POSTERIOR CRLB FOR TRACKING A MOBILE STATION IN NLOS MULTIPATH ENVIRONMENTS

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ABSTRACT

In this paper we investigate the posterior Cramer-Rao lower bound for tracking a mobile station in a non-line-of-sight multipath localization scenario. The posterior Cramer-Rao lower bound is the mean-square estimation error lower bound of a random scalar or vector parameter in linear/nonlinear filtering problems. A practical system model is chosen to represent the non-line-of-sight multipath propagations. A recursive formulation of the posterior Cramer-Rao lower bound is developed for tracking a mobile station in a non-line-of-sight multipath localization scenarios when the observations are the time-of-arrival, the angle-of-departure, the angle-of-arrival and the Doppler shift. The lower bound is also shown for a subset of these observations. The developed lower bound can be used to analyze the achievable performance of a tracking system designed for use in a non-line-of-sight multipath localization scenarios.

Index Terms— NLOS multipath Localization, CRLB, Posterior CRLB, NLOS multipath propagations

1. INTRODUCTION

Localization has a number of potential applications like augmented reality, context-aware services, guided tour of museums etc. However, localization in non-line-of-sight (NLOS) multipath environments is rather a difficult problem. Conventional satellite and cellular network positioning systems, which are often based on a certain line-of-sight (LOS) assumption, fail to meet the performance requirements. This is due to the fact that the NLOS multipath propagations often result in unreliable pseudo-range observations. Here, we consider the single bounce scattering model introduced in [1] to model the NLOS multipath propagations. The single bounce scattering model helps to exploit the NLOS multipath propagations and thus improves the localization accuracy [2, 3].

Given an unknown deterministic scalar or vector parameter, the Cramer-Rao lower bound (CRLB) sets the lower bound of the variance of an unbiased estimator given sequences of observations. The lower bound of the mean-square estimation error of a random scalar or vector parameter is referred to as the Van Trees bound [4] or the posterior CRLB (PCRLB) [5]. In [5], a recursive method for determining the PCRLB of a linear/nonlinear filtering problem is proposed. In this paper, we use the proposed method to determine the PCRLB of an estimator which tracks a mobile station (MS) moving randomly in an NLOS multipath environments.

The CRLB for localizing an MS in an NLOS multipath propagation environments where the observation consists of the range, the bearing and the Doppler shift (DS) has been presented in [6]. However, the CRLB presented in [6] is essentially similar to the CRLB presented in [1]. The improvement from the CRLB presented in [1] is the assumption of a linear deterministic mobility model for a short period of time which helps to exploit the correlation between successive observations. The linear deterministic mobility model simplifies the localization process as only the initial position and the initial velocity estimates of the MS are required for subsequent position estimation of the MS. In addition, the CRLB calculation in [6] involves the inversion of a very large matrix whose dimension grows with the number of observations which makes it computationally intensive. The PCRLB for tracking an MS in mixed LOS/NLOS conditions has been presented in [7] and [8] for time-of-arrival (TOA) observations and received signal strength (RSS)/TOA observations, respectively. In both [7] and [8], the scatterers are not explicitly considered. Hence it is assumed in [7] that the LOS and NLOS transition history is known. Whereas in [8] the NLOS bias is modeled as positive uniform randomly distributed variable. In this paper we present the PCRLB for tracking an MS in an NLOS multipath propagation environments, under the explicit consideration of the scatterers, which allows for a random movement of the MS. The PCRLB is computed recursively for each step of the MS path and hence it does not suffer from the dimensionality problem. The observations considered are the range, the bearing and the DS.

The present paper is organized as follows. In section 2, the PCRLB is discussed. The system, mobility and observation models of the localization scenario are discussed in section 3. The PCRLB of tracking an MS in an NLOS multipath environments is presented in section 4 followed by simulation results and discussions in section 5. Finally, a conclusion of this paper is drawn in section 6.

Notation: $\mathbf{I}_m$ and $\mathbf{0}_{m\times m}$ are identity and zero matrices of size $m \times m$, respectively. $\mathbf{1}_m$ and $\mathbf{0}_m$ are column vectors of ones and zeros of size m, respectively. A superscript "T" denotes a transpose. $\mathrm{diag}\,(\mathbf{a})$ denotes a diagonal matrix with the diagonal elements defined by vector $\mathbf{a}$.

2. POSTERIOR CRLB

Given an unknown random r-dimensional parameter $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_r)$ and an observation vector $\mathbf{z}$, the PCRLB is the lower bound of the mean-square estimation error of an estimator $\hat{\boldsymbol{\theta}}$ which estimates the parameter $\boldsymbol{\theta}$. Let $\mathrm{p}(\boldsymbol{\theta}, \mathbf{z})$ denote the joint probability density of $(\boldsymbol{\theta}, \mathbf{z})$, the PCRLB of the estimation error is

$$\mathrm{E}\{[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}][\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}]^{\mathrm{T}}\} \succeq \mathbf{J}^{-1}, \quad (1)$$

where $\mathbf{J}$ is the Fisher information matrix with the elements

$$\mathbf{J}_{ij} = \mathrm{E}\left[-\frac{\partial^2 \ln \mathrm{p}(\boldsymbol{\theta}, \mathbf{z})}{\partial \boldsymbol{\theta}_i \partial \boldsymbol{\theta}_j}\right] \quad i, j = 1, \ldots, r. \quad (2)$$

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In the following we denote the first- and second-order partial derivatives with ∇ and Δ:

$$\nabla_{\boldsymbol{\theta}} = \left[\frac{\partial}{\partial \boldsymbol{\theta}_1}, \cdots, \frac{\partial}{\partial \boldsymbol{\theta}_r}\right]^{\mathrm{T}} \tag{3}$$

$$\Delta_{\boldsymbol{\theta}}^{\boldsymbol{\varphi}} = \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\varphi}}^{\mathrm{T}}. \tag{4}$$

Using the operator $\Delta_{\boldsymbol{\theta}}^{\boldsymbol{\theta}}$, equation (2) can be written as

$$\mathbf{J} = \mathrm{E}\left[-\Delta_{\boldsymbol{\theta}}^{\boldsymbol{\theta}} \ln \mathrm{p}(\boldsymbol{\theta}, \mathbf{z})\right]. \tag{5}$$

Let's consider the vector parameter $\boldsymbol{\theta}_n$ and the observation vector $\mathbf{z}_n$ at time instant n. In [5], a recursive computation of the PCRLB for each $\boldsymbol{\theta}_n$ is proposed. A frequent singular case in which the conditional probability density function (pdf) $\mathrm{p}(\boldsymbol{\theta}_{n+1}|\boldsymbol{\theta}_n)$ is not defined is also considered. A singular conditional pdf $\mathrm{p}(\boldsymbol{\theta}_{n+1}|\boldsymbol{\theta}_n)$ arises if there are variables in the vector parameter which remain constant or which are not directly driven by the system driving noise. In the latter case the variables change as a result of being a function of the variables being driven by the system driving noise. The PCRLB for tracking an MS in an NLOS multipath environments has a singular conditional pdf $\mathrm{p}(\boldsymbol{\theta}_{n+1}|\boldsymbol{\theta}_n)$ and hence we consider this special case of the PCRLB. To this end, the part of the vector parameter $\boldsymbol{\theta}_n$ which makes the conditional pdf $\mathrm{p}(\boldsymbol{\theta}_{n+1}|\boldsymbol{\theta}_n)$ singular is separated from the part of the vector parameter which is nonsingular. Let the nonsingular part be $\boldsymbol{\theta}_n^{(1)}$ and the singular part be $\boldsymbol{\theta}_n^{(2)}$. Hence the vector parameter $\boldsymbol{\theta}_n$ is rewritten as

$$\boldsymbol{\theta}_n = ((\boldsymbol{\theta}_n^{(1)})^{\mathrm{T}}, (\boldsymbol{\theta}_n^{(2)})^{\mathrm{T}})^{\mathrm{T}}. \tag{6}$$

The main idea of finding the recursive PCRLB is to compute first the PCRLB for the nonsingular part of the vector parameter $\boldsymbol{\theta}_n^{(1)}$ and then update the PCRLB by considering the singular part of the vector parameter $\boldsymbol{\theta}_n^{(2)}$ by using the rule for change of coordinates of parameters. The relationship between the nonsingular part of the vector parameter $\boldsymbol{\theta}_n^{(1)}$ and the singular part of the vector parameter $\boldsymbol{\theta}_n^{(2)}$ has to be defined to apply the rule for change of coordinates of parameters.

Considering a linear state transition function and a nonlinear observation function which are independent of n, we will get the following system model:

$$\boldsymbol{\theta}_{n+1}^{(1)} = \boldsymbol{\theta}_n^{(1)} + \mathbf{u}_n, \tag{7}$$

$$\boldsymbol{\theta}_{n+1}^{(2)} = \mathbf{G}^{(2)} \boldsymbol{\theta}_n^{(2)} + \mathbf{G}^{(3)} \boldsymbol{\theta}_{n+1}^{(1)}, \tag{8}$$

$$\mathbf{z}_n = \mathbf{h}(\boldsymbol{\theta}_n) + \mathbf{v}_n, \tag{9}$$

where $\mathbf{u}_n$ and $\mathbf{v}_n$ are independent and identically distributed system and observation noises, respectively.

Let $\mathbf{K}_n$ be the information matrix for the vector $((\boldsymbol{\theta}_{n-1}^{(1)})^{\mathrm{T}}, (\boldsymbol{\theta}_n^{(1)})^{\mathrm{T}}, (\boldsymbol{\theta}_n^{(2)})^{\mathrm{T}})$. The decomposition of the matrix $\mathbf{K}_n$ into blocks corresponding to the vectors $\boldsymbol{\theta}_{n-1}^{(1)}, \boldsymbol{\theta}_n^{(1)}$ and $\boldsymbol{\theta}_n^{(2)}$ reads

$$\mathbf{K}_n = \begin{pmatrix} \mathbf{K}_n^{(11)} & \mathbf{K}_n^{(12)} & \mathbf{K}_n^{(13)} \\ \mathbf{K}_n^{(21)} & \mathbf{K}_n^{(22)} & \mathbf{K}_n^{(23)} \\ \mathbf{K}_n^{(31)} & \mathbf{K}_n^{(32)} & \mathbf{K}_n^{(33)} \end{pmatrix}. \tag{10}$$

Let $\mathbf{J}_n$ be the information matrix for the vector $((\boldsymbol{\theta}_n^{(1)})^{\mathrm{T}}, (\boldsymbol{\theta}_n^{(2)})^{\mathrm{T}})$. The decomposition of the matrix $\mathbf{J}_n$ into blocks corresponding to the vectors $\boldsymbol{\theta}_n^{(1)}$ and $\boldsymbol{\theta}_n^{(2)}$ reads

$$\mathbf{J}_n = \begin{pmatrix} \mathbf{J}_n^{(11)} & \mathbf{J}_n^{(12)} \\ \mathbf{J}_n^{(21)} & \mathbf{J}_n^{(22)} \end{pmatrix}. \tag{11}$$

The matrix $\mathbf{K}_n$ is computed from the matrix $\mathbf{J}_{n-1}$ as follows:

$$\mathbf{K}_n = \mathbf{M}^{-\mathrm{T}} \mathbf{N}_{n-1} \mathbf{M}^{-1}, \tag{12}$$

where

$$\mathbf{M} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{G}^{(2)} & \mathbf{G}^{(3)} \end{pmatrix}, \tag{13}$$

and

$$\mathbf{N}_{n-1} = \begin{pmatrix} \mathbf{J}_{n-1}^{(11)} + \mathbf{H}_{n-1}^{(11)} & \mathbf{J}_{n-1}^{(12)} + \mathbf{H}_{n-1}^{(12)} & \mathbf{H}_{n-1}^{(13)} \\ (\mathbf{J}_{n-1}^{(12)} + \mathbf{H}_{n-1}^{(12)})^{\mathrm{T}} & \mathbf{J}_{n-1}^{(22)} + \mathbf{H}_{n-1}^{(22)} & \mathbf{H}_{n-1}^{(23)} \\ (\mathbf{H}_{n-1}^{(13)})^{\mathrm{T}} & (\mathbf{H}_{n-1}^{(23)})^{\mathrm{T}} & \mathbf{H}_{n-1}^{(33)} \end{pmatrix}. \tag{14}$$

The matrix $\mathbf{H}_n$ is defined as follows:

$$\mathbf{H}_n^{(11)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_n^{(1)}}^{\boldsymbol{\theta}_n^{(1)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{15}$$

$$\mathbf{H}_n^{(12)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_n^{(1)}}^{\boldsymbol{\theta}_n^{(2)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{16}$$

$$\mathbf{H}_n^{(13)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_n^{(1)}}^{\boldsymbol{\theta}_{n+1}^{(1)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{17}$$

$$\mathbf{H}_n^{(22)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_n^{(2)}}^{\boldsymbol{\theta}_n^{(2)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{18}$$

$$\mathbf{H}_n^{(23)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_n^{(2)}}^{\boldsymbol{\theta}_{n+1}^{(1)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{19}$$

$$\mathbf{H}_n^{(33)} = \mathrm{E}\left\{-\Delta_{\boldsymbol{\theta}_{n+1}^{(1)}}^{\boldsymbol{\theta}_{n+1}^{(1)}} \ln \overline{\mathrm{p}}_n\right\}, \tag{20}$$

where

$$\overline{\mathrm{p}}_n = \mathrm{p}\left(\boldsymbol{\theta}_{n+1}^{(1)}|\boldsymbol{\theta}_n^{(1)}\right) \cdot \mathrm{p}\left(\mathbf{z}_{n+1}|\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}\right). \tag{21}$$

Using the matrix inversion Lemma, the matrix $\mathbf{J}_n$ can be computed recursively by using the matrix $\mathbf{K}_n$ as follows:

$$\mathbf{J}_n = \begin{pmatrix} \mathbf{K}_n^{(22)} & \mathbf{K}_n^{(23)} \\ \mathbf{K}_n^{(32)} & \mathbf{K}_n^{(33)} \end{pmatrix} - \begin{pmatrix} \mathbf{K}_n^{(21)} \\ \mathbf{K}_n^{(31)} \end{pmatrix} (\mathbf{K}_n^{(11)})^{-1} \begin{pmatrix} \mathbf{K}_n^{(21)} \\ \mathbf{K}_n^{(31)} \end{pmatrix}^{\mathrm{T}}. \tag{22}$$

3. SYSTEM MODEL

3.1. NLOS multipath propagation model

The NLOS multipath propagations impair the accuracy of conventional localization techniques which work under the assumption of LOS propagation. In the following we model the NLOS multipath propagations by considering single bounce reflections only. Signals from multiple bounce reflections are not considered as they are assumed to have small power due to the severe attenuation caused by the multiple scattering. In practice, the two step proximity detection algorithm presented in [9] can be applied to detect and discard multiple bounce scattering.

We consider a single BS of known position whereas the position of the stationary scatterers and the trajectory of the MS are unknown. Since large scale mobility is not expected from the MS, the localization scenario is assumed to remain unchanged with respect to the visibility of the scatterers. Fig. 1 shows the system model of a single bounce scattering scenario. The communication between the BS and the MS is established via the NLOS propagation path through the scatterers. The scatterers are denoted by S_l, $l \in \{1, 2, \ldots, L\}$. The transmitted signal propagates from the BS at position $(x_\mathrm{B}, y_\mathrm{B})$ to the l-th scatterer at position $(x_{\mathrm{S},l}, y_{\mathrm{S},l})$ and then to the MS at position $(x_{\mathrm{M},n}, y_{\mathrm{M},n})$ which travels with a velocity $(v_{x,n}, v_{y,n})$ at discrete time instant n, , $n \in \{1, 2, \ldots, N\}$. The total propagation path length is denoted by $d_{n,l}$ and it consists of the path length from the BS to the l-th scatterer denoted by $d_{\mathrm{B},l}$ and the path length from the l-th scatterer to the MS denoted by $d_{\mathrm{M},n,l}$. The angle-of-departure (AOD), the angle-of-arrival (AOA) and the DS are denoted by ψ_l, $\phi_{n,l}$ and $f_{n,l}$, respectively.

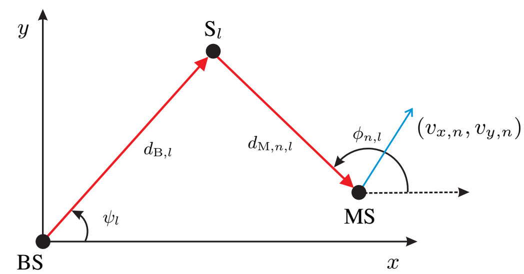

Fig. 1. System model for one NLOS propagation path

The range $d_{n,l}$, the AOD ψ_l, the AOA $\phi_{n,l}$ and the DS $f_{n,l}$ are calculated as

$$d_{n,l} = d_{\mathrm{B},l} + d_{\mathrm{M},n,l} \tag{23}$$

$$d_{\mathrm{B},l} = \sqrt{(x_{\mathrm{S},l} - x_\mathrm{B})^2 + (y_{\mathrm{S},l} - y_\mathrm{B})^2} \tag{24}$$

$$d_{\mathrm{M},n,l} = \sqrt{(x_{\mathrm{S},l} - x_{\mathrm{M},n})^2 + (y_{\mathrm{S},l} - y_{\mathrm{M},n})^2} \tag{25}$$

$$\psi_l = \frac{\pi}{2}(1 - \mathrm{sgn}(x_{\mathrm{S},l} - x_\mathrm{B})) + \tan^{-1}\frac{y_{\mathrm{S},l} - y_\mathrm{B}}{x_{\mathrm{S},l} - x_\mathrm{B}} \tag{26}$$

$$\phi_{n,l} = \frac{\pi}{2}(1 - \mathrm{sgn}(x_{\mathrm{S},l} - x_{\mathrm{M},n})) + \tan^{-1}\frac{y_{\mathrm{S},l} - y_{\mathrm{M},n}}{x_{\mathrm{S},l} - x_{\mathrm{M},n}} \tag{27}$$

$$f_{n,l} = \frac{f_\mathrm{c}}{c}\frac{v_{x,n}(x_{\mathrm{S},l} - x_{\mathrm{M},n}) + v_{y,n}(y_{\mathrm{S},l} - y_{\mathrm{M},n})}{d_{\mathrm{M},n,l}}, \tag{28}$$

where f_c and c are the carrier frequency of the signal and the speed of light, respectively. The state vector is $\boldsymbol{\theta}_n = (v_{x,n}, v_{y,n}, x_{\mathrm{M},n}, y_{\mathrm{M},n}, \mathbf{x}_\mathrm{S}^\mathrm{T}, \mathbf{y}_\mathrm{S}^\mathrm{T})^\mathrm{T}$ where $\mathbf{x}_\mathrm{S} = (x_{\mathrm{S},1}, \ldots, x_{\mathrm{S},L})^\mathrm{T}$ and $\mathbf{y}_\mathrm{S} = (y_{\mathrm{S},1}, \ldots, y_{\mathrm{S},L})^\mathrm{T}$. The range $d_{n,l}$, the bearing ψ_l and $\phi_{n,l}$ and the DS $f_{n,l}$ can be estimated using the SAGE algorithm [10] and the localization can be performed either at the BS or at the MS.

3.2. Observation model

The observations corresponding to the l-th scatterer are assumed to be noisy versions of the range $d_{n,l}$, the bearing ψ_l and $\phi_{n,l}$ and the DS $f_{n,l}$. Let the vectors

$$\mathbf{d}_n = (d_{n,1}, d_{n,2}, \ldots, d_{n,L})^\mathrm{T}, \tag{29}$$

$$\boldsymbol{\psi} = (\psi_1, \psi_2, \ldots, \psi_L)^\mathrm{T}, \tag{30}$$

$$\boldsymbol{\phi}_n = (\phi_{n,1}, \phi_{n,2}, \ldots, \phi_{n,L})^\mathrm{T}, \tag{31}$$

$$\mathbf{f}_n = (f_{n,1}, f_{n,2}, \ldots, f_{n,L})^\mathrm{T}, \tag{32}$$

denote the range, the bearing and the DS of all the scatterers at time instant n. The relation between the $4L$-dimensional observation vector $\mathbf{z}_n$ and the state vector $\boldsymbol{\theta}_n$ is defined by equation (9) where $\mathbf{h}(\boldsymbol{\theta}_n)$ is defined as

$$\mathbf{h}(\boldsymbol{\theta}_n) = (\mathbf{d}_n^\mathrm{T}(\boldsymbol{\theta}_n), \boldsymbol{\psi}^\mathrm{T}(\boldsymbol{\theta}_n), \boldsymbol{\phi}_n^\mathrm{T}(\boldsymbol{\theta}_n), \mathbf{f}_n^\mathrm{T}(\boldsymbol{\theta}_n))^\mathrm{T} \tag{33}$$

and

$$\mathbf{v}_n \sim \mathcal{N}(\mathbf{0}_{4L}, \mathbf{R}_n). \tag{34}$$

$\mathbf{R}_n$ is the $4L \times 4L$ covariance matrix defined as

$$\begin{aligned}\mathbf{R}_n = \; & \mathrm{diag}(\sigma^2_{d_{n,1}}, \ldots, \sigma^2_{d_{n,L}}, \sigma^2_{\psi_1}, \ldots, \sigma^2_{\psi_L}, \\ & \sigma^2_{\phi_{n,1}}, \ldots, \sigma^2_{\phi_{n,L}}, \sigma^2_{f_{n,1}}, \ldots, \sigma^2_{f_{n,L}}),\end{aligned} \tag{35}$$

i.e., the observation noises are assumed to have independent distribution. Thus the conditional pdf $\mathrm{p}(\mathbf{z}_n|\boldsymbol{\theta}_n)$ can be calculated as

$$\mathrm{p}(\mathbf{z}_n|\boldsymbol{\theta}_n) = \frac{\exp\left(-\frac{1}{2}(\mathbf{z}_n - \mathbf{h}(\boldsymbol{\theta}_n))^\mathrm{T}\mathbf{R}_n^{-1}(\mathbf{z}_n - \mathbf{h}(\boldsymbol{\theta}_n))\right)}{(2\pi)^{\frac{4L}{2}}\det^{\frac{1}{2}}(\mathbf{R}_n)}. \tag{36}$$

3.3. Mobility model

The mobility of the MS is described by the nearly constant velocity model in which the MS is assumed to move with a constant velocity during the sampling interval δt. The random movement of the MS is modeled by the random changes in the velocity in every sampling time interval which is modeled by a Gaussian noise. Thus the state vector is partitioned as

$$\boldsymbol{\theta}_n^{(1)} = (v_{x,n}, v_{y,n})^\mathrm{T}, \tag{37}$$

$$\boldsymbol{\theta}_n^{(2)} = (x_{\mathrm{M},n}, y_{\mathrm{M},n}, \mathbf{x}_\mathrm{S}^\mathrm{T}, \mathbf{y}_\mathrm{S}^\mathrm{T})^\mathrm{T}. \tag{38}$$

The mobility model is defined by equation (7) and (8) with

$$\mathbf{u}_n \sim \mathcal{N}(\mathbf{0}_2, \mathbf{Q}_n), \tag{39}$$

$$\mathbf{Q}_n = \mathrm{diag}\left(\sigma^2_{u_{x,n}}, \sigma^2_{u_{y,n}}\right), \tag{40}$$

$$\mathbf{G}^{(2)} = \mathbf{I}_{2(L+1)}, \tag{41}$$

$$\mathbf{G}^{(3)} = \left(\mathrm{diag}\left(\delta t, \delta t\right), \mathbf{0}_{2\times 2L}\right)^\mathrm{T}. \tag{42}$$

Thus the matrix $\mathbf{M}$ in equation (13) can be constructed using the actual values of $\mathbf{G}^{(2)}$ and $\mathbf{G}^{(3)}$. The conditional pdf $\mathrm{p}\left(\boldsymbol{\theta}_{n+1}^{(1)}|\boldsymbol{\theta}_n^{(1)}\right)$ can be calculated as

$$\mathrm{p}\left(\boldsymbol{\theta}_{n+1}^{(1)}|\boldsymbol{\theta}_n^{(1)}\right) = \frac{\exp\left(-\frac{1}{2}(\boldsymbol{\theta}_{n+1}^{(1)} - \boldsymbol{\theta}_n^{(1)})^\mathrm{T}\mathbf{Q}_n^{-1}(\boldsymbol{\theta}_{n+1}^{(1)} - \boldsymbol{\theta}_n^{(1)})\right)}{(2\pi)^{\frac{2}{2}}\det^{\frac{1}{2}}(\mathbf{Q}_n)}. \tag{43}$$

The initial state vector $\boldsymbol{\theta}_0$ is independent of $\mathbf{u}_n$ and has the Gaussian distribution $\boldsymbol{\theta}_0 \sim \mathcal{N}(\mathbf{c}_\mathrm{s}, \mathbf{Q}_s)$.

4. PCRLB FOR NLOS MULTIPATH ENVIRONMENTS

To evaluate the PCRLB, we need to find the partial derivatives of $\ln \overline{\mathrm{p}}_n$ which is a sum of $\ln \mathrm{p}\left(\boldsymbol{\theta}_{n+1}^{(1)}|\boldsymbol{\theta}_n^{(1)}\right)$ and $\ln \mathrm{p}\left(\mathbf{z}_{n+1}|\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}\right)$. Using equations (43) and (36) we find

$$-\ln \mathrm{p}\left(\boldsymbol{\theta}_{n+1}^{(1)}|\boldsymbol{\theta}_n^{(1)}\right) = \mathrm{c}_1 + \frac{1}{2}(\boldsymbol{\theta}_{n+1}^{(1)} - \boldsymbol{\theta}_n^{(1)})^\mathrm{T}\mathbf{Q}_n^{-1}(\boldsymbol{\theta}_{n+1}^{(1)} - \boldsymbol{\theta}_n^{(1)}), \tag{44}$$

and

$$\begin{aligned}-\ln \mathrm{p}\left(\mathbf{z}_{n+1}|\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}\right) = \; & \mathrm{c}_2 + \frac{1}{2}(\mathbf{z}_{n+1} - \mathbf{h}(\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}))^\mathrm{T} \\ & \mathbf{R}_n^{-1}(\mathbf{z}_{n+1} - \mathbf{h}(\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)})).\end{aligned} \tag{45}$$

c_1 and c_2 are constants. Thus equations (15) – (20) can be calculated using equation (44) and (45) as

$$\mathbf{H}_n^{(11)} = \mathbf{Q}_n^{-1}, \tag{46}$$

$$\mathbf{H}_n^{(12)} = \mathbf{0}_{2\times 2(L+1)}, \tag{47}$$

$$\mathbf{H}_n^{(13)} = -\mathbf{Q}_n^{-1}, \tag{48}$$

$$\mathbf{H}_n^{(22)} = \mathbf{V}_n\mathbf{R}_n^{-1}\mathbf{V}_n^{\mathrm{T}}, \tag{49}$$

$$\mathbf{H}_n^{(23)} = \mathbf{V}_n\mathbf{R}_n^{-1}\mathbf{W}_n^{\mathrm{T}}, \tag{50}$$

$$\mathbf{H}_n^{(33)} = \mathbf{Q}_n^{-1} + \mathbf{W}_n\mathbf{R}_n^{-1}\mathbf{W}_n^{\mathrm{T}}, \tag{51}$$

where

$$\mathbf{V}_n = \nabla_{\boldsymbol{\theta}_n^{(2)}}\mathbf{h}^{\mathrm{T}}(\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}), \tag{52}$$

$$\mathbf{W}_n = \nabla_{\boldsymbol{\theta}_{n+1}^{(1)}}\mathbf{h}^{\mathrm{T}}(\boldsymbol{\theta}_n^{(2)}, \boldsymbol{\theta}_{n+1}^{(1)}). \tag{53}$$

The derivations of the Jacobian matrices $\mathbf{V}_n$ and $\mathbf{W}_n$ are not shown here for brevity.

The initial recursion value $\mathbf{J}_0$ is defined by the Jacobian of the a-priori pdf $\mathrm{p}\left(\boldsymbol{\theta}_0^{(1)}, \boldsymbol{\theta}_0^{(2)}, \mathbf{z}_1\right) = \mathrm{p}\left(\boldsymbol{\theta}_0^{(1)}, \boldsymbol{\theta}_0^{(2)}\right)$. Since a Gaussian distributed initial state vector is assumed

$$\mathbf{J}_0 = \mathbf{Q}_{\mathrm{s}}^{-1}. \tag{54}$$

The PCRLB of the MS position $\gamma_{\mathrm{M},n}$ at time instant n can thus be determined as

$$\gamma_{\mathrm{M},n} = \sqrt{\frac{[\mathbf{J}_n^{-1}]^{(33)} + [\mathbf{J}_n^{-1}]^{(44)}}{2}}. \tag{55}$$

5. RESULTS AND DISCUSSION

Let's consider a pico-cell scenario which covers indoor areas like offices, museums etc which are characterized by NLOS multipath propagations. The MS has an initial velocity of $(v_{x,0}, v_{y,0}) = (0.5, -0.5)$ m/s which is assumed to be an average indoor walking speed. The initial position of the MS is generated with the distribution $\mathcal{N}((20, 20)\mathrm{m}, \mathrm{diag}\left(20^2, 20^2\right)\mathrm{m}^2)$. $L = 3$ scatterers are located at $\mathbf{x}_{\mathrm{S}} = (31, 40, 19)^{\mathrm{T}}$ m and $\mathbf{y}_{\mathrm{S}} = (18, 14, 26)^{\mathrm{T}}$ m. The BS is positioned at $\mathbf{p}_{\mathrm{B}} = (0, 0)$ m. The sampling interval is $\delta t = 0.5$ s and the total path length is $N = 20$. The signal carrier frequency is $f_{\mathrm{c}} = 2.4$ GHz. The system noise covariance matrix is $\mathbf{Q} = \mathrm{diag}\left(0.5^2, 0.5^2\right)\mathrm{m}^2/\mathrm{s}^2$. The covariance matrix of the initial state vector is $\mathbf{Q}_{\mathrm{s}} = \mathrm{diag}\left(20^2 \cdot \mathbf{1}_{2(L+2)}^{\mathrm{T}}\right)$. The standard deviation of the noise for the range measurement from the TOA is σ_d, for the AOD measurement is σ_ψ, for the AOA measurement is σ_ϕ and for the DS measurement is σ_f for each of the scatterers.

Fig. 2 shows the PCRLB $\gamma_{\mathrm{M},n}$ versus the time instant n for the parameters $\sigma_d = 1$ m, $\sigma_\psi = \sigma_\phi = 5^\circ$ and $\sigma_f = 1$ Hz. The TOA/AOD/AOA/DS (d, ψ, ϕ, f) method has the lowest PCRLB as it utilizes all the observations. The PCRLB of the TOA/AOD/AOA (d, ψ, ϕ) method is also shown in the figure. At $n = 1$ there is little difference between the PCRLB of the TOA/AOD/AOA/DS (d, ψ, ϕ, f) method and the TOA/AOD/AOA (d, ψ, ϕ) method as the DS observation provides little more information about the position of the MS than what is already extracted from the AOA observation. However, after a few iterations, an almost constant gap between the PCRLB of the TOA/AOD/AOA/DS (d, ψ, ϕ, f) method and the TOA/AOD/AOA (d, ψ, ϕ) method develops which accounts for the extra information obtained from the sequences of the DS observations. A higher PCRLB is obtained for the estimation based on the TOA/AOD/DS (d, ψ, f) method which, however, significantly improves as more and more observations are considered. At $n = 1$ the TOA/AOD/DS (d, ψ, f) method has a PCRLB value which depends on the covariance matrix $\mathbf{Q}_{\mathrm{s}}$ as position estimation using the TOA/AOD/DS (d, ψ, f) requires at least $n = 2$ time instants. The performance of a TOA/AOD/AOA tracking algorithm using the extended Kalman filter [11] proposed in [12] is also shown in Fig. 2. It can be seen that the performance is bounded by the PCRLB.

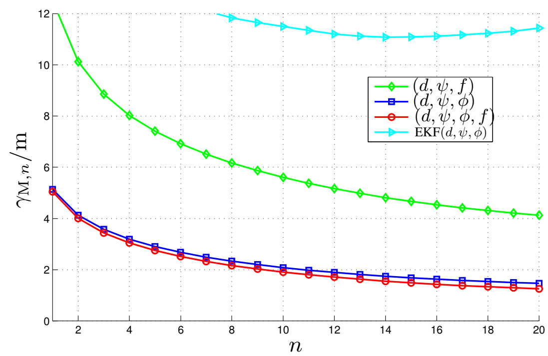

Fig. 2. The PCRLB of the MS position $\gamma_{\mathrm{M},n}$ and the extended Kalman filter (EKF) algorithm versus the time instant n.

The Jacobian matrices of the AOA $\boldsymbol{\phi}_n$ and the DS shift $\mathbf{f}_n$ are linearly related as follows

$$\nabla_{\boldsymbol{\theta}_n^{(2)}}\mathbf{f}_n = \left(\nabla_{\boldsymbol{\theta}_n^{(2)}}\boldsymbol{\phi}_n\right)\mathbf{U}_n, \tag{56}$$

$$\nabla_{\boldsymbol{\theta}_{n+1}^{(1)}}\mathbf{f}_n = \left(\nabla_{\boldsymbol{\theta}_{n+1}^{(1)}}\boldsymbol{\phi}_n\right)\mathbf{U}_n + \frac{f_{\mathrm{c}}}{c}\begin{pmatrix} \mathbf{1}_L^{\mathrm{T}}\mathbf{C}_{\phi,n} \\ \mathbf{1}_L^{\mathrm{T}}\mathbf{S}_{\phi,n} \end{pmatrix}, \tag{57}$$

where

$$\mathbf{U}_n = \frac{f_{\mathrm{c}}}{c}\left(-v_{x,n}\mathbf{S}_{\phi,n} + v_{y,n}\mathbf{C}_{\phi,n}\right), \tag{58}$$

$$\mathbf{C}_{\phi,n} = \mathrm{diag}\left(\cos\phi_{n,1}, \cos\phi_{n,2}, \ldots, \cos\phi_{n,L}\right), \tag{59}$$

$$\mathbf{S}_{\phi,n} = \mathrm{diag}\left(\sin\phi_{n,1}, \sin\phi_{n,2}, \ldots, \sin\phi_{n,L}\right). \tag{60}$$

It is thus possible to get the same performance from the TOA/AOD/DS (d, ψ, f) sequences of observations as the TOA/AOD/AOA (d, ψ, ϕ) sequences of observations. However, it is difficult to determine analytically the relation between the σ_ϕ and the σ_f which result in the same performance. It can also be seen from Fig. 2 that, given sufficiently long sequences of observations, the PCRLB of the TOA/AOD/DS (d, ψ, f) method approaches the PCRLB of the TOA/AOD/AOA (d, ψ, ϕ) method. The TOA/AOD/DS (d, ψ, f) method is attractive in that all the observations can be obtained by employing multiple antennas at the BS only. Estimation of the AOA at the MS requires multiple antennas at the MS which is difficult to implement in hand-held MSs. Accommodating multiple antennas in hand-held MSs is difficult due to physical space constraint. It is also a challenging task to estimate the AOA while accounting for the orientation of the MS. Besides, the antenna calibration process for measuring the AOA which accounts for the usage conditions of the hand–held MS is a daunting task. The downside of the TOA/AOD/DS (d, ψ, f) method is that its performance depends on the mobility of the MS.

6. CONCLUSIONS

We have presented the PCRLB for tracking an MS in an NLOS multipath environment. The developed PCRLB can be used to evaluate the performance of a tracking algorithm which tracks an MS in NLOS multipath environments. It has also been shown that the DS observation can be exploited to yield a PCRLB comparable with that when considering the AOA observations.

7. REFERENCES

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