COOPERATIVE DAY-AHEAD BIDDING STRATEGIES FOR DEMAND-SIDE EXPECTED COST MINIMIZATION

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ABSTRACT

The envisioned smart grid aims to improve the interaction between the supply- and the demand-side of the electricity network, resulting in a great optimization potential. In this paper, we propose a holistic-based, distributed day-ahead demand-side management method that is suitable for energy markets subject to an external regulation. Here, active subscribers solve the nonconvex problem of deriving the bidding strategies that minimize their overall expected monetary expense and simultaneously optimize eventual dispatchable energy generation and storage strategies. We show that, when such users collaborate, they achieve greater saving with respect to the corresponding user-oriented, selfish optimization. In this setting, we propose a cooperative, distributed, and iterative algorithm providing the optimal bidding, production, and storage strategies of the users, along with its convergence properties.

***Index Terms*—** Distributed Dynamic Pricing Algorithm, Smart Grid, Cooperative Demand-Side Management.

1. INTRODUCTION

Demand-side management (DSM) in the smart grid introduces advanced mechanisms for encouraging the demand-side to participate actively in the network optimization process by modifying the time pattern and the magnitude of the energy load demand [1]. DSM, distributed energy generation, and distributed storage are considered increasingly essential elements for implementing the smart grid concept and balancing massive energy production from renewable sources. These concepts open up unprecedented possibilities for optimizing the energy grid and energy usage at different network levels.

In particular, day-ahead DSM techniques provide the supply-side with an estimation of the amount of energy to be delivered to the demand-side during the upcoming day. Nonetheless, pure day-ahead approaches prove incapable of accommodating real-time fluctuations from the expected energy consumption by the demand-side users, as well as the randomness of their renewable sources. On top of that, additional costs are incurred by the supply-side when the consumption schedule is not correctly predicted by the users, and are transferred to the demand-side in the form of penalty charges [2, 3].

This paper proposes a day-ahead demand-side bidding process whereby the end users, possibly with dispatchable generation and storage capabilities, minimize their expected monetary expense. In order to do so, these users are connected not only to the power distribution grid, but also to a communication infrastructure that enables bidirectional communication between their smart meters and the independent regulator of the day-ahead market. Building on the previous work [4], a two-stage pricing model is used, which combines: i) a price per unit of energy related to the day-ahead bid energy needs of the demand-side, and ii) a penalty system that aims at narrowing potential real-time deviations from the negotiated energy loads, providing thus an incentive for a more accurate demand prediction.

DSM techniques have been traditionally formulated from an end-user oriented point of view. However, a collaborative approach minimizing, e.g., the peak-to-average ratio (PAR) of the energy demand or the total energy cost [5], can be more beneficial for all actors in the energy grid. In this paper, we formulate the DSM design as a nonlinear programming that minimizes the overall expected expense incurred by the active demand-side of the network. In contrast to the noncooperative method discussed in [4], which assumes a free market behavior, the present approach needs to be externally regulated in order to promote the cooperative bidding of demand-side users in favor of a better performance. To solve the resulting nonconvex optimization problem, we resort to the recent results in [6, 7] and introduce a distributed dynamic pricing-based algorithm (DDPA) that converges to a stationary solution of the problem under very mild assumptions (always satisfied in practice). Nonetheless, the resulting best-response-based update must be imposed as a protocol to the demand-side users so as to avoid selfish deviations from it.

The rest of the paper is organized as follows. In Section 2 we describe the smart grid model, whereas the demand-side bidding process is introduced in Section 3. In Section 4 we focus on the proposed cooperative DSM method, which is compared with the corresponding noncooperative game-theoretical formulation in Section 5. Finally, Section 6 draws some concluding remarks.

2. SMART GRID MODEL

The modern power grid is a complex network that can be conveniently divided into [8, 9]: i) supply-side (energy producers and providers), ii) central unit (regulation authority that coordinates the day-ahead market and the proposed demand-side bidding process), and iii) demand-side (end users). In this paper, we focus our attention on the demand-side of the smart grid, whereas the supply-side and the central unit are modeled as simply as possible.

2.1. Demand-Side Model

Demand-side users are characterized in the first place by the *per-slot net energy consumption* $e_n(h)$, which indicates the energy needed by user n to supply his appliances at time-slot h in the time period of analysis, taking into account eventual non-dispatchable (renewable) energy resources that he may adopt. In order to tackle with the uncertainties associated with future load demands and with renewable sources (when available), $e_n(h)$ is modeled as a random variable with pdf $f_{e_n(h)}(x)$ and cdf $F_{e_n(h)}(x)$.

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Our demand-side model distinguishes between *passive* and *active* users: the former resemble traditional energy consumers, whereas the latter indicate those users participating in the demand-side bidding process, i.e., reacting to changes in the cost per unit of energy by modifying their day-ahead bidding strategies. For convenience, we group the N active users into the set $\mathcal{N}$ and we suppose that each one of them can derive his individual load statistics from past energy consumption data measurements. Lastly, we conveniently divide the time period of analysis into H time-slots.

2.2. Energy Generation and Storage Model

Let $\mathcal{G} \subseteq \mathcal{N}$ be the subset of dispatchable energy producers. For users $n \in \mathcal{G}$, $g_n(h) \geq 0$ represents the *per-slot energy production profile* at time-slot h. Introducing the *energy production scheduling vector* $\mathbf{g}_n \triangleq \big(g_n(h)\big)_{h=1}^H$, we have that $\mathbf{g}_n \in \Omega_{\mathbf{g}_n}$, where $\Omega_{\mathbf{g}_n}$ is the strategy set for dispatchable energy producers (see an example in [10, Sec. II-B]). Besides, the *production cost function* $W_n\big(g_n(h)\big)$ gives the variable production costs incurred by user $n \in \mathcal{G}$ for generating the amount of energy $g_n(h)$ at time-slot h, with $W_n(0) = 0$.

Likewise, let $\mathcal{S} \subseteq \mathcal{N}$ denote the subset of energy storers. Users $n \in \mathcal{S}$ are characterized by the *per-slot energy storage profile* $s_n(h)$ at time-slot h: $s_n(h) > 0$ when the storage device is to be charged, $s_n(h) < 0$ when the device is to be discharged, and $s_n(h) = 0$ when the device is inactive. Introducing the *energy storage scheduling vector* $\mathbf{s}_n \triangleq \big(s_n(h)\big)_{h=1}^H$, it holds that $\mathbf{s}_n \in \Omega_{\mathbf{s}_n}$, being $\Omega_{\mathbf{s}_n}$ the strategy set for energy storers (see, e.g., [10, Sec. II-C]).

Finally, let us introduce the individual *per-slot energy load*

$$l_n(h) \triangleq e_n(h) - g_n(h) + s_n(h) \tag{1}$$

being the real-time energy flow between user $n \in \mathcal{N}$ and the grid at time-slot h, with $l_n(h) > 0$ when the user purchases energy from the grid and $l_n(h) < 0$ when he sells energy to the grid.

3. DEMAND-SIDE BIDDING SYSTEM

We describe next the proposed demand-side bidding process, through which active users determine in advance their bidding, dispatchable production, and storage strategies for the upcoming time period of analysis.

3.1. Energy Load Bidding Model

Let us denote by $\hat{e}_n(h)$ the *per-slot bid net energy consumption*, i.e., the day-ahead amount of energy (to be optimized) that user $n \in \mathcal{N}$ commits to consume at time-slot h. Defining the *bidding strategy vector* as $\hat{\mathbf{e}}_n \triangleq \big(\hat{e}_n(h)\big)_{h=1}^H$, we express the bidding strategy set as

$$\Omega_{\hat{\mathbf{e}}_n} \triangleq \big\{\hat{\mathbf{e}}_n \in \mathbb{R}^H : \chi_n^{(\min)}(h) \leq \hat{e}_n(h) \leq \chi_n^{(\max)}(h),\ \forall h\big\} \tag{2}$$

with $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ denoting the minimum and maximum per-slot bid net energy consumption, respectively.

Then, let us define the *per-slot bid energy load* as

$$\hat{l}_n(h) \triangleq \hat{e}_n(h) - g_n(h) + s_n(h) \tag{3}$$

and the strategy vector of a generic user $n \in \mathcal{N}$ as $\mathbf{x}_n \triangleq \big(\mathbf{x}_n(h)\big)_{h=1}^H$, with $\mathbf{x}_n(h) \triangleq \big(\hat{e}_n(h), g_n(h), s_n(h)\big)^{\mathrm{T}}$. Given the bidding strategy set $\Omega_{\hat{\mathbf{e}}_n}$ defined as in (2), and the sets $\Omega_{\mathbf{g}_n}$ and $\Omega_{\mathbf{s}_n}$ introduced in Section 2.2, the corresponding strategy set for a generic user $n \in \mathcal{N}$ is given by

$$\Omega_{\mathbf{x}_n} \triangleq \big\{\mathbf{x}_n \in \mathbb{R}^{3H} : \hat{\mathbf{e}}_n \in \Omega_{\hat{\mathbf{e}}_n}, \mathbf{g}_n \in \Omega_{\mathbf{g}_n}, \mathbf{s}_n \in \Omega_{\mathbf{s}_n}\big\} \tag{4}$$

with $\mathbf{g}_n = \mathbf{0}$ if $n \notin \mathcal{G}$ and $\mathbf{s}_n = \mathbf{0}$ if $n \notin \mathcal{S}$.

3.2. Energy Cost and Pricing Model

Let us now describe the cost model regulating the energy prices. First, we introduce the *cost per unit of energy* $C_h(\cdot)$ indicating the cost function at time-slot h set by the supply-side before the day-ahead market. Within the day-ahead bidding process, demand-side users agree the *per-slot aggregate bid energy load* $\hat{L}(h)$, and the price per unit of energy $C_h\big(\hat{L}(h)\big)$ remains fixed during the time period of analysis, while real-time penalties for load deviations are subsequently applied. In this paper, we adopt the cost function

$$C_h\big(\hat{L}(h)\big) = K_h \hat{L}(h) \tag{5}$$

which corresponds to the non-normalized quadratic grid cost function widely used in the smart grid literature (e.g., in [4, 5, 10, 11]). In general, the grid coefficients $K_h > 0$ vary along the time period of analysis according to the aggregate energy demand and to the availability of intermittent energy sources.

The per-slot aggregate bid energy load satisfies

$$L^{(\min)}(h) \leq \hat{L}(h) \triangleq \hat{L}^{(\mathcal{P})}(h) + \sum_{n \in \mathcal{N}} \hat{l}_n(h) \leq L^{(\max)}(h) \tag{6}$$

where $\hat{L}^{(\mathcal{P})}(h)$ is the predicted per-slot aggregate energy consumption associated with the passive users, and $L^{(\min)}(h), L^{(\max)}(h) > 0$ denote the minimum and maximum per-slot aggregate energy load, respectively. In this regard, we suppose that the central unit can predict these amounts based on available past statistics (see [12] for an overview on load forecasting techniques). Moreover, we suppose that, once $\hat{L}(h)$ has been fixed in the day-ahead bidding process, the real-time aggregate energy load is always guaranteed by the supply-side.

Each active user $n \in \mathcal{N}$ derives his *bid energy load vector* $\hat{\mathbf{l}}_n \triangleq \big(\hat{l}_n(h)\big)_{h=1}^H$ in the day-ahead demand-side bidding process. Nonetheless, he can possibly deviate from such strategy in real time by purchasing/selling a different amount of energy $l_n(h)$, for which he pays/perceives $K_h \hat{L}(h) l_n(h)$, while incurring in the penalties given by $\vartheta_h\big(l_n(h) - \hat{l}_n(h)\big)$, where the penalty function $\vartheta_h(x)$ is defined as

$$\vartheta_h(x) \triangleq \alpha_h (x)^+ + \beta_h (-x)^+ \tag{7}$$

with $(x)^+ = \max(x, 0)$, and where $\alpha_h, \beta_h \in (0, 1]$ are the penalty parameters for exceeding and for falling behind $\hat{l}_n(h)$, respectively.

Given the bid energy load vector $\hat{\mathbf{l}}_n$, the cumulative monetary expense incurred by user $n \in \mathcal{N}$ for exchanging the energy loads $\big\{l_n(h)\big\}_{h=1}^H$ with the grid, taking into account the aforementioned penalties for deviations and the energy produced $\big\{g_n(h)\big\}_{h=1}^H$, is denoted by the *cumulative expense* over the time period of analysis

$$\mathsf{p}_n(\hat{\mathbf{l}}_n, \hat{\mathbf{l}}_{-n}) \triangleq \sum_{h=1}^H \Big(K_h\big(\hat{l}_{-n}(h) + \hat{l}_n(h)\big)\big(l_n(h) + \vartheta_h\big(l_n(h) - \hat{l}_n(h)\big)\big) + W_n\big(g_n(h)\big)\Big) \tag{8}$$

where $\hat{\mathbf{l}}_{-n} \triangleq \big(\hat{l}_{-n}(h)\big)_{h=1}^H$ is the aggregate bid energy load vector of the other demand-side users, with

$$\hat{l}_{-n}(h) \triangleq \hat{L}(h) - \hat{l}_n(h) = \hat{L}^{(\mathcal{P})}(h) + \sum_{m \in \mathcal{N} \setminus \{n\}} \hat{l}_m(h). \tag{9}$$

On the other hand, here we are not interested in how the passive users are billed (this issue is discussed in [4]).

3.3. Expected Cost Minimization

Once the grid coefficients $\{K_h\}_{h=1}^H$ and the penalty parameters $\{\alpha_h, \beta_h\}_{h=1}^H$ are fixed in the day-ahead market [8, 9] and broadcast to the demand-side, active users react to the prices provided by the central unit by iteratively adjusting their bid energy load vectors $\hat{\mathbf{l}}_n$ (as in multi-round auctions [13]). Their final objective is to jointly minimize the aggregate expected cumulative expense $\mathsf{f}(\mathbf{x}) \triangleq \sum_{n\in\mathcal{N}} \mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}) = \sum_{n\in\mathcal{N}} \mathsf{E}\{\mathsf{p}_n\}$, with $\mathbf{x} \triangleq (\mathbf{x}_n)_{n=1}^N$ and

$$\mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}) \triangleq \sum_{h=1}^{H} \Big(K_h\big(\hat{l}_{-n}(h) + \boldsymbol{\delta}^{\mathrm{T}}\mathbf{x}_n(h)\big) \times \phi_{e_n(h)}\big(\mathbf{x}_n(h)\big) + W_n\big(\boldsymbol{\delta}_g^{\mathrm{T}}\mathbf{x}_n(h)\big)\Big) \quad (10)$$

denoting the individual expected cumulative expense of user $n \in \mathcal{N}$ over the time period of analysis (see [4, Lem. 1]), where we have defined $\boldsymbol{\delta} \triangleq (1, -1, 1)^{\mathrm{T}}$, $\boldsymbol{\delta}_{\mathbf{g}} \triangleq (0, 1, 0)^{\mathrm{T}}$, and

$$\phi_{e_n(h)}\big(\mathbf{x}_n(h)\big) \triangleq (1+\alpha_h)\bar{e}_n(h) - g_n(h) + s_n(h) - \alpha_h \hat{e}_n(h) + (\alpha_h + \beta_h)\big(\hat{e}_n(h) F_{e_n(h)}\big(\hat{e}_n(h)\big) - G_{e_n(h)}\big(\hat{e}_n(h)\big)\big) \quad (11)$$

with $\bar{e}_n(h) \triangleq \mathsf{E}\{e_n(h)\}$ and $G_{e_n(h)}(x) \triangleq \int_{-\infty}^{x} t f_{e_n(h)}(t)dt$.

4. COOPERATIVE DSM APPROACH FOR EXPECTED COST MINIMIZATION

In our previous works [4, 10], we modeled the active users as players of a noncooperative game, acting thus selfishly to reduce their individual monetary expenses. In this paper, instead, we follow an holistic-based approach, which is more desirable from the perspective of both the active users and the supply-side since one jointly optimizes the overall energy consumption of the active users, resulting thus in a more efficient demand-side management.

Stated in mathematical terms, we formulate our cooperative DSM optimization problem as

$$\begin{array}{lll} \min\limits_{\mathbf{x}} & \mathsf{f}(\mathbf{x}) = \sum\limits_{n\in\mathcal{N}} \mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}) & \\ \text{s.t.} & \mathbf{x}_n \in \Omega_{\mathbf{x}_n}, \qquad \forall n \in \mathcal{N} & \end{array}. \quad (12)$$

Note that a centralized solution of (12) by the central unit is not adequate because of users' privacy and scalability issues. Therefore, we focus in the following on *distributed* solution methods for (12).

We study the social problem (12) making the following blanket assumptions, which are very reasonable and easily satisfied in practice [10, Rem. 1.1], [4, Rem. 1.1].

Assumption 1. (a) The strategy sets $\Omega_{\mathbf{g}_n}$ and $\Omega_{\mathbf{s}_n}$ are compact and convex; (b) each production cost function $W_n(\cdot)$ is convex; and (c) all $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ are chosen such that the pdf of the per-slot net energy consumption satisfies

$$f_{e_n(h)}(x) \geq \frac{1}{L^{(\min)}(h)} \frac{(\alpha_h + 1)^2}{\alpha_h + \beta_h}, \ \forall x \in \big[\chi_n^{(\min)}(h), \chi_n^{(\max)}(h)\big]. \quad (13)$$

Condition (a) ensures that the individual strategy sets $\Omega_{\mathbf{x}_n}$ are compact and convex, whereas conditions (b)–(c) guarantee the convexity of $\mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n})$ on $\Omega_{\mathbf{x}_n}$, for any feasible $\hat{\mathbf{l}}_{-n}$ (see [4, Th. 2]).

4.1. Distributed Dynamic Pricing Algorithm

Traditionally, nonconvex optimization problems in the form of (12) have been tackled by using gradient descent algorithms, which solve a sequence of convex problems obtained by convexifying the whole social function; because of that, they generally suffer from slow convergence. A faster algorithm can be obtained by following the approach recently proposed in [6, 7]: since each $\mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n})$ is convex for any feasible $\hat{\mathbf{l}}_{-n}$ (under Assumption 1), we then convexify only the nonconvex part, i.e., $\sum_{m\in\mathcal{N}\setminus\{n\}} \mathsf{f}_m(\mathbf{x}_m, \hat{\mathbf{l}}_{-m})$, and solve the sequence of resulting parallel optimization subproblems, one for each user. A formal description of the algorithm is given next.

We give first the following preliminary definitions. Let $\mathbf{x}^{(i)} \triangleq (\mathbf{x}_n^{(i)})_{n=1}^N$ be the joint strategy vector at iteration i; the resulting aggregate load at time-slot h is

$$\hat{L}^{(i)}(h) \triangleq \hat{L}^{(\mathcal{P})}(h) + \sum_{m\in\mathcal{N}} \hat{l}_m^{(i)}(h) \quad (14)$$

where $\hat{l}_n^{(i)}(h)$ is the per-slot bid energy load of user $n \in \mathcal{N}$ at iteration i, and $\hat{\mathbf{l}}_{-n}^{(i)} \triangleq \big(\hat{l}_{-n}^{(i)}(h)\big)_{h=1}^H$ with $\hat{l}_{-n}^{(i)}(h) \triangleq \hat{L}^{(i)}(h) - \hat{l}_n^{(i)}(h)$. We then introduce the best-response mapping $\Omega_{\mathbf{x}} \ni \mathbf{x}^{(i)} \to \widehat{\mathbf{x}}_{\tau_n}(\mathbf{x}^{(i)}) \triangleq \big(\widehat{\mathbf{x}}_{\tau_n,n}(\mathbf{x}^{(i)})\big)_{n=1}^N$, where each $\widehat{\mathbf{x}}_{\tau_n,n}(\mathbf{x}^{(i)})$ is given by

$$\widehat{\mathbf{x}}_{\tau_n,n}(\mathbf{x}^{(i)}) \triangleq \underset{\mathbf{x}_n\in\Omega_{\mathbf{x}_n}}{\operatorname{argmin}} \Big\{ \mathsf{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}^{(i)}) + \boldsymbol{\pi}_n^{\mathrm{T}}\big(\{\Phi_h(\mathbf{x}^{(i)})\}_{h=1}^H\big)(\mathbf{x}_n - \mathbf{x}_n^{(i)}) + \frac{\tau_n}{2}\|\mathbf{x}_n - \mathbf{x}_n^{(i)}\|^2 \Big\} \quad (15)$$

with

$$\boldsymbol{\pi}_n\big(\{\Phi_h(\mathbf{x}^{(i)})\}_{h=1}^H\big) \triangleq \sum_{m\in\mathcal{N}\setminus\{n\}} \nabla_{\mathbf{x}_n} \mathsf{f}_m(\mathbf{x}_m, \hat{\mathbf{l}}_{-m}^{(i)}) = \Big(\boldsymbol{\delta} K_h\big(\Phi_h(\mathbf{x}^{(i)}) - \phi_{e_n(h)}\big(\mathbf{x}_n^{(i)}(h)\big)\big)\Big)_{h=1}^H \quad (16)$$

and $\Phi_h(\mathbf{x}^{(i)}) \triangleq \sum_{n\in\mathcal{N}} \phi_{e_n(h)}\big(\mathbf{x}_n(h)\big)$, where $\phi_{e_n(h)}\big(\mathbf{x}_n(h)\big)$ is defined in (11). The proximal term $\tau_n\|\mathbf{x}_n - \mathbf{x}_n^{(i)}\|^2/2$ in (15) has a motivation equivalent to that of [4, Alg. 2], since it makes (15) strongly convex; $\widehat{\mathbf{x}}_{\tau_n,n}(\mathbf{x}^{(i)})$ is thus well-defined.

The proposed algorithm solving (12) is a Jacobi scheme based on the best-response (15), i.e., all the users solve in parallel the subproblems in (15). The formal description of the algorithm is given in Algorithm 1, and its convergence conditions in Theorem 1 (whose proof is omitted because of the space limitation, see [7]).

Algorithm 1 Distributed Dynamic Pricing Algorithm (DDPA)

`Data` : $\{K_h\}_{h=1}^H$, $\{\tau_n\}_{n\in\mathcal{N}} > 0$, $\{\gamma^{(i)}\} > 0$,
$\Omega_{\mathbf{x}_n} \ni \mathbf{x}^{(0)} = (\mathbf{x}_n^{(0)})_{n=1}^N$; set $i = 0$.
`(S.1)` : `If` a suitable termination criterion is satisfied: `STOP`.
`(S.2)` : The central unit calculates $\{\Phi_h(\mathbf{x}^{(i)})\}_{h=1}^H$. `For` $n \in \mathcal{N}$, compute $\mathbf{x}_n^{(i+1)} = \mathbf{x}_n^{(i)} + \gamma^{(i)}\big(\widehat{\mathbf{x}}_{\tau_n,n}(\mathbf{x}^{(i)}) - \mathbf{x}_n^{(i)}\big)$
`(S.4)` : $i \leftarrow i + 1$; `Go to (S.1)`.

Theorem 1. *Given the social problem* (12)*, suppose that Assumption 1 holds,* $\{\tau_n\}_{n\in\mathcal{N}} > 0$*, and* $\{\gamma^{(i)}\}$ *are chosen such that*

$$\gamma^{(i)} \in (0, 1], \qquad \gamma^{(i)} \to 0, \qquad \sum_{i=1}^{\infty} = +\infty. \quad (17)$$

Then, either Algorithm 1 *converges in a finite number of iterations to a stationary solution of* (12) *or every limit point of the sequence* $\{\mathbf{x}^{(i)}\}_{i=1}^{\infty}$ *is a stationary solution of* (12).

Note that Algorithm 1 is guaranteed to converge whenever a solution to the social problem (12) exists. Therefore, its convergence conditions are consistently milder than those required by the noncooperative approach based on the proximal decomposition algorithm (PDA) recently proposed in [4, Alg. 2]. However, differently

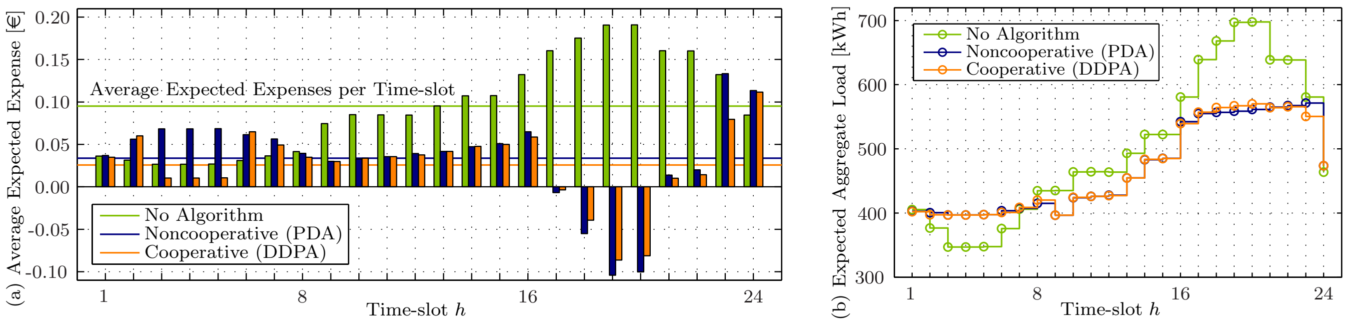

Fig. 1. (a) Average per-slot expected expenses resulting from the PDA in [4, Alg. 2] and from the DDPA in Algorithm 1; (b) Expected per-slot aggregate energy loads resulting from the PDA in [4, Alg. 2] and from the DDPA in Algorithm 1.

from the PDA, Algorithm 1 is not incentive compatible, in the sense that active users need to reach an agreement in following the best-response protocol (15). Moreover, it is characterized by the synchronous update of the users' strategies, whereas the PDA in [4, Alg. 2] is totally asynchronous. Lastly, the signaling required by Algorithm 1 is slightly more than that of [4, Alg. 2] since, at each iteration i, the users need to provide $\{\phi_{e_n(h)}(\mathbf{x}_n^{(i)}(h))\}_{h=1}^{H}$ to the central unit, in addition to the bid energy load vectors $\hat{\mathbf{l}}_n$. For these reasons, the proposed method needs to be coordinated by an external regulator in order to promote the cooperative bidding of demand-side users. The incentive in using such an algorithm is that it yields greater savings for all the costumers, as shown in our numerical results in the next section.

5. SIMULATION RESULTS

In this section, we illustrate numerically the performance of the proposed cooperative day-ahead bidding process.

We consider a smart grid of $N = 100$ active users and 900 passive users, evaluating a time period of analysis of $H = 24$ time-slots of one hour each. With the same setup of [4], all demand-side users $n \in \mathcal{D}$ have random energy consumption curves with daily average of $\sum_{h=1}^{24} \bar{e}_n(h) = 12$ kWh, with higher consumption during day-time hours (from 08:00 to 24:00) than during night-time hours (from 00:00 to 08:00), and reaching its peak between 17:00 and 23:00. The grid coefficients are chosen such that $\{K_h\}_{h=1}^{8} = K_{\text{night}}$ and $\{K_h\}_{h=9}^{24} = K_{\text{day}}$, with $K_{\text{day}} = 1.5K_{\text{night}}$ as in [5, 10], so as to obtain an initial price of 0.15 €/kWh when real-time penalties are neglected. Furthermore, we set $\{\alpha_h\}_{h=1}^{8} = 0.2$ and $\{\alpha_h\}_{h=9}^{24} = 0.9$, with $\beta_h = 1 - \alpha_h, \forall h$.

We model $e_n(h)$ as a normal random variable with mean $\bar{e}_n(h)$ and standard deviation $\sigma_n(h) = 0.75|\bar{e}_n(h)|$, with $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ chosen to satisfy Assumption 1(c). Furthermore, we suppose that active users with dispatchable generation and storage capabilities follow the production and storage models proposed in [10, Sec. II], with the same parameters used in [10, Sec. IV].

Fig. 1(a) illustrates the average per-slot expected expense derived from the PDA in [4, Alg. 2] and the DDPA in Algorithm 1 with $\{\tau_n\}_{n\in\mathcal{N}} = 0.1$, $\gamma^{(0)} = 1$, and the following step-size rule:

$$\gamma^{(i)} = \gamma^{(i-1)}(1 - \epsilon\,\gamma^{(i-1)}), \quad i = 1, \ldots \tag{18}$$

with $\epsilon = 10^{-3}$ (c.f. [7]). The resulting average expected cumulative expense $\mathsf{f}(\mathbf{x})/N$ decreases from its initial value of €2.33 to €0.82 with the PDA and to €0.62 with the DDPA. Hence, the proposed cooperative approach allows a 24.6% save with respect to the corresponding noncooperative method. On the other hand, Fig. 1(b)

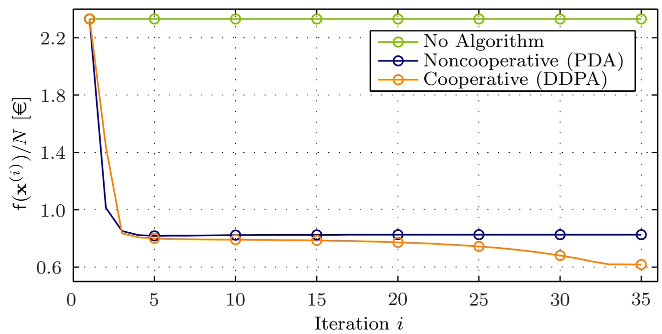

Fig. 2. Aggregate expected cumulative expenses at each iteration i, obtained using the PDA in [4, Alg. 2] and DDPA in Algorithm 1.

depicts the expected per-slot aggregate energy loads obtained with the noncooperative and with the cooperative approach: the resulting expected demand curves are overall similar and show a substantial flattening compared to the initial one, which considers the aggregate average per-slot net energy consumptions $\sum_{n\in\mathcal{N}} \bar{e}_n(h)$, due to the employment of distributed energy generation and storage.

As termination criterion in `(S.1)` of Algorithm 1, we require the modification in the bid energy load vector of each user between two consecutive iterations to be sufficiently small (10^{-2}). Algorithm 1 converges after 32 iterations whereas, under an equivalent setup, the PDA in [4, Alg. 2] converges after 11 iterations, although the former has already reached a better result than the latter at $i = 11$. Moreover, it is worth remarking that [4, Alg. 2] is a double-loop algorithm where each iteration implies several updates of the users' strategies. Fig. 2 compares the evolution of the aggregate expected cumulative expense resulting from both algorithms over the first 35 iterations. It is evident that, despite the more iterations needed, the proposed DDPA allows to achieve a consistently lower value of $\mathsf{f}(\mathbf{x})$.

6. CONCLUSIONS

In this paper, we propose a cooperative day-ahead bidding process for smart grid users based on a pricing model with real-time penalties. We provide a distributed and iterative algorithm that allows to compute the optimal bidding, production, and storage strategies of the users with limited information exchange between the central unit and the demand-side of the grid. Despite being slower in convergence and requiring slightly more signaling, the proposed algorithm converges under less stringent conditions and reduces considerably the aggregate expected expense of the active users with respect to the the corresponding noncooperative (selfish) approach.

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