TARGET TRACKING WITH DISTANCE-DEPENDENT MEASUREMENT NOISE IN WIRELESS SENSOR NETWORKS

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ABSTRACT

A distributed extended Kalman filter (EKF) algorithm is developed for tracking moving targets in a wireless sensor network equipped with distance estimating sensors. In particular, a distance-dependent measurement error of range-estimating sensors is modeled as a multiplicative noise in the observation model. A new formulation of EKF, called generalized EKF (GEKF) based on the multiplicative noise model is developed. Compared to conventional EKF formulation, it is shown that GEKF can achieve smaller estimation error than traditional EKF. Simulation results also demonstrated superior performance of GEKF.

Index Terms— Wireless sensor networks, extended Kalman filtering, target tracking, distance-dependent

1. INTRODUCTION

A wireless sensor network (WSN) is an enabling technology of cyber physical systems. It has found numerous applications such as infrastructure monitoring, habitat sensing, and battlefield surveillance [1]. A critical task that is required by many WSN applications is the ability to track moving targets within the sensing field based on distributed sensor measurements [2, 3].

Traditionally, target tracking is performed using the Kalman Filter (KF) algorithm or its variants such as the Extended Kalman filter (EKF), or Unscented Kalman filter (UKF). These algorithms assume the target movement can be described by a dynamic system model where the state (location, speed, etc.) can be observed via a measurement model. The traditional Kalman filter provides optimal estimates of target states if both the dynamic system model and the measurement model are linear and the system driving noise and observation noise are additive independent, identically distributed normal random variables.

In a WSN, however, sensor measurements have multiple modalities and the measurement model is often non-linear or even signal dependent. For instance, in a bearings-only tracking application [4], the measurement noise is a function of the signal to noise ratio(SNR) and the incident angle of the signal. In a target tracking application using distance measurements[5], the measurement noise covariance matrix increases as the relative distance to the target increases. To deal with state-dependent measurement noise, a recursive extended Kalman filter using maximum likelihood (ML) estimation with Newton-Raphson nonlinear optimization is proposed[6]. In [7], a separate ML localization step is first performed to estimate the target location based only on present observations. Then the estimated target locations are treated as an observation to facilitate KF-based tracking.

In this work, the task of tracking a moving target in a WSN using observations of sensor to target distance is considered. In particular, it is assumed that the distance estimate is contaminated by distance-dependent multiplicative observation noise. Using this nonlinear, signal dependent noise model, a generalized EKF (GEKF) algorithm is derived. Compared to conventional EKF solution, it is proved that the estimation error of GEKF is smaller than that of the conventional EKF. This finding is further justified with extensive simulation comparing the performance of GEKF against those of traditional EKF as well as the method presented in [7].

The rest of this paper is organized as follows. In Section 2, the nonlinear observation model with multiplicative noise is formulated. The proposed GEKF Tracking algorithm with state-dependent distance measurement noise is derived in Section 3. Performance evaluations comparing the simulation performance of GEKF against those of existing algorithms are reported in Section 4. Conclusions and discussions are presented in section 5.

2. PROBLEM FORMULATION

Assume that N identical sensor nodes with known positions are deployed over a 2-D sensing field and form a WSN. The position of the n^{th} $(1 \leq n \leq N)$ sensor node is denoted by r_n. Each sensor node is equipped with an ultrasonic range sensor with a known detection range (sensing range)

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d. When the sensor to target distance $h_n(k)$ is less than d, the target will be detected and the distance $h_n(k)$ will be reported by the sensor. The subset of sensor nodes within the sensing range of the target at time k will be denoted by $L_k = \{n | h_n(k) \leq d, 1 \leq n \leq N\}$. Therefore, if $n \notin L_k$, the n^{th} sensor will not reported a reading in time k.

When the target moves through the monitored area, the sensors which have detected the target will organized to form a cluster [8, 9, 3]. A sensor in the cluster will be selected as the cluster head which also serves as the fusion center of signal and information processing. Cluster members measure their respective sensor-to-target distances and transmit the distance to the cluster head via wireless channels. For convenience, transmission delay or packet loss are ignored.

The trajectory of a moving target within the WSN at a position $\rho_k = [x(k)\ y(k)]^{\mathrm{T}}$ and a speed $\dot{\rho}(k) = [\dot{x}(k)\ \dot{y}(k)]^{\mathrm{T}}$ can be described by a discrete time dynamic state transition equation[10]:

$$X_k = F_{k-1} X_{k-1} + G_{k-1} W_{k-1}, \tag{1}$$

where $X_k = [x(k)\ \dot{x}(k)\ y(k)\ \dot{y}(k)]^{\mathrm{T}}$ is the state vector. F_{k-1} and G_{k-1} are the state transition matrix and input matrix respectively. The input noise W_{k-1} is a 2×1 Gaussian random vector with zero mean and a covariance matrix Q_{k-1}.

The sensor measurement $z_n(k)$ of the n^{th} sensor at time k consists of the sensor to target distance

$$h_n(k) = \|r_n - \rho_n(k)\| \tag{2}$$

and a distance dependent observation noise [7]:

$$z_n(k) = [1 + u_n(k)] h_n(k) + v_n(k) \quad n \in L_k \tag{3}$$

$u_n(k)$ and $v_n(k)$ are independent Gaussian noises such that $\mathbb{E}\{u_n(k)\} = \mu_u$, $\mathbb{E}\{v_n(k)\} = \mu_v$, $n \in L_k$, and for $i, j \in L_k$

$$\begin{aligned}
\mathbb{E}\{(u_i(k) - \mu_u)(v_j(k) - \mu_v)\} &= 0, \\
\mathbb{E}\{(u_i(k) - \mu_u)(u_j(k) - \mu_u)\} &= \sigma_u^2 \delta_{i,j}, \\
\mathbb{E}\{(v_i(k) - \mu_v)(v_j(k) - \mu_v)\} &= \sigma_v^2 \delta_{i,j},
\end{aligned} \tag{4}$$

where $\delta_{i,j} = 1$ if $i = j$, and $= 0$ otherwise.

The use of $u_n(k)$ as a multiplicative noise is motivated by the fact that the measurement error on a distance measuring sensor increases roughly linearly as a function of the sensor to target distance [11].

Denote ℓ_k to be the cardinal number of L_k, the sensor measurements at the k^{th} time step may be represented in a matrix form:

$$Z_k = A_k \cdot H_k(X_k) + V_k \tag{5}$$

where $A_k = diag\{1 + u_n(k); 1 \leq n \leq \ell_k\}$, $H_k(X_k) = [h_1(k) \cdots h_{\ell_k}(k)]^T$, and $V_k = [v_1(k) \cdots v_{\ell_k}(k)]^T$. Moreover, $\mathbb{E}\{V_k\} = \mu_v \mathbf{1} = \mu_v [1\ 1\ \cdots 1]^T_{1 \times \ell_k}$, and $R_k = \sigma_v^2 \mathbf{I}$.

The objective of this work is to develop a generalized extended Kalman filter (GEKF) tracking algorithm for the given observation model to be executed at the cluster head. More specifically, given the previous target state X_{k-1} and noisy sensor observations Z_k, our goal is to obtain a sequential Bayesian estimate of X_k using a Kalman filter formulation.

3. TRACKING ALGORITHM WITH DISTANCE-DEPENDENT OBSERVATION NOISE

Using Bayes rule, one has

$$p(X_k | Z_{1:k}) \propto p(Z_k | X_k) p(X_k | Z_{1:k-1}). \tag{6}$$

where $Z_{1:k} = \{Z_1, Z_2, \cdots, Z_k\}$ and $p(X_{k-1} | Z_{1:k-1}) = \mathrm{N}(\hat{X}_{k-1|k-1}, P_{k-1|k-1})$ [12] . Thus

$$p(X_k | Z_{1:k-1}) = \mathrm{N}(\hat{X}_{k|k-1}, P_{k|k-1}) \tag{7}$$

where $\hat{X}_{k|k-1} = F_{k-1} \hat{X}_{k-1|k-1}$, and $P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^{\mathrm{T}} + G_{k-1} Q_{k-1} G_{k-1}^{\mathrm{T}}$.

To update the state estimate using latest observation Z_k, consider a first order Taylor series expansion of $H_k(X_k)$ around $\hat{X}_{k|k-1}$:

$$H_k(X_k) \approx H_k(\hat{X}_{k|k-1}) + \dot{H}_k (X_k - \hat{X}_{k|k-1}) \tag{8}$$

where $\dot{H}_k = \left[\left. \frac{\partial H_k(X_k)}{\partial X_k^T} \right|_{X_k = \hat{X}_{k|k-1}} \right]$ is a $\ell_k \times 4$ matrix. Thus,

$$\tilde{Z}(k) \approx A_k [H_k + \dot{H}_k (X_k - \hat{X}_{k|k-1})] + V_k. \tag{9}$$

In eq. (9), H_k and $\dot{H}_k$ may be estimated from the expected value of $\hat{X}_{k|k-1}$ and treated as known quantities. Then $\tilde{Z}(k)$ becomes a *linear* combination of two normal random variables $\hat{X}_{k|k-1}$ and V_k. Therefore, the joint probability distribution

$$\begin{aligned}
p(X_k, Z_k | Z_{1:k-1}) &= p(Z_k | X_k) p(X_k | Z_{1:k-1}) \\
&\approx \mathrm{N}\left(\begin{bmatrix} \hat{X}_{k|k-1} \\ \Sigma_E(k) \end{bmatrix}, \begin{bmatrix} P_{k|k-1} & C_E(k) \\ C_E^{\mathrm{T}}(k) & S_E(k) \end{bmatrix} \right)
\end{aligned} \tag{10}$$

where

$$\Sigma_E(k) = \mathbb{E}\{\tilde{Z}_k\} = (1 + \mu_u) H_k(\hat{X}_{k|k-1}) + \mu_v \mathbf{1}, \tag{11}$$

$$C_E(k) = \mathrm{Cov}\{X_K, \tilde{Z}_k\} = (1 + \mu_u) P_{k|k-1} \dot{H}_k^{\mathrm{T}}. \tag{12}$$

Denote $B_k = diag\{u_1 - \mu_u, \cdots, u_{\ell_k} - \mu_u\}_{\ell_k \times \ell_k}$, one has

$$\begin{aligned}
S_E(k) = \mathrm{Cov}\{\tilde{Z}_k, \tilde{Z}_k\} &= \mathbb{E}\{A_k \dot{H}_k P_{k|k-1} \dot{H}_k^{\mathrm{T}} A_k^{\mathrm{T}}\} \\
&+ \mathbb{E}\{B_k H_k(\hat{X}_{k|k-1}) H_k^{\mathrm{T}}(X_{k|k-1}) B_k^{\mathrm{T}}\} + \sigma_v^2 \mathbf{I}.
\end{aligned}$$

Using eq. (4), $S_E(k)$ may be simplified as:

$$S_E(k) = (1 + \mu_u)^2 \dot{H}_k P_{k|k-1} \dot{H}_k^{\mathrm{T}} + \sigma_u^2 \mathbf{M}_k + \sigma_v^2 \mathbf{I}, \tag{13}$$

where $\mathbf{M}_k = diag\{\dot{H}_k P_{k|k-1} \dot{H}_k^{\mathrm{T}} + H_k H_k^{\mathrm{T}}\}$ is a diagonal matrix consisting of the diagonal elements of the matrix inside the brackets.

From eq. (10), the posterior distribution of X_k is

$$p(X_k | Z_k, Z_{1:k-1}) = \mathrm{N}(\hat{X}_{k|k}, P_{k|k}). \tag{14}$$

This leads to the GEKF update equations:

$$\begin{aligned}\hat{X}_{k|k} &= \hat{X}_{k|k-1} + K_E(k)[Z_k - \Sigma_E(k)],\\ P_{k|k} &= P_{k|k-1} - K_E(k)C_E^{\mathrm{T}}(k),\\ K_E(k) &= C_E(k)S_E(k)^{-1}.\end{aligned} \tag{15}$$

3.1. Traditional EKF Formulation

Note that the observation model in eq. (3) can also be expressed as the sum of a nonlinear observation and an additive, signal-dependent noise:

$$\begin{aligned}\mathbf{z}_n(k) &= [1 + u_n(k)]h_n(k) + v_n(k) \quad n \in L_k\\ &= h_n(k) + v'_n(k)\end{aligned} \tag{16}$$

where $v'_n(k) = u_n(k)h_n(k) + v_n(k)$. The corresponding Taylor series expansion leads to a first order linear approximation of the nonlinear measurement model as follows:

$$Z'(k) \approx H_k + \dot{H}_k(X_k - \hat{X}_{k|k-1}) + V'_k, \tag{17}$$

and $V'_k = (A_k - \mathbf{I}) \cdot [H_k + \dot{H}_k(X_k - \hat{X}_{k|k-1})] + V_k$ is an equivalent (data dependent) additive noise term such that

$$\begin{aligned}&\Sigma_{V'}(k) = \mathbb{E}\{V'_k\} = \mu_u H_k(\hat{X}_{k|k-1}) + \mu_u \mathbf{1},\\ &R'_k = Cov\{V'_k, V'_k\} = \mu_u^2 \dot{H}_k P_{k|k-1} \dot{H}_k^T + \sigma_u^2 \mathbf{M}_k + \sigma_v^2 \mathbf{I}.\end{aligned}$$

$\mathbf{M}_k$ is defined in eq. (13). Now, the joint distribution of X_k and Z_k may be approximated as

$$p(X_k, Z_k | Z_{1:k-1}) \approx \mathrm{N}(\begin{bmatrix} \hat{X}_{k|k-1} \\ \Sigma_E(k) \end{bmatrix}, \begin{bmatrix} P_{k|k-1} & C'_E(k) \\ C_E^{'\mathrm{T}}(k) & S'_E(k) \end{bmatrix}) \tag{18}$$

where $C'_E(k) = \mathrm{Cov}\{X_K, Z'_k\} = P_{k|k-1}\dot{H}_k^{\mathrm{T}}$, and $S'_E(k) = \mathrm{Cov}\{Z'_k, Z'_k\} = \dot{H}_k P_{k|k-1} \dot{H}_k^{\mathrm{T}} + R'_k$. This leads to the state update equations in a traditional EKF formulation:

$$\begin{aligned}&\hat{X}'_{k|k} = \hat{X}_{k|k-1} + K'_E(k)^{-1}[Z_k - \Sigma_E(k)],\\ &K'_E(k) = C'_E(k)S'_E(k)^{-1}.\end{aligned} \tag{19}$$

3.2. Performance Comparison

The trace (sum of diagonal elements) of the covariance matrix tr$\{P_{k|k}\}$ corresponds to the mean squared error (MSE) of the updated state and can be used to measure the tracking performance[8, 3]. In general, smaller tr$\{P_{k|k}\}$ value implies more accurate tracking.

Now, from above derivations, one has the following lemma:

Lemma 1.

$$P'_{k|k} = P_{k|k} + [K'_E(k) - C_E(k)S_E^{-1}]S_E[K'_E(k) - C_E(k)S_E^{-1}]^{\mathrm{T}}. \tag{20}$$

Proof. From eq. (19) and eq. (10), the covariance of the state updated by the measurements Z_k in the traditional EKF is

$$\begin{aligned}&P'_{k|k} = \mathbb{E}\{[X_k - \hat{X}'_{k|k}][X_k - \hat{X}'_{k|k}]^{\mathrm{T}}\}\\ &= P_{k|k-1} - C_E(k)K'_E(k)^{\mathrm{T}} - K'_E(k)C_E^{\mathrm{T}}(k) + K'_E(k)S_E(k)K'_E(k)^{\mathrm{T}},\\ &= P_{k|k-1} - C_E(k)S_E^{-1}C_E^{\mathrm{T}}(k)\\ &+[K'_E(k) - C_E(k)S_E^{-1}]S_E[K'_E(k) - C_E(k)S_E^{-1}]^{\mathrm{T}}\\ &= P_{k|k} + [K'_E(k) - C_E(k)S_E^{-1}]S_E[K'_E(k) - C_E(k)S_E^{-1}]^{\mathrm{T}}.\end{aligned}$$

□

Lemma 1 relates to state covariance matrices of the GEKF and traditional EKF. Using this lemma, the following theorem can be easily verified:

Theorem 2.

$$tr\{P_{k|k}\} < tr\{P'_{k|k}\}. \tag{21}$$

Proof. Since $[K'_E(k) - C_E(k)S_E^{-1}]S_E[K'_E(k) - C_E(k)S_E^{-1}]^{\mathrm{T}}$ is a positive-definite matrix, its matrix diagonal entries are real and positive[13]. Therefore, tr$\{P_{k|k} - P'_{k|k}\} < 0$. Or equivalently, eq. (21) is verified. □

Theorem 2 states that the GEKF yields more accurate estimate than that of the traditional EKF under the multiplicative measurement model eq. (3).

4. SIMULATION AND DISCUSSION

To validate the expected performance advantage of the proposed GEKF tracking algorithm, Monte Carlo simulations are conducted. In these simulations, the performance of GEKF is compared against three competing Kalman filter formulations: the traditional EKF, UKF algorithms, and the two-phase KF+ML algorithm proposed in [7].

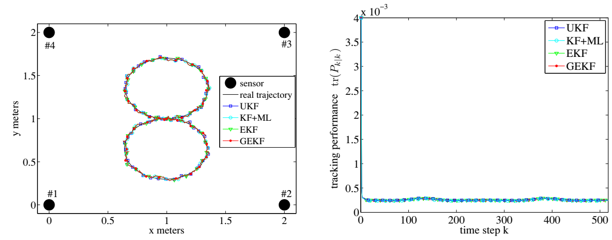
In these experiments, four sensors are deployed at four corners of a square sensing field of size 2 meters by 2 meters. Each sensor has a detection radius $\mathbf{d} \geq 2\sqrt{2}$ meters. A single target travels within the sensing field at a constant angular velocity of 0.122 rad/s along a spiral trajectory with radius 0.35 meters. The sampling interval is 0.2 second (5 Hz). For this motion, the process noise W_{k-1} can be approximated by a variable acceleration with $Q_{k-1} = \mathrm{diag}\{0.0027, 0.0027\}$.

The parameter values for Kalman filters are set to

$$\hat{X}_{0|0} = [\ 1.0 \quad 0.0428 \quad 1.0 \quad 0.0\ ]^{\mathrm{T}}, P_{0|0} = 0.001 \times \mathrm{diag}\{\ 1 \quad 1 \quad 1 \quad 1\ \}$$

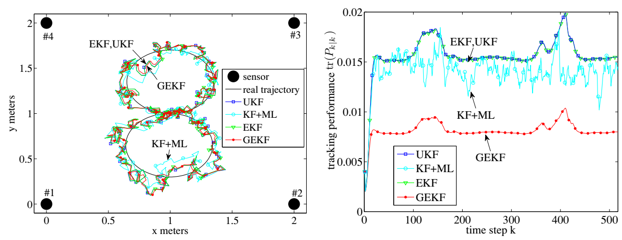
Two different noise conditions are simulated in these experiments: (a) Low Noise condition with $\mu_u = 0.0174$, $\sigma_u^2 = 2.916 \times 10^{-4}$, $\mu_v = -0.0386$, $\sigma_v^2 = 7.97 \times 10^{-5}$. (b) High Noise condition with $\mu_u = 0.54$, $\sigma_u^2 = 0.056$, $\mu_v = 0.11$, $\sigma_v^2 = 0.017$.

100 sets realizations of sensor observation noise are generated for each of the two noise conditions. Each of the four Kalman filter algorithms then are applied to track the moving target. Average of the results of 100 trials are reported.



(a) Tracking trajectories (b) Tracking performances

Fig. 1. Tracking results under low noise environment.



(a) Tracking trajectories (b) Tracking performances

Fig. 2. Tracking results under high noise environment.

Figure 1,2 show the tracking trajectories and tracking performances of different tracking algorithms under the low and high noise conditions. In the low noise condition, all four tracking algorithms attain similar tracking performance. This is quite reasonable as the impact of the multiplicative distance dependent noise term has negligible magnitude under the low noise condition.

However, under the high noise condition, as shown in figure 2, the inadequate noise model of the EKF, UKF and KF+ML significantly degraded their tracking performance compared to that of the GEKF algorithm.

5. CONCLUSION

In this paper, a practical distance-dependent, multiplicative observation noise model is proposed to enhance the observation model for distance-measuring sensors. Based on this new noise model, a generalized extended Kalman filter (GEKF) algorithm is proposed. Extensive simulation comparing GEKF agains traditional EKF, UKF and a previously proposed KF+ML method yield very favorable results for GEKF.

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