DISTRIBUTED DEMAND-SIDE OPTIMIZATION WITH LOAD UNCERTAINTY

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ABSTRACT

Demand-side management will play a crucial role in balancing the energy generation and demand in future smart grids. In this paper, game-theoretic demand-side management algorithms are proposed for energy consumption scheduling under load uncertainty. The demand-side optimization and scheduling problem is formulated as a noncooperative cost minimization game among the end-users and an iterative algorithm that averages over the load uncertainty is proposed for solving it. The proposed algorithm is proven to converge to a Nash equilibrium. Simulation results show that taking into account the uncertainty in the load reduces significantly the load peak-to-average ratio and the hourly variation of the aggregate load profile.

Index Terms— Distributed demand-side management, load uncertainty, noncooperative game theory, smart grid.

1. INTRODUCTION

Demand-side management is based on the premise of giving the consumers an incentive to alter their energy usage by offering lower electricity prices at off-peak hours. The goal is to shift the load from peak hours to off-peak hours, thus, providing a smoother and more even aggregate hourly load profile. This facilitates utility's task of dispatching generation to match the demand and reduces the overall cost of generating electricity. In this paper, we propose a distributed game-theoretic cost minimization based demand-side optimization and load scheduling method that takes into account load uncertainty.

Game-theoretic and other distributed demand-side management schemes have been proposed in [1–5]. A recent review of game theory-based demand-side management methods can be found in [6]. However, most of the above works assume that the end-users have full knowledge of their future load in the scheduling horizon. This is not a very realistic assumption. In this paper, we relax this assumption regarding the users' non-adjustable loads, such as refrigerators, freezers, TVs, computers, and various other domestic appliances and devices, by assuming that each user employs a prediction algorithm to predict its hourly aggregate non-adjustable load for the scheduling horizon. In particular, we assume that each predictor provides a set of samples from the non-adjustable load distribution, thus, providing also information about the uncertainty in the predicted load.

Distributed demand-side management with load uncertainty has been considered in [2]. Moreover, single user demand-side management with price uncertainty has been considered in [7]. In [2] the utility sets the amount of electricity provided to the customers and the electricity prices according to the customer demands and satisfaction while taking into account the uncertainty in the load. However, the customers are price takers and hence do not take into account the influence of their actions on the prices. In this paper, we extend the distributed demand-side management framework in [1] by introducing a method that takes into account the uncertainty in each user's non-adjustable hourly load. In this proposed method the users are price anticipators and thus they consider the influence of their actions on the overall cost.

The contributions of this paper are as follows. We formulate the demand-side optimization problem as a noncooperative electricity cost minimization game among the end-users. In our formulation the user payoffs depend on the expected value of the overall cost of all users. The proposed payoff functions average over the non-adjustable load uncertainty. We consider scheduling of two different types of adjustable loads: loads whose instantaneous power is freely adjustable within some interval and loads following a fixed operating and power cycle. We propose an iterative optimization algorithm for solving the game. We prove that it converges to a Nash equilibrium starting from any feasible initial loads. We provide simulation results showing that taking into account the uncertainty in the non-adjustable load reduces significantly the peak-to-average ratio (PAR) and variation of the aggregate daily load profile while at the same time reducing the cost for the end-users.

The paper is organized as follows. In Section 2 we formulate the noncooperative electricity minimization game, propose an iterative algorithm for solving it, and prove that the algorithm converges to a Nash equilibrium. Simulation results are presented in Section 3 and finally concluding remarks are given in Section 4.

2. DISTRIBUTED DEMAND-SIDE MANAGEMENT WITH LOAD UNCERTAINTY

We assume that each end-user has a set of non-adjustable and adjustable loads. Non-adjustable loads are loads whose instantaneous power or starting time cannot be adjusted. Loads belonging to this category are, e.g., refrigerator, freezer, oven, microwave and other cooking appliances, as well as TVs, computers and other entertainment devices. In addition to non-adjustable loads, we consider loads whose instantaneous power, starting time, or both can be adjusted. In particular, we consider two different types of adjustable loads:

1. Loads whose instantaneous power can be freely adjusted within some interval $[p_{\min}, p_{\max}]$. Examples of such loads are plug-in vehicle (PEV) charging and electric water heaters. These loads will denoted by $\mathcal{T}_1$.
2. Loads following a fixed operating cycle such that the instantaneous power is predetermined during operation, i.e., p_a^t is

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fixed for $t = 1, \ldots, T_a$ where T_a is the time it takes to complete the task. Thus, the only adjustable parameter is the starting time of the load. Examples of such loads are washing machines, dishwashers, and clothes dryers. These loads will be denoted by $\mathcal{T}_2$.

We assume that the cost of electricity for the end-users is proportional to the cost of generating the electricity for the utility, hence, it is in the interest of the end-users to minimize the generating cost. Moreover, we assume that the price each end-user pays is proportional to the amount of electricity it uses, i.e., the overall price charged by the utility is divided among the end-users according to the proportion of electricity each has used. The problem can be formulated as a noncooperative electricity cost minimization game consisting of (without any loss of generality we assume an hourly scheduling resolution throughout this paper):

- A finite set of end-users $n = 1, \ldots, N$.
- A set of energy usage strategies $\boldsymbol{X}_n$ for each end-user. A strategy $\boldsymbol{x}_n \in \boldsymbol{X}_n$ defines the adjustable load x_n^h of end-user n for each hour $h = 1, \ldots, H$. Let $\boldsymbol{x}_{-n}$ denote the strategies of all the other users except n.
- Payoff functions $\rho_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n})$, $n = 1, \ldots, N$, that define the user payoffs for the joint strategies:

$$\rho_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n}) = - E\left[\frac{\sum_{h=1}^{H}(x_n^h + l_n^h)}{\sum_{n=1}^{N}\sum_{h=1}^{H}(x_n^h + l_n^h)} \sum_{h=1}^{H} C_h\left(\sum_{n=1}^{N} x_n^h + l^h\right)\right], \tag{1}$$

where l_n^h is the non-adjustable load of user n for hour h, $l^h = \sum_{n=1}^{N} l_n^h$, and $C_h(\cdot)$, $h = 1, \ldots, H$, are convex functions, e.g., piecewise linear or quadratic functions, representing the cost of generating the electricity for each hour.

We assume that each user has a predetermined amount of adjustable load to schedule for each particular day, i.e., $\sum_{h=1}^{H} x_n^h$ is predetermined for all n. Hence, the users cannot influence the value of $\sum_{h=1}^{H}(x_n^h + l_n^h)/\sum_{n=1}^{N}\sum_{h=1}^{H}(x_n^h + l_n^h)$ with the choice of their strategy. Consequently, in order to maximize their payoff the goal of the end-users is to minimize the expected overall cost of generating the electricity:

$$\begin{aligned}&\min_{\boldsymbol{x}} E\left[\sum_{h=1}^{H} C_h\left(\sum_{n=1}^{N} x_n^h + l^h\right)\right]\\ &= \min_{\boldsymbol{x}} \sum_{h=1}^{H} \int C_h\left(\sum_{n=1}^{N} x_n^h + l^h\right) f_h(l^h) dl^h,\end{aligned} \tag{2}$$

where $f_h(\cdot)$, $h = 1, \ldots, H$, are distributions of the aggregate non-adjustable load for each hour.

However, in practice, the non-adjustable load distributions $f_h(l)$, $h = 1, \ldots, H$, are unknown. Here, we assume that each end-user employs a load prediction algorithm that provides samples from its non-adjustable load distribution. Bayesian neural networks [8], Gaussian processes [9], and particle filters [10] are examples of such prediction techniques. Let $\hat{l}_{n,k}^h$, $n = 1, \ldots, N$, $k = 1, \ldots, K$, denote samples from the local non-adjustable load distributions of the end-users. The utility (or the end-users) obtain samples $\hat{l}_m^h$, $m = 1, \ldots, M$, from the aggregate non-adjustable load distribution by drawing a random sample of size M independently from each local set $\hat{l}_{n,k}^h$, $n = 1, \ldots, N$, $k = 1, \ldots, K$, with replacement and then summing these samples item-wise (i.e., sum over the N local loads). Given the random sample $\hat{l}_m^h$, $m = 1, \ldots, M$, the cost function in (2) can be approximated as follows:

$$\begin{aligned}&\min_{\boldsymbol{x}} E\left[\sum_{h=1}^{H} C_h\left(\sum_{n=1}^{N} x_n^h + l^h\right)\right]\\ &\approx \min_{\boldsymbol{x}} \frac{1}{M}\sum_{h=1}^{H}\sum_{m=1}^{M} C_h\left(\sum_{n=1}^{N} x_n^h + \hat{l}_m^h\right).\end{aligned} \tag{3}$$

Since the sum of convex functions is convex, the cost function in (3) is convex given that $C_h(\cdot)$'s are convex.

However, each appliance or task may have time constraints on its usage. Consequently, the minimization problem is, in practice, a constrained optimization problem with equality and inequality constraints:

$$\begin{aligned}&\min_{x_{n,a}} \frac{1}{M}\sum_{h=1}^{H}\sum_{m=1}^{M} C_h\left(\sum_{n=1}^{N}\sum_{a=1}^{A_n} x_{n,a}^h + \hat{l}_m^h\right),\\ &\text{s.t.}\quad p_{a,\min} \le x_{n,a}^h \le p_{a,\max}, \forall a \in \mathcal{T}_1, \forall h \in \mathcal{H}_{n,a},\\ &\qquad x_{n,a}^h = 0, \forall a \in \mathcal{T}_1, \forall h \notin \mathcal{H}_{n,a},\\ &\qquad x_{n,a}^{h+t-1} = p_a^t, t = 1, \ldots, T_a, \forall a \in \mathcal{T}_2, \text{for one } h \in \mathcal{H}_{n,a},\end{aligned} \tag{4}$$

where $\mathcal{H}_{n,a}$ is the set of hours when the load a of user n, such as PEV charging, can be executed ($\mathcal{T}_1$) or started ($\mathcal{T}_2$). In this paper, we solve this problem separately for each load a. For $\mathcal{T}_1$ loads this is a constrained convex optimization problem that can be solved, e.g., using interior-point methods [11]. For $\mathcal{T}_2$ loads the problem is a binary integer programming problem that can, in this case, be solved using brute force search since the number of feasible strategies is relatively small.

Following the distributed approach in [1], the end-users minimize the optimization problem in (4) in sequential order by constructing a better reply path $\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^L$ in which the objective function, $g(\boldsymbol{x}) = \frac{1}{M}\sum_{h=1}^{H}\sum_{m=1}^{M} C_h\left(\sum_{n=1}^{N}\sum_{a=1}^{A_n} x_{n,a}^h + \hat{l}_m^h\right)$, in (4) is strictly decreasing, i.e., $g(\boldsymbol{x}^{t+1}) < g(\boldsymbol{x}^t)$. In other words, each end-user n minimizes (4) with respect to $x_{n,a}^h$, $h = 1, \ldots, H$, $\forall a \in A_n$, while the adjustable loads of the other end-users $\boldsymbol{x}_{-n,a}^h = \{x_{u,a}^h \mid u = 1, \ldots, N, u \ne n, \forall a\}$, $h = 1, \ldots, H$, are kept fixed. User n then transmits its new local adjustable loads x_n^h, $h = 1, \ldots, H$, where $x_n^h = \sum_{a \in A_n} x_{n,a}^h$, to the other users provided that the objective function is decreased. The next user in the sequential order then minimizes (3) with respect to its local adjustable load. This iterative process is continued until none of the users can improve its reply and decrease the objective function.

Theorem 1. *The distributed sequential iterative optimization process converges to a Nash equilibrium, $\rho_n(\boldsymbol{x}_n^*; \boldsymbol{x}_{-n}^*) \ge \rho_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n}^*)$, $\forall n, \boldsymbol{x}_n \in \boldsymbol{X}_n$, of the noncooperative electricity cost minimization game starting from any feasible random initial loads. Moreover, if the cost functions $C_h(\cdot)$ are strictly convex, the weighted sum of payoff functions is diagonally strictly concave, and there are no $\mathcal{T}_2$ loads, the Nash equilibrium is unique.*

Proof. Let us consider a situation in which all the $\mathcal{T}_2$ loads have been scheduled and thus the chosen strategies for $\mathcal{T}_2$ loads are fixed. The remaining $\mathcal{T}_1$ loads form a concave N-person game since the negative of the cost function, i.e., $-C_h$, is concave and the feasible set for $\mathcal{T}_1$ loads is convex. The existence of a Nash equilibrium for concave N-person game follows from Theorem 1 in [12]. Moreover, if the cost functions are strictly convex and the weighted sum of payoff functions is diagonally strictly concave [12], the uniqueness of the Nash equilibrium follows from Theorem 2 in [12]. Thus, there exists a Nash equilibrium for the concave game among $\mathcal{T}_1$ loads for each given $\mathcal{T}_2$ load strategy. Now, since the strategy set for $\mathcal{T}_2$ loads is finite, the better reply path for the game involving both $\mathcal{T}_1$ and $\mathcal{T}_2$ loads cannot be infinitely continued or cycle back to a previous strategy profile which concludes the proof. □

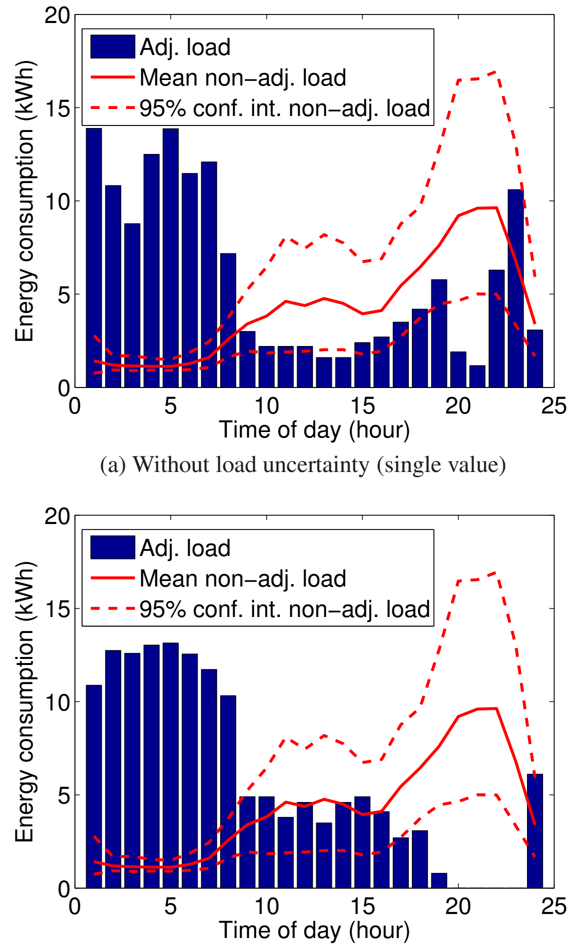
Consequently, the distributed algorithm converges to a solution in which no user can unilaterally decrease its own cost.

3. SIMULATION RESULTS

We consider distributed demand-side optimization in a scenario consisting of $N = 10$ domestic dwellings (i.e., end-users) each having a number of different adjustable and non-adjustable daily loads. Each dwelling has 2 residents and various appliances including TVs, computer, refrigerator/freezer, oven, microwave, electric sauna heater, among others. We model the daily load in each dwelling using the usage statistics based load model proposed in [13]. This model simulates the daily load with one minute resolution through simulation of appliance use and by taking into account simulated resident activity. In addition to the above non-adjustable loads, each dwelling has the following adjustable daily loads: PEV charging ($\mathcal{T}_1$: 9.9 kWh, $p_{\min} = 0$, $p_{\max} = 3.3$ kW), washer/dryer ($\mathcal{T}_2$: 2.4 kWh, $p = 0.8$ kW, 3 h), and dishwasher ($\mathcal{T}_2$: 2.2 kWh, $p = 1.1$ kW, 2 h). PEV charging can be scheduled to any hour of the day while the other two loads can be scheduled to start at any hour of the day provided that they can be completed within the same day.

We compare the proposed approach to an algorithm in which the predicted non-adjustable load is assumed to be exact. This corresponds to a scenario where the load predictor provides a single load value that is then assumed to be the actual non-adjustable load. In essence, the compared algorithm is a variant of the proposed scheme with $M = 1$. Now, instead of restricting our results to a particular predictor, we simulate the load prediction using the aforementioned domestic dwelling load model to generate appropriate number of load profiles for both methods. For the proposed algorithm, each dwelling has $K = 100$ predicted hourly load values from which the $M = 10000$ random samples from the aggregate non-adjustable load profile are generated according to the procedure described in Section 2. For the comparison method, each dwelling has one predicted hourly load which are all summed to obtain the aggregate non-adjustable load.

Fig. 1 illustrates example hourly load profiles for both with and without taking into account the uncertainty in the non-adjustable load while scheduling the adjustable load. The cost function is a piecewise linear function $C_h(x) = 0.12x$, for $x \leq 15$, and $C_h(x) = 0.24x - 1.8$, for $x > 15$, $h = 1, \ldots, 24$. We can see that the resulting daily profile is significantly smoother when the uncertainty in the non-adjustable load is taken into account. This reduces the utility's cost for generating the electricity and makes the dispatching of generation much easier. This is further illustrated by Fig. 2 that depicts the PAR and variance of the hourly loads for 100 different days. Moreover, Table 1 lists the mean cost, PAR, and load variance values for the 100 days. The results show that taking the load uncertainty into account reduces the PAR on average by over 20% and the average load variance by over 48% for the piecewise linear cost function. Table 1 lists the performance measures also for a quadratic cost function $C_h(x) = 0.02x^2$, $h = 1, \ldots, 24$. In this case, the average PAR is reduced by over 6.5% and average load variance by



(a) Without load uncertainty (single value)

(b) With load uncertainty

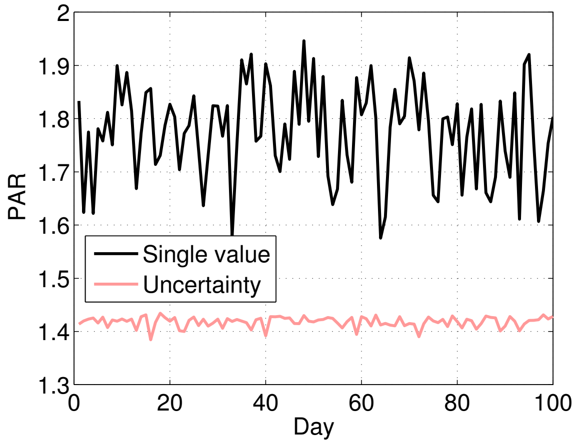
Fig. 1: Example hourly adjustable load profile for the algorithm without load uncertainty (a) and with load uncertainty (b) taken into account. The red curves show the mean (solid curve) and 95% confidence interval (dashed curves) of the aggregate non-adjustable load. The hourly load profile is smoother if the uncertainty in the non-adjustable load is taken into account in the load scheduling.

34% when uncertainty in the non-adjustable load is taken into account.

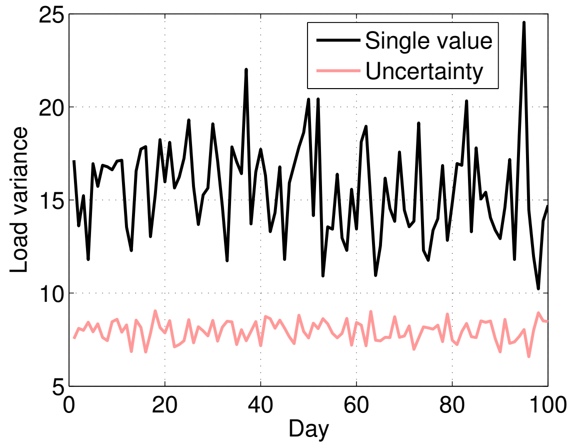
Table 1 shows also that the overall cost of electricity for the end-users is reduced by roughly 2% and 1.4%, respectively, for the piecewise linear and quadratic cost functions when the load uncertainty is taken into account. Although the reduction in the cost appears to be rather small for the end-users, the full reduction in the cost to the utility due to smaller PAR and load variance is not entirely reflected by the chosen cost model. Hence, the actual reductions in the cost could, in practice, be taken into account in the cost function parameters over a longer time span and thus the end-users could see a larger decrease in their cost as well.

4. CONCLUSION

In this paper, we have proposed a distributed cost minimization based demand-side optimization and scheduling scheme under load



(a) Load PAR



(b) Load variance

Fig. 2: Load peak-to-average ratio (PAR) and load variance for 100 different days for the piecewise linear cost function. Both the PAR and load variance are significantly lower when the load uncertainty is taken into account. In practice, this would result in much lower generation cost and easier generation dispatching for the utility.

Table 1. Mean cost, peak-to-average ratio (PAR), and variance of the daily load profile with and without (single value) load uncertainty taken into account. Taking the uncertainty into account in the demand-side optimization algorithm reduces significantly the variance and PAR of the daily load profiles. In addition, the cost of electricity to the end-users is reduced as well.

Algorithm	Cost function	Mean cost (EUR)	Mean PAR	Mean variance
Uncertainty	Piecewise linear	29.87	1.42	7.96
Single value	Piecewise linear	30.46	1.78	15.54
Uncertainty	Quadratic	52.95	1.41	3.03
Single value	Quadratic	53.69	1.51	4.65

uncertainty. We formulated the problem as a noncooperative game among the end-users and proposed an iterative algorithm for solving it. The proposed optimization algorithms average over the uncertainty in the load distribution. The proposed iterative algorithm was proven converge to a Nash equilibrium. Our results show that taking into account the uncertainty in the load reduces significantly the PAR and variance of the hourly load profile. This reduces the cost of generating the electricity and facilitates dispatching generation to match the demand. Moreover, the cost for the end-users is reduced as well.

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