy storage management with renewable energy integration by designing online control policy to minimize the long-term time-averaged cost. We take into account system input dynamics, and incorporate the battery operation cost for energy storage into the control optimization. Applying Lyapunov optimization technique, we design an on-line control policy that jointly optimizes the decisions for storage from two energy sources and supply to the consumer, which has bounded performance from the optimal scheme. We provide a close-form solution to our control optimization which renders our policy implementation with minimum complexity. Simulations show that introducing renewable energy can effectively reduce the long-term cost and improve the efficiency of energy storage relative to the battery operation cost.

**Index Terms**— Smart Grid, Renewable Energy, Energy Storage, Lyapunov Optimization, Online Control

## 1. INTRODUCTION

The incorporation of energy storage in smart grid has the promising potential to increase the grid reliability by reducing voltage fluctuation, and at the same time reduce energy bill to the consumers. The wider adoption of renewable energy into the grid system as green energy source enables us to harness the free energy source but also imposes challenge for grid operation due to its nature of uncertainty. Building effective energy storage integrating the renewable energy source is particularly important to realize both benefits. In smart grid, information, communication, and control technologies are essential to improve the power grid stability and reliability, and to effectively manage the integrated renewable energy sources and energy storage devices. The most challenging problem in energy storage and management is concerning the high cost of storage itself, *i.e.*, the limited battery life, the number of recharges/discharges, and the cost associated to them. To manage storage efficiently, an optimized control policy for energy management at the battery and from both renewable source and power grid is needed. The design challenges involved in this problem include stochastic behaviors of renewable energy and grid power pricing, the double effects of energy cost saving and operating cost by storing energy or using stored energy, and the finite storage capacity coupling the control decisions over time.

There has been rising interests on control policy design to manage power for energy storage recently. Different techniques are sought to address this problem. With statistical information of the dynamics available, [1], [2] use dynamic programming formulation to derive threshold-based control policies. Quadratic

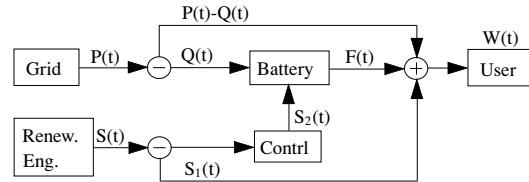


Fig. 1. An energy storage and management system

control techniques [3] and convex optimization [4] are used to formulate the storage management problem, where the system is assumed static or no battery capacity constraints. Some recent works apply Lyapunov optimization techniques [5] to design online control policy for energy storage [6]–[8]. However, they either only consider power grid without renewable energy source [6], or with renewable energy but without considering the battery recharging or discharging cost [7], [8]. The latter cost is in fact crucial in determining the battery actions for storage management, and making the problem more involved. In addition, these existing works do not provide explicit control solution.

In this paper, we aim at designing an online control policy for battery storage management with integrated renewable energy, with the goal of meeting consumer demand while minimizing the long-term cost. Both energy purchase cost and battery operating cost are modeled. Through Lyapunov optimization technique, we develop an online control scheme which only depends on the current system dynamics. We provide explicit solution of the control actions as functions of the current system dynamics, thus the complexity is minimum for implementation. The explicit control solution also enables us to see a clear priority among grid, renewable, and battery, for energy supply to the consumer. The performance bound of the proposed online control policy to the optimal performance is also provided.

## 2. SYSTEM MODEL

We consider an energy storage and management system that supplies the electricity to a power consuming entity, such as residential household, through the power sources and energy storage. We consider two types of power source generators: the traditional grid and the renewable energy (e.g. solar). In addition, a DC battery is used for energy storage and supply. Both generators are able to directly supply electricity to the user, and recharge the storage battery, as shown in Fig. 1<sup>1</sup>. We assume a time-slotted system model

<sup>1</sup>Fig. 1 shows a simplified model. In a real system, DC/AC converters are applied for charging/discharging to/from the battery.

where the operation is performed per time slot  $t$ . We denote the power demand at the user at time slot  $t$  as  $W(t)$ .

### 2.1. Power Source

*Traditional Grid:* A user is able to purchase power  $P(t)$  from the traditional power grid at a real-time price  $C(t)$  set by the utility. The amount  $P(t)$  is bounded by

$$0 \leq P(t) \leq P_{\max} \quad (1)$$

where  $P_{\max}$  is the maximum amount of power a user can purchase from the grid. The amount  $P(t)$  can be used to supply the user demand  $W(t)$  and/or charge into the battery for storage. The unit price  $C(t)$  is bounded by  $C_{\min} \leq C(t) \leq C_{\max}$ . The value of  $C(t)$  is known to the user and remain unchanged during the same slot. We assume  $C(t)$  is determined by the utility and is not a function of  $P(t)$ .

*Renewable Energy:* We consider solar energy as a typical example of our renewable energy source at the user-end. Let  $S(t)$  be the amount of renewable energy generated at time  $t$ . It is used for both direct power supply to meet the demand and energy storage in the battery. Let  $S_1(t)$  and  $S_2(t)$  be the amount used for direct power supply and battery recharging, respectively. We assume that  $S(t)$  will have the priority to first supply the demand  $W(t)$ , i.e.,

$$S_1(t) = \min\{W(t), S(t)\}. \quad (2)$$

A controller will then decide whether the remaining part will be charged into the battery, as shown in Fig. 1. Thus, the amount charged into the battery,  $S_2(t)$ , is bounded by

$$0 \leq S_2(t) \leq S(t) - S_1(t). \quad (3)$$

### 2.2. Battery Storage

We consider a simple model for the battery recharging/discharging, where no energy loss during recharging/discharging nor leakage of stored energy over time. Due to the battery property, the battery cannot be both recharged and discharged at same time. However, there can be multiple sources for recharging, e.g., from both the grid and renewable sources. Let  $Q(t)$  be the portion of  $P(t)$  from the traditional grid to be stored into the battery. The total amount of energy can be recharged into battery per time slot is bounded by  $R_{\max}$

$$0 \leq Q(t) + S_2(t) \leq R_{\max}. \quad (4)$$

Similarly, the discharging amount, denoted by  $F(t)$ , is bounded by:

$$0 \leq F(t) \leq D_{\max} \quad (5)$$

where  $D_{\max}$  denotes the maximum amount of discharge amount from battery per time slot. Since there is no simultaneous recharging or discharging, we have

$$Q(t) + S_2(t) > 0 \Rightarrow F(t) = 0, \quad F(t) > 0 \Rightarrow Q(t) + S_2(t) = 0. \quad (6)$$

Denote the state of battery (SOB) at time slot  $t$  as  $Y(t)$ . It is upper bounded by a finite capacity  $Y_{\max}$  and lower bounded by  $Y_{\min}$

$$Y_{\min} \leq Y(t) \leq Y_{\max}. \quad (7)$$

The values of  $Y_{\max}$  and  $Y_{\min}$  depend on the size and type of battery. The dynamics of SOB due to recharging and discharging activities are modeled as

$$Y(t+1) = Y(t) + Q(t) + S_2(t) - F(t). \quad (8)$$

We take into account the costs of battery recharging and discharging, which are denoted by  $C_{rc}$  and  $C_{dc}$ , respectively. They are determined by the battery lifetime characteristics and the price of the battery. Note that since there are two possible sources  $Q(t)$  and  $S_2(t)$  for recharging, the cost of  $C_{rc}$  will be paid if either of them is positive. Finally, we define two indicator functions for recharging and discharging activities:  $1_R(t) = \{1 : \text{if } Q(t) + S_2(t) > 0; 0 : \text{otherwise}\}$  and  $1_D(t) = \{1 : \text{if } F(t) > 0; 0 : \text{otherwise}\}$ .

### 2.3. Power Demand Balancing

At each time slot  $t$ , required energy from different sources that need to be determined are  $P(t)$ ,  $Q(t)$ ,  $F(t)$ , and  $S_2(t)$ . From the supply-demand relationship shown in Fig. 1, these quantities must meet the user's demand as

$$W(t) = P(t) - Q(t) + S_1(t) + F(t). \quad (9)$$

The cost incurred to the system at each time slot  $t$  includes the power purchase cost from the grid and the cost of battery recharging/discharging for energy storage or consumption, defined as  $J(t)$ , given by

$$J(t) = P(t)C(t) + 1_R(t)C_{rc} + 1_D(t)C_{dc}. \quad (10)$$

## 3. ENERGY MANAGEMENT CONTROL ALGORITHM

At each time slot, we assume  $C(t)$ ,  $W(t)$  and  $S(t)$  are known. Our objective is to design a control policy  $\pi(t) = \{P(t), Q(t), F(t), S_2(t)\}$  to minimize the long-term time-averaged cost, provided that all the constraints given in Section 2 are satisfied. The optimization problem can be described as follow

$$\mathbf{P1} : \min_{\pi(t)} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{J(t)\} \quad (11)$$

s.t. (1), (6), (9), and

$$0 \leq S_2(t) \leq \min\{S(t) - S_1(t), R_{\max}, Y_{\max} - Y(t)\} \quad (12)$$

$$0 \leq Q(t) + S_2(t) \leq \min\{R_{\max}, Y_{\max} - Y(t)\} \quad (13)$$

$$0 \leq F(t) \leq \min\{D_{\max}, Y(t) - Y_{\min}\} \quad (14)$$

where  $\mathbb{E}\{\cdot\}$  is taken with respect to  $S(t)$  and  $W(t)$ . Note that constraints (12)-(14) are the results of combining (3)-(5) with (7).

Note that if the distributions of  $W(t)$ ,  $C(t)$  and  $S(t)$  are known, it is possible to solve the optimization problem **P1** through Dynamic Programming, of which we have to face the curse of dimensionality in terms of complexity. Instead, we are interested in designing an online control policy that does not rely on the statistics of  $W(t)$ ,  $C(t)$  and  $S(t)$ . To do this, we adopt Lyapunov optimization technique [5] to obtains a sub-optimal solution while satisfying all constraints in **P1**. The Lyapunov approach provides an on-line algorithm for control decision  $\pi(t)$  with given system input  $W(t)$ ,  $C(t)$ , and  $S(t)$  at time slot  $t$ . Furthermore, we can bound its performance gap to the optimal solution by a system design parameter.

### 3.1. Online Control Policy via Lyapunov Optimization

Due to the dependency of  $\{Q(t), S_2(t), F(t)\}$  on the SOB  $Y(t)$  in constraints (12)-(14), **P1** is difficult to solve. We first relax these constraints to a long-term time-averaged relation between  $Q(t)$ ,  $S_2(t)$  and  $F(t)$ . It can be shown that the following condition holds

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q(t) + S_2(t) - F(t)\} = 0. \quad (15)$$

We modify **P1** to the following problem with relaxed constraints.

$$\begin{aligned} \mathbf{P1}_r : \quad & \min_{\pi(t)} \lim_{t \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{J(t)\} \\ \text{s.t.} \quad & (1), (4), (5), (6), (9), (15) \text{ and} \\ & 0 \leq S_2(t) \leq \min\{S(t) - S_1(t), R_{\max}\}. \end{aligned} \quad (16)$$

Note that, constraints (12)-(14) in **P1** are replaced by (4), (5), (15), and (16) in **P1<sub>r</sub>**, which remove the dependence on  $Y(t)$ . Note that since the constraints are relaxed, solving **P1<sub>r</sub>** will not give a feasible solution to **P1**.

Define  $X(t)$  as a virtual queue

$$X(t) \triangleq Y(t) - VC_{\max} - D_{\max} - Y_{\min} \quad (17)$$

where  $V > 0$  is a constant to be explained later. Note that  $X(t)$  is a shifted version of  $Y(t)$  which can be negative. Due to (8), the dynamic of  $X(t)$  is given by

$$X(t+1) = X(t) + Q(t) + S_2(t) - F(t). \quad (18)$$

Define the Lyapunov function for  $X(t)$  as  $L(X(t)) \triangleq X^2(t)/2$ , and the conditional Lyapunov drift [5] for  $X(t)$  at time  $t$  as  $\Delta X(t) \triangleq \mathbb{E}\{L(X(t+1)) - L(X(t)) | X(t)\}$ . With (9), we can show that  $\Delta X(t)$  is bounded by

$$\Delta X(t) \leq B - X(t) \mathbb{E}\{W(t) - P(t) - S_1(t) - S_2(t) | X(t)\}$$

where  $B \triangleq \max\{D_{\max}^2, R_{\max}^2\}/2$ .

The Lyapunov approach intends to minimize a drift-plus-penalty metric. The drift-plus-penalty is expressed as a weighted sum of the Lyapunov drift and the expected cost, defined as  $U(t) \triangleq \Delta X(t) + V \mathbb{E}\{J(t)\}$ , where  $V$  serves as a weighting factor providing the relative weight between the cost and the drift in the metric. Using the bound above for  $\Delta X(t)$ , we have the bound on the drift-plus-penalty as

$$\begin{aligned} U(t) & \leq B - X(t) \mathbb{E}\{W(t) - P(t) - S_1(t) - S_2(t) | X(t)\} \\ & \quad + V \mathbb{E}\{J(t)\}. \end{aligned} \quad (19)$$

We design our online control algorithm to minimize the upper bound of the drift-plus-penalty  $U(t)$  in (19), with given system states  $\{W(t), C(t), S(t), X(t)\}$  at time  $t$ . The resulting minimization problem can be shown to have the following expression

$$\begin{aligned} \mathbf{P2} : \quad & \min_{\pi(t)} X(t)[P(t) + S_2(t)] + VJ(t) \\ \text{s.t.} \quad & (1), (4), (5), (6), (9), \text{ and } (16). \end{aligned} \quad (20)$$

We will show later that the solution to **P2** will meet the constraints (12)-(14) of **P1**.

As shown in (6), the battery is either in a recharging/discharging state, i.e.,  $1_R(t) + 1_D(t) = 1$ , or in an idle state, i.e.,  $1_R(t) = 1_D(t) = 0$ . Let  $\xi(t)$  denote the value of the objective in **P2** when battery is in the idle state, then we have

$$\xi(t) = [W(t) - S_1(t)][X(t) + VC(t)].$$

By analyzing the objective and constraints in the problem **P2**, we arrive at the following optimal solution  $\{P^*(t), S_2^*(t), Q^*(t), F^*(t)\}$ :

1) When  $X(t) < X(t) + VC(t) \leq 0$ : Let

$$\begin{cases} F'(t) = 0 \\ S_2'(t) = \min\{S(t) - S_1(t), R_{\max}\} \\ Q'(t) = \min\{R_{\max} - S_2'(t), P_{\max} - W(t) + S_1(t)\} \\ P'(t) = \min\{W(t) + R_{\max} - S_1(t) - S_2'(t), P_{\max}\}. \end{cases} \quad (21)$$

If  $P'(t)[X(t) + VC(t)] + S_2'(t)X(t) + VC_{rc} < \xi(t)$ , then  $\{F^*(t), S_2^*(t), Q^*(t), P^*(t)\} = \{F'(t), S_2'(t), Q'(t), P'(t)\}$ ; Otherwise,  $F^*(t) = S_2^*(t) = Q^*(t) = 0, P^*(t) = W(t) - S_1(t)$ .

2) When  $X(t) < 0 \leq X(t) + VC(t)$ : Let

$$\begin{cases} F'(t) = \min\{W(t) - S_1(t), D_{\max}\} \\ S_2'(t) = \min\{S(t) - S_1(t), R_{\max}\} \\ Q'(t) = 0 \\ P'(t) = [W(t) - S_1(t) - D_{\max}]^+ \end{cases} \quad (22)$$

If  $P'(t)[X(t) + VC(t)] + S_2'(t)X(t) + V(1_R(t)C_{rc} + 1_D(t)C_{dc}) < \xi(t)$ , then  $\{F^*(t), S_2^*(t), Q^*(t), P^*(t)\} = \{F'(t), S_2'(t), Q'(t), P'(t)\}$ ; Otherwise,  $F^*(t) = S_2^*(t) = Q^*(t) = 0, P^*(t) = W(t) - S_1(t)$ .

3) When  $0 \leq X(t) < X(t) + VC(t)$ : Let

$$\begin{cases} F'(t) = \min\{W(t) - S_1(t), D_{\max}\} \\ S_2'(t) = Q'(t) = 0 \\ P'(t) = [W(t) - S_1(t) - D_{\max}]^+. \end{cases} \quad (23)$$

If  $P'(t)[X(t) + VC(t)] + VC_{dc} < \xi(t)$ , then  $\{F^*(t), S_2^*(t), Q^*(t), P^*(t)\} = \{F'(t), S_2'(t), Q'(t), P'(t)\}$ ; Otherwise,  $F^*(t) = S_2^*(t) = Q^*(t) = 0, P^*(t) = W(t) - S_1(t)$ .

Note that due to  $S_1(t)$  in (2), the expressions in (22) for  $F'(t)$  and  $S_2'(t)$  imply that  $F'(t) \cdot S_2'(t) = 0$ , i.e., the battery is either in recharging or discharging state, but not both.

The above solution can be intuitively explained as follows: case 1) corresponds to the state when energy stored in the battery is relatively low, and (21) reflects the incentive to recharge the battery, provided that the recharging cost  $C_{rc}$  is not high. To the opposite, case 3) indicates the scenario when the energy stored in the battery is high, and there is an incentive to discharge the battery to supply energy if  $C_{dc}$  is not high. Case 2) corresponds to the case when the battery is moderately charged, and recharging or discharging is only based on the balance between supply  $S(t)$  and demand  $W(t)$ .

### 3.2. Performance of the Online Control Policy

We first show that the online control policy developed in Section 3.1 meets the constraints of the original problem.

**Proposition 1.** *The optimal control policy  $\pi^*(t)$  for the problem **P2** is a feasible policy for the problem **P1**.*

*Proof:* It is easy to verify that  $Q^*(t)$ ,  $S_2^*(t)$ , and  $F^*(t)$  satisfy constraints (4) and (5), respectively. Moreover, if we can prove (7) holds, then as a direct result,  $Q^*(t)$ ,  $S^*(t)$ , and  $F^*(t)$  will meet constraints (12) and (14), respectively. Let  $V$  in (17) be upper bounded by  $V_{\max}$ , where  $V_{\max} = \frac{Y_{\max} - Y_{\min} - R_{\max} - D_{\max}}{C_{\max}}$ . Then, for any fixed  $V \in (0, V_{\max}]$  at time slot  $t$ , we have the following bounds for  $X(t)$ :  $-VC_{\max} - D_{\max} \leq X(t) \leq Y_{\max} - Y_{\min} - D_{\max} - VC_{\max}$ . Combining the above and (17), we have  $Y(t)$  satisfies (7) at each time  $t$ . Therefore, control decisions  $P^*(t)$ ,  $Q^*(t)$ ,  $S_2^*(t)$ , and  $F^*(t)$  are feasible for the original problem **P1**. ■

Next, we show that under the i.i.d. assumption of system inputs, the performance of the online control policy is bounded from that of the optimal policy as follow.

**Proposition 2.** Assume  $W(t)$ ,  $C(t)$ , and  $S(t)$  i.i.d. over time slot  $t$ . Under the online control policy  $\pi^*(t)$  given in Section 3.1, the resulting long-term time averaged cost is bounded from the optimal objective value  $\xi^o$  in the problem **P1** by

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{J(t)\} \leq \xi^o + B/V. \quad (24)$$

*Proof:* We adopt the Lyapunov optimization technique in [5] to derive the above bound. Details omitted. ■

#### 4. SIMULATION RESULTS

To realistically set the price  $C(t)$ , we use the data collected from Ontario Energy Board [9], where  $C(t)$  consists of three-stage prices and is periodic every 24 hours. Fig. 2 shows the value of  $C(t)$  within the period of 24 hours, where the three-stage prices are given as  $C_h = \$0.118$ ,  $C_m = \$0.099$ , and  $C_l = \$0.063$ . We set the time slot duration to be 10 mins, and approximate the renewable energy  $S(t)$  and user demand  $W(t)$  within each slot to be constant. We generate  $S(t)$  and  $W(t)$  per slot using uniform distribution within interval  $[0.1/6 \text{ kWh}, 1.5/6 \text{ kWh}]$  and  $[0.5/6 \text{ kWh}, 2/6 \text{ kWh}]$ , respectively. Other parameters are chosen as follows:  $R_{\max} = 1/3 \text{ kWh}$ ,  $D_{\max} = 1/3 \text{ kWh}$ ,  $P_{\max} = 1/3 \text{ kWh}$ ,  $C_{rc} = C_{dc}$ , and  $V = V_{\max} = \frac{Y_{\max} - Y_{\min} - R_{\max} - D_{\max}}{C_h}$ .

First, we look at the effect of renewable energy on the cost saving. We assume  $Y_{\max} = 3 \text{ kWh}$ , and  $Y_{\min} = 0.2Y_{\max}$ . As shown in Fig. 3, the integration of renewable energy offers about 70% off in the total cost that a user needs to pay in a grid-only system (i.e., the traditional grid is the only energy source). The benefit is mainly contributed by a deduction from grid cost.

For the same battery capacity, in Fig. 4, we study the battery recharging/discharging activities by plotting the number of recharges (and/or discharges) vs.  $C_{rc}$  (or  $C_{dc}$ ). We compare the performance of the system with or without renewable energy. As shown, with the renewable energy, higher cost associated with recharging/discharging becomes more tolerable for long-term cost minimization. For example, in a grid-only system, the control decision suggests no recharging (discharging) actions at a cost of  $C_{rc}$  (or  $C_{dc}$ )  $> \$0.008$ , while this cost becomes “affordable” when the renewable energy is added, reflected by the positive number of recharges/discharges. In general, the total number of recharges and discharges decreases when the cost  $C_{rc}$  (or  $C_{dc}$ ) increases. Thus, the battery recharging/discharging cost directly affect the battery participation, and thus the effectiveness of the energy management system.

Finally, in Fig. 5, we show how the battery capacity affects the relative proportions of purchased power from grid into the battery at different prices. We plot the total amount of purchased energy at a specific price that is charged into the battery,  $E_Q(C_i) \triangleq \sum_{t \in \{t: C(t)=C_i\}} Q^*(t)$ , for  $C_i \in \{C_l, C_m, C_h\}$ . Intuitively, as the capacity increases, the optimal decision will let the battery buy energy from the grid only if  $C(t)$  is the cheapest. For our control policy, as  $Y_{\max}$  increases,  $V$  increases, and the performance approaches to the optimal one as indicated in Proposition 2. This result is verified on Fig. 5, where we see when  $Y_{\max}$  is large, most energy purchased from the grid is when  $C(t) = C_l$ .

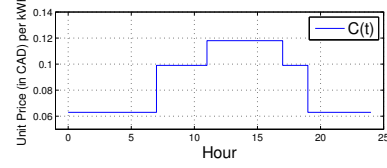


Fig. 2. Power grid real-time price  $C(t)$ .

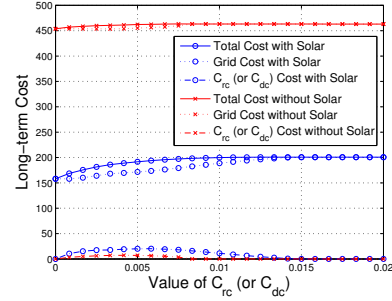


Fig. 3. Long-term averaged cost vs.  $C_{rc}$  (or  $C_{dc}$ )

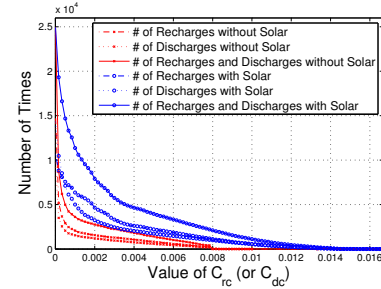


Fig. 4. # of recharges (discharges) vs.  $C_{rc}$  (or  $C_{dc}$ )

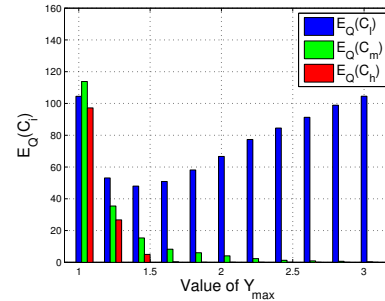


Fig. 5. Proportion of total stored energy purchased from the grid at different price  $E_Q(C_i)$  vs. battery capacity

#### 5. CONCLUSION

In this work, we proposed an online control policy to minimize the long-term time-averaged cost for energy storage management with renewable energy integration. We incorporated the system dynamics and the battery operation cost in the problem formulation, and applied Lyapunov optimization technique to design an online control policy with a bounded performance from the optimal scheme. Our control decision was derived in closed-form resulting in minimum implementation complexity. Simulations showed the effectiveness of integrating renewable energy for energy storage in reducing the long-term cost as well as improving the efficiency of energy storage relative to the battery operation cost.

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