DISTRIBUTED FREQUENCY CONTROL VIA DEMAND RESPONSE IN SMART GRIDS

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ABSTRACT

Frequency control is essential to maintain the stability and reliability of power grids. For decades, generation side controllers, e.g., isochoronous governors and automatic generation controllers, have been used to stabilize the frequency of power systems, which, however, incur high operational costs. In smart grids, demand response can be used to control frequency and thus reduce the grids' dependency on expensive controllers. Despite of its economic advantages, the synchronization problem, which is due to the simultaneous responses of smart appliances, becomes the main barrier to implementing frequency responsive demand control in reality. In this paper, we propose a distributed control algorithm for smart appliances, based on the randomized frequency monitoring and a baseline hysteresis algorithm, to solve the synchronization problem. We provide analytical results to characterize the influence of distributed demand response on the system frequency dynamics. Finally, we validate our analysis and demonstrate the effectiveness of our proposed algorithm via simulations on the Ireland power system.

***Index Terms*—** Smart grid, demand response, frequency control, distributed algorithm.

1. INTRODUCTION

Through computer-based control and automation, smart grid offers technical, economic and environmental advantages by improving reliability, lowering operational costs, and reducing greenhouse gas emissions of power systems. In particular, demand response is an effective mechanism to manage users' electricity consumptions in response to supply conditions, e.g., reducing power consumptions at critical times or varying them in response to market prices [1]. In this paper, we focus on exploring demand response for frequency control, a kind of emergency service for the system operator during system failures and/or unplanned outages when the system frequency deviates from its nominal value. For instance, when the system loses part of its generation power, or the demand increases drastically, the system frequency will drop below its nominal value and the system experiences an under-frequency event. To restore to the nominal frequency, traditional controllers, e.g., isochoronous governors and automatic generation controllers are used to automatically increase the output power of generation units [2]. However, employing these controllers is very costly, since the system operator needs to procure expensive regulatory and spinning reserve services [3]. Besides the generation side control, smart appliances in the demand side can also be used to restore the frequency to its nominal value [4], by detecting frequency deviation and shedding their loads accordingly. Using demand response to recover the system frequency is more advantageous than traditional controllers by two main reasons. First, traditional controllers recover the system frequency in tens of seconds, while demand response reduces this time to a couple of seconds [5]. Second, demand response reduces the required amount of reserves and thus the system operational costs [4].

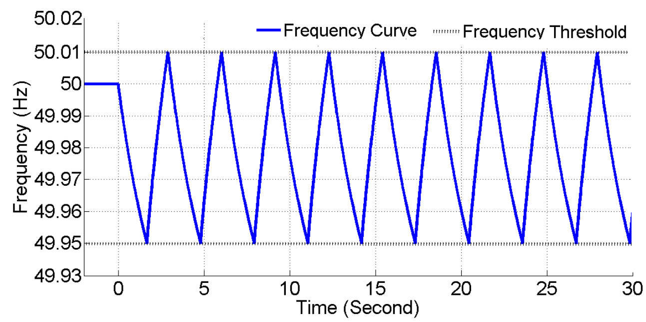

Fig. 1: Synchronization problem.

Despite of the advantages of frequency control via demand response, a *synchronization* problem needs to be addressed. The synchronization problem refers to simultaneous responses of appliances upon a contingency which could potentially lead to the system frequency oscillations. In order to illustrate this problem, we consider the algorithm reported by Pacific Northwest National Laboratory [6], and implement it on the Ireland power system (for which the more details will be given in Section 6). In this algorithm, smart appliances continuously monitor the system frequency and shed their loads when the system frequency drops below a lower threshold, i.e., 49.95Hz, and remain in the off state until the system frequency goes up to an upper threshold, i.e., 50.01Hz. Fig. 1 shows the system frequency dynamics after a contingency, from which the frequency oscillation is observed. This phenomenon is due to the fact that all appliances sense and

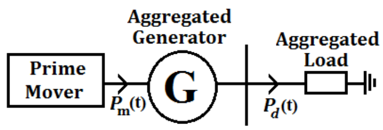

Fig. 2: Schematic of an aggregated power system model.

respond at the same time when the frequency reaches each of the two frequency thresholds. In this paper, we propose a novel distributed control algorithm to tackle the synchronization problem by randomizing the responses of smart appliances. It is worth noting that a handful of randomized algorithms have been investigated in the literature, e.g., [7]-[9]; however, these works are all based on simulations while no analytical results on the algorithm performance are provided therein. To implement randomized demand responses in practice, we need to choose appropriate randomization parameters given the system of interest, which is not feasible without a rigorous analysis on the system frequency dynamics.

2. FREQUENCY DYNAMICS IN POWER GRID

In this paper, we study the power system dynamics under the so-called *synchronous* operating regime where the whole system operates under a single system-wide frequency, even during contingencies [10]. Therefore, we can model the power system in an aggregated form, as illustrated in Fig. 2, where $P_m(t)$ and $P_d(t)$ represent the aggregated mechanical power from the prime movers and demand power consumption, respectively. Let $f(t)$ denote the system frequency with the nominal value of f_0 at time $t = 0$. According to the introduced model in [10], we have

$$P_d(t) = P_a(t) + \left(\frac{f(t) - f_0}{f_0}\right) K_f P_0, \tag{1}$$

where $P_a(t)$ represents the aggregated demand power consumption under the nominal frequency and voltage with an initial value of P_0 at time $t = 0$. The second term on the right hand side of (1) represents the demand power change due to the system frequency deviation, where K_f represents the frequency damping coefficient that measures the sensitivity [12].

At the generation side, the system dynamics are governed by the physics of motion and described by a so-called *swing equation*, which is given by [10]:

$$\frac{2H}{f_0}\frac{df(t)}{dt} = P_m(t) - P_d(t), \tag{2}$$

where H denotes the stored energy in the generator in Joule. By substituting (1) into (2), we obtain

$$\frac{2H}{f_0}\frac{df(t)}{dt} = P_m(t) - P_a(t) - \left(\frac{f(t) - f_0}{f_0}\right) K_f P_0. \tag{3}$$

Next, we consider the frequency dynamics upon a system contingency, during which the generation power and demand become imbalanced. Without loss of generality, we assume that the generator loses A_0 amount of mechanical power at time $t = 0$, which is a typical scenario in practice. Suppose that the aggregated demand power consumption under the nominal situation is not changed over time, i.e., appliances do not respond by shedding themselves. We thus have $P_m(t) - P_a(t) = -A_0 \mathbf{1}_{\{t>0\}}$, where $\mathbf{1}_{\{\cdot\}}$ denotes an indicator function. Consequently, (3) can be re-written as

$$\frac{2H}{f_0}\frac{df(t)}{dt} = -A_0 - \left(\frac{f - f_0}{f_0}\right) K_f P_0, \;\; t \geq 0. \tag{4}$$

By solving the above differential equation, we obtain

$$f(t) = f_0 - \frac{A_0 f_0}{K_f P_0}(1 - e^{-\alpha t}), \; t \geq 0, \tag{5}$$

where $\alpha = (K_f P_0)/(2H)$. The above frequency equation indicates that a generation power loss of A_0 eventually results in an $(A_0 f_0)/(K_f P_0)$ amount of frequency loss in steady state; and it changes the system frequency exponentially fast. Note that in (5), we have assumed $A_0/(K_f P_0) < 1$ since in practice $A_0 \ll P_0$. For convenience, we define $g_\alpha(A, s)$ to quantify the effect of generation capacity loss on the system frequency, i.e.,

$$g_\alpha(A, s) \triangleq \frac{f_0}{K_f P_0} A(1 - e^{-\alpha s}), \;\; s \geq 0. \tag{6}$$

Thus, without considering the response of appliances, the system frequency in (5) is expressed as $f(t) = f_0 - g_\alpha(A_0, t)$.

3. DEMAND RESPONSE AND FREQUENCY CONTROL

In this section, we introduce our distributed frequency control algorithm for frequncy responsive smart appliances.

3.1. Hysteresis Algorithm

We consider a power system consisting of M smart appliances denoted by the set $\mathcal{M} = \{1, \cdots, M\}$, with power consumption of A_i for the ith appliance, $i = 1, \ldots, M$. It is assumed that all smart appliances detect frequency deviations in a distributed manner and respond independently based on a hysteresis algorithm similar to that proposed in [7, 13], which is described as follows. Let $S_i(t) \in \{0, 1\}$ denote the state of appliance i at time t, where 0 and 1 indicate the off and on states, respectively. The hysteresis algorithm with a given pair of frequency lower and upper thresholds, denoted by $f_{\min} \leq f_0$ and $f_{\max} \geq f_0$, respectively, is given by

$$S_i(t^+) = \begin{cases} 0 & \text{if } S_i(t) = 1 \text{ and } f(t) < f_{\min}; \\ 1 & \text{if } S_i(t) = 0 \text{ and } f(t) > f_{\max}; \\ S_i(t) & \text{otherwise.} \end{cases} \tag{7}$$

where t^+ denotes the time immediately after monitoring the system frequency at time t. Therefore, each appliance will introduce a positive (negative) effect on the system frequency when it switches from an on (off) state to an off (on) state.

3.2. Randomized Inter-Response Time

Existing works [6]-[9] have assumed that all smart appliances continuously monitor the system frequency, which cause the synchronization problem as shown in Fig. 1. In our design, we let each appliance i monitor and respond to frequency deviations according to a discrete-time sequence $\mathbf{t}_i = (t_i^1, t_i^2, \cdots)$, where t_i^j denotes the jth response time of appliance i. Thus, the behavior of appliance i can be tracked by a continuous-time counting process $\{N_i(t) : t \geq 0\}$, defined by $N_i(t) = \sum_{j=1}^{\infty} \mathbf{1}_{\{t_i^j < t\}}$, where the indicator function $\mathbf{1}_{\{t_i^j < t\}}$ indicates whether the jth response of appliance i occurs before time t or not. Furthermore, to desynchronize responses, we impose the constraint that each appliance must wait a random time between any two consecutive responses. Specifically, we define the jth inter-response time of appliance i as $T_i^j = t_i^j - t_i^{j-1}$, where $t_i^0 \triangleq 0$ by default. Then, we design the inter-response time $\{T_i^j : j = 1, 2, \cdots\}$ for appliance i to be independent and identically distributed (i.i.d.) exponential random variables with mean λ_i^{-1}. As a result, $N_i(t)$ becomes a Poisson process with the rate of λ_i. Thus, (5) is modified as

$$f(t) = f_0 - g_\alpha(A_0, t) + \sum_{i=1}^{M} \sum_{j=1}^{N_i(t)} X(t_i^j) g_\alpha(A_i, t - t_i^j). \quad (8)$$

Now, we explain the impact of the generation power loss A_0 and the aggregated power consumption of smart appliances, denoted by $A_a = \sum_{i=1}^{M} A_i$, on the system frequency. First, consider the effect of A_0 on $f(t)$ by defining

$$\overline{A}_0 \triangleq \frac{K_f P_0}{f_0}(f_0 - f_{\min}). \quad (9)$$

Form (5) and (7), it follows that if $A_0 > \overline{A}_0$, smart appliances will respond by shedding loads after the system frequency drops below $f_{\min}$ at time $t_0 > 0$, which is given by

$$t_0 = -\frac{1}{\alpha} \ln\left(1 - \frac{\overline{A}_0}{A_0}\right). \quad (10)$$

Next, we study the effect of A_a on $f(t)$ by defining

$$A_{\min} \triangleq A_0 - \overline{A}_0. \quad (11)$$

From (8), it follows that if $A_a \geq A_{\min}$, the system frequency recovers back to $f_{\min}$ at a certain time $T_r \geq t_0$, which is termed the *frequency recovery time* and is defined as the smallest $t > t_0$ that solves the frequency equation $f(t) = f_{\min}$.

4. MEAN AND VARIANCE ANALYSIS OF THE SYSTEM FREQUENCY

With the randomized frequency control algorithm proposed in the previous section, the system frequency given in (8) is a random process over time. To characterize $f(t)$ for $t \geq t_0$, we derive its mean and variance as a function of time with the given system parameters. Due to the space limitations, we provide the results without the proofs, which will be given in the journal version of this paper.

Proposition 4.1 *Given $A_0 > \overline{A}_0$ and $A_a \geq A_{\min}$, the mean value of the system frequency for $t_0 \leq t \leq T_r$ is given by*

$$E[f(t)] = f_0 - g_\alpha(A_0, t) + \sum_{i=1}^{M} g_\alpha^i(A_i, t - t_0), \quad (12)$$

where $g_\alpha^i(A_i, s)$ is given by

$$g_\alpha^i(A_i, s) = \frac{f_0}{K_f P_0} A_i (1 - h(\alpha, \lambda_i, s)), \quad (13)$$

with $h(\alpha, \lambda, s)$ defined as

$$h(\alpha, \lambda, s) \triangleq \begin{cases} \dfrac{\lambda e^{-\alpha s} - \alpha e^{-\lambda s}}{\lambda - \alpha} & \text{if } \lambda \neq \alpha \text{ and } s \geq 0; \\ (\lambda s + 1) e^{-\lambda s} & \text{if } \lambda = \alpha \text{ and } s \geq 0. \end{cases} \quad (14)$$

The mean frequency given in (12) is due to both the deterministic frequency dynamics without demand response and that contributed by responses of all smart appliances, where $g_\alpha^i(A_i, t - t_0)$ denotes the contribution of smart appliance $i \in \mathcal{M}$. The contribution of each appliance to frequency recovery takes effect only after t_0 given in (10). This is due to the fact that prior to t_0, although smart appliances monitor the system frequency, they do not respond by shedding themselves since $f(t) > f_{\min}$. From $g_\alpha^i(A_i, s)$ given in (13), it follows that the frequency restoration contribution of appliance i is proportional to its power consumption A_i, but discounted over time by a factor of $h(\alpha, \lambda_i, s)$.

Proposition 4.2 *Given $A_0 > \overline{A}_0$ and $A_a \geq A_{\min}$, the variance of the system frequency for $t_0 \leq t \leq T_r$ is given by*

$$Var(f(t)) = \sum_{i=1}^{M} \left(\frac{f_0 A_i}{K_f P_0}\right)^2 v(\alpha, \lambda_i, t - t_0), \quad (15)$$

where $v(\alpha, \lambda, s) \triangleq h(2\alpha, \lambda, s) - h^2(\alpha, \lambda, s)$.

Proposition 4.2 characterizes the variance of the system frequency as a function time t, which is the weighted sum of individual appliances' variance contributions $v(\alpha, \lambda_i, t - t_0)$.

5. MEAN RECOVERY TIME AND EXPECTED NUMBER OF RESPONDED APPLIANCES

In the section, we reveal how the different response rates of smart appliances will impact on the frequency recovery time as well as the number of appliances that shed loads during a contingency.

First, we characterize the mean frequency recovery time $\overline{T}_r = E[T_r]$ by utilizing the frequency mean function $E[f(t)]$

obtained in Proposition 4.1. This is justified since in practical power systems, although there are many smart appliances, their individual power consumptions are usually much smaller than the aggregated system load; therefore, the variance of the system frequency given in (15) is practically very small and thus can be safely ignored in our analysis . Hence, we have the following proposition.

Proposition 5.1 *Given $A_0 > \overline{A}_0$ and $A_a \geq A_{\min}$, the mean frequency recovery time $\overline{T}_r$ is approximately equal to the smallest $t > t_0$ which is the solution of the frequency equation: $E[f(t)] = f_{\min}$, where $E[f(t)]$ is given in (12).*

Next, we investigate the expected number of responded smart appliances upon frequency recovery. For the convenience of analysis, we further assume that the power system consists of a set of different classes of smart appliances, denoted by $\mathcal{C} = \{1, \ldots, C\}$. Let M_c, A_c, and λ_c be the number of appliances in Class $c \in \mathcal{C}$, the power consumption and response rate of each appliance in Class c, respectively. To be consistent with our previous notation, we consider $\sum_{c=1}^{C} M_c = M$. Moreover, let $N_c(t)$ denote the number of appliances in Class $c \in \mathcal{C}$ which have responded by shedding themselves by time $t \geq t_0$. We then state our result in the following proposition.

Proposition 5.2 *Given $A_0 > \overline{A}_0$ and $A_a \geq A_{\min}$, the expected number of responded appliances from Class $c \in \mathcal{C}$ for $t_0 \leq t \leq T_r$ is given by*

$$E[N_c(t)] = M_c(1 - e^{-\lambda_c(t-t_0)}). \tag{16}$$

From Proposition 5.2, it immediately follows that the expected total amount of demand shed in Class $c \in \mathcal{C}$ at time $t_0 \leq t \leq T_r$ is $M_c A_c(1 - e^{-\lambda_c(t-t_0)})$.

6. SIMULATION RESULTS

W consider the Ireland power system [14] with a peak power of $P_0 = 5000$MW, the nominal frequency $f_0 = 50$Hz, frequency coefficient $K_f = 2.5$, and $\alpha = 0.4$. Suppose there are two classes of smart appliances: Class 1 consists of $M_1 =$

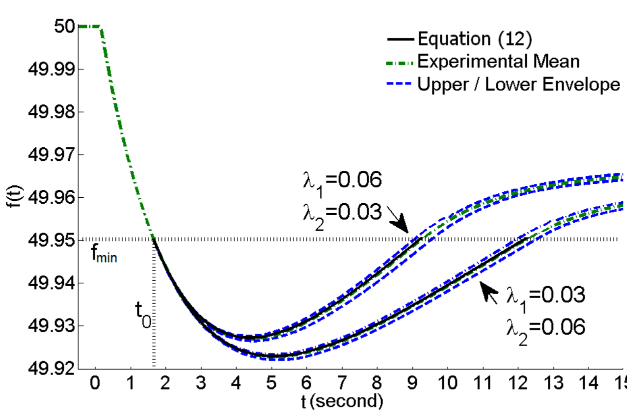

Fig. 3: Simulated system frequency dynamics.

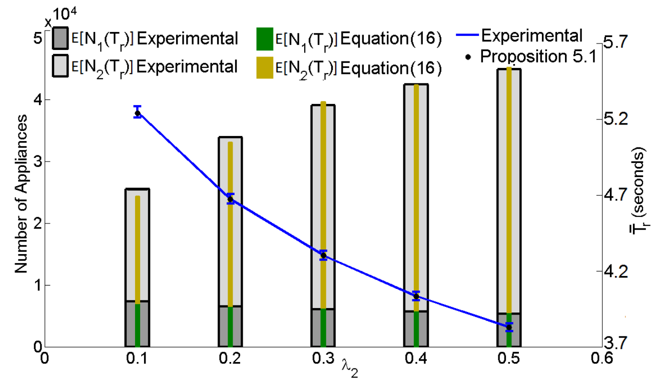

Fig. 4: Recovery time and number of responded appliances.

20000 water heaters and/or cloth dryers with a typical power consumption $A_1 = 2$KW and a response rate λ_1; and Class 2 consists of $M_2 = 50000$ refrigerators and/or freezers with a typical power consumption $A_2 = 200$W and a response rate λ_2. We set the frequency thresholds of the hysteresis algorithm as $f_{\min} = 49.95$Hz and $f_{\max} = 50.01$Hz. Suppose that the power system experiences a generation power loss of $A_0 = 25$MW at $t = 0$, which may correspond to e.g. the failure of a large wind farm, e.g., Gruig plant in Antrim.

We simulate the system frequency dynamics after the generation power loss with $(\lambda_1, \lambda_2) = (0.03, 0.06)$ or $(0.06, 0.03)$. We perform the simulation for each of the two settings and plot the mean frequency curve as well as the upper and lower extreme values of the frequency in Fig. 3. We observe that for both settings, the system frequency is recovered in less than 13 seconds. It is also observed that the analytical frequency mean based on Proposition 4.1 closely matches the experimental mean.

Next, with fixed $\lambda_1 = 0.1$ and by varying λ_2 from 0.1 to 0.5, we verify the mean frequency recovery time $\overline{T}_r$ and the expected number of responded appliances $E[N_1(T_r)]$ and $E[N_2(T_r)]$ given in Propositions 5.1 and 5.2 by simulations. As shown in Fig. 4, the simulation results closely fit to our analytical results.

7. CONCLUSION

In this paper, we proposed a randomized demand response algorithm to help the main grid stabilize the system frequency during contingencies. The proposed algorithm is completely distributed in smart grids, and thus does not require any centralized control or overlaid communication infrastructure. We develop a theoretical framework to analyze the system frequency dynamics based on the aggregated power system model. In particular, we derive the closed-form mean and variance of the system frequency as a function of time given the smart appliances' response rates. Based on these results, we characterize the mean recovery time and the expected number of responded appliances. We hope that our results will provide useful guidance to implement distributed demand response for reliable frequency control in power systems.

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