EFFICIENT DOA, DOD, AND TARGET ESTIMATION FOR BISTATIC MIMO SONAR

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ABSTRACT

In this paper, we consider the extension of the capabilities of autonomous underwater vehicles (AUV's) to operate in pairs, thus representing each pair of sonars as a bistatic sonar. We utilize principles from the emerging MIMO radar theory, considering both the sonar transmitter and the sonar receiver as systems of closely spaced antennas that transmit/receive multiple linearly independent waveforms. Our objective is to formulate a covariance matrix that allows independent direction of arrival (DOA), direction of departure (DOD), and target strength estimation with reduced complexity. Simulation results demonstrate the effectiveness of the proposed method.

***Index Terms*—** AUV, radar, sonar, MIMO, DOA, DOD.

1. INTRODUCTION

Underwater sensing has received considerable attention over the past years, due to the increased use of unmanned marine vehicles, both underwater vehicles known as autonomous underwater vehicles (AUV's) and surface vehicles [1], and the evolving potential for sensor network deployment for underwater exploration. Although seawater is much more complicated as a medium for sound propagation [2] than air is for electromagnetic wave propagation, sonar development [3–6] has been based on the principles of radar theory [7–9].

In particular, sound speed propagation in seawater is almost constant in shallow water regions, with mild temperature/ salinity variations from the surface to the bottom, relatively low water pollution, and relatively high carrier frequency. AUV's thrive in these regions, equipped with sensors like multibeam and sidescan sonars [10]. However, the increasing demands for underwater exploration present new challenges that shall increase the overall effectiveness of AUV's, such as operation in pairs or in swarm mode. The potential for use of AUV's in pairs leads to the consideration of bistatic sonar (borrowing ideas from bistatic radar, [11]) as a sonar whose transmitter and receiver are widely separated arrays of transceiver hydrophones.

A wealth of works have addressed the problem of bistatic radar, a number of them using the emerging MIMO radar theory [12]. Unlike a phased-array radar, which transmits scaled versions of a single waveform, a MIMO radar system transmits multiple diverse (possibly linearly independent) waveforms via its antennas. This waveform diversity enables superior capabilities compared with a standard phased-array radar [13].

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A basic problem with bistatic radar is the joint estimation at the receiver of the RCS (target strength in sonar terminology) $\beta(\theta_t, \theta_r)$, for each direction-of-arrival (DOA) / direction-of-departure (DOD) pair of interest (θ_t, θ_r) from the observed data matrix. The estimates of $\beta(\theta_t, \theta_r)$ are used to form a two-dimensional spatial spectrum. The locations of the targets are estimated, together with the complex amplitudes of their target strengths, by searching the peaks of the obtained spectrum. In the following, we assume that the transmit array has M_t elements, the receive array has M_r elements, there are K targets at every range bin, and each pulse consists of a train of N subpulses.

A few works have addressed the issue of formation and manipulation of the observed data matrix of the bistatic MIMO radar. In [14], Bekkerman et al. form a data vector of size $M_t M_r \times 1$ after matching the received array samples with the transmitted signal vector, rearrange columnwise, and form the covariance matrix by multiplication of the data vector with its conjugate transpose, however a bistatic case is not considered explicitly. In [15], Yan et al. assume a 2-D model, form a data matrix of size $M_t M_r \times N$ as in [14], and perform Capon estimation using the resulting correlation matrix of size $M_t M_r \times M_t M_r$. In [16], Jiang et al. assume a 3-D model where the receive array is a rectangular grid and use the ESPRIT algorithm on a correlation matrix that is formed on the same principles. ESPRIT-based methods are also used in [17], [18], [19], and [20]. In [21], Liu et al. reduce the problem of DOA and DOD estimation to two 1-D searches and compute a mutual coupling coefficient. In [22], Nion and Sidiropoulos use the parallel factor (PARAFAC) model to perform target localization. In [23], Cheng et al. formulate the matched filter output to a 5th-order tensor using a PARAFAC model. In [24], Hassanien et al. form the data vector in the same manner as in [14], although their main consideration is target estimation with Doppler shift in noncoherent MIMO radars.

1.1. Our Contribution

In this paper we perform independent DOA, DOD, and target estimation of a bistatic MIMO sonar considered as a pair of uniform linear arrays (ULAs) in a 2-D environment by forming two separate covariance matrices and performing two 1-D searches with reduced complexity and in a form that is suitable for previous estimation methods. We also assume that transmitter and receiver are widely separated and are moving independently, with no means of synchronization between them. The proposed algorithm is suboptimal by nature, however it exhibits near-optimal performance with an increasing number of array elements at the transmitter and the receiver. Furthermore, the problem of determining the target position based on the estimated target direction is addressed. Future papers topics would be the performance comparison between var-

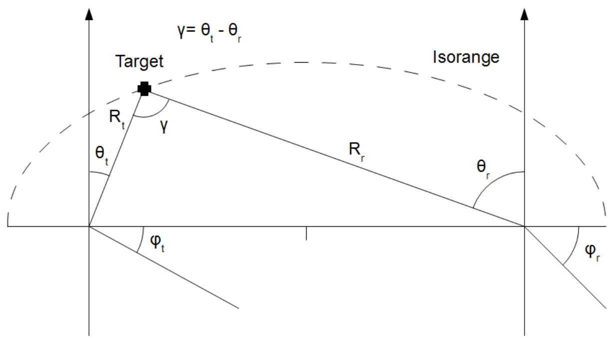

Fig. 1. Bistatic radar.

ious nonparametric estimators including the Capon estimator (also with Doppler shift), the performance comparison between particular transmit signal designs, and the generalization in 3 dimensions.

2. BISTATIC MIMO SONAR SIGNAL MODEL

We consider a two-dimensional bistatic sonar system where both the Transmitter (Tx) and the Receiver (Rx) are located underwater in a shallow water region, where the maximum sea-bottom depth and the distance between Tx and Rx are in the same order of magnitude. Furthermore the wavelength λ of the acoustic transmission is much smaller than the maximum depth, therefore the ray theory is applicable. We are working in two dimensions and assume that there is no motion of either targets or the transmitter-receiver pair, thus no Doppler shift.

As it is illustrated in Fig. 1, the baseline $2\alpha e$ is the distance between Tx and Rx. The factor e is the eccentricity of an ellipse ($0 < e < 1$) which is defined as an isorange contour and α is the semi-major axis of the ellipse. Angle γ is the bistatic angle. Both angles θ_t and θ_r are measured clockwise positive ($-\pi/2 \leq \theta_t, \theta_r \leq \pi/2$) relative to the baseline normal. Angles ϕ_t and ϕ_r are measured relative to the baseline and are also clockwise positive.

The discrete-time vector output signal at the receiver array can be formed as

$$\mathbf{y}(n) = \sum_{k=1}^{K} \beta_k \mathbf{b}_k^* \mathbf{a}_k^H \mathbf{x}(n) + \mathbf{w}(n) \tag{1}$$

where β_k is the target strength (the equivalent of RCS in radar theory, considered as Swerling type I) of target $k = 1, 2, \ldots, K$ and $\mathbf{w}(n)$ denotes additive white noise at the receive array with $E\left[\mathbf{w}\mathbf{w}^H\right] = \sigma_w^2 \mathbf{I}_{M_r}$ and

$$\mathbf{a}_k \triangleq \left[\begin{array}{ccc} e^{j2\pi f_0 \tau_{0,k}} & \ldots & e^{j2\pi f_0 \tau_{M_t-1,k}} \end{array} \right]^T$$

and

$$\mathbf{b}_k \triangleq \left[\begin{array}{ccc} e^{j2\pi f_0 \tilde{\tau}_{0,k}} & \ldots & e^{j2\pi f_0 \tilde{\tau}_{M_r-1,k}} \end{array} \right]^T$$

are the vectors of the acoustic wave travel from Tx to the target and from the target to Rx, respectively. The terms $\tau_{m_t,k}$ and $\tilde{\tau}_{m_r,k}$ are the time intervals for the acoustic ray to travel from the m_t element of the transmit array to the k-point target and then to the m_r element of the receive array, that is,

$$\tau_{m_t,k} \triangleq \frac{R_{t,k}}{v_s} - \frac{m_t D_t}{v_s} \sin\left(\theta_{t,k} - \phi_t\right) \tag{2}$$

and

$$\tilde{\tau}_{m_r,k} \triangleq \frac{R_{r,k}}{v_s} - \frac{m_r D_r}{v_s} \sin\left(\theta_{r,k} - \phi_r\right) \tag{3}$$

for $\phi_t - \pi/2 < \theta_{t,k} < \phi_t + \pi/2$ and for $\phi_r - \pi/2 < \theta_{r,k} < \phi_r + \pi/2$, respectively. The term v_s is the speed of sound in seawater (assumed constant), f_0 is the sound frequency, $R_{t,k}$ is the distance from the first element of the transmit array to the k-target, $R_{r,k}$ is the distance from the k-target to the first element of the receiver, D_t is the Tx inter-element spacing, D_r is the Rx inter-element spacing, $m_t = 0, 1, \ldots, M_t - 1$, and $m_r = 0, 1, \ldots, M_r - 1$. Then,

$$b_{m_r,k}^* \cdot a_{m_t,k}^* = e^{-j2\pi f_0 \tilde{\tau}_{m_r,k}} \cdot e^{-j2\pi f_0 \tau_{m_t,k}} = \exp\left[-j2\pi \frac{R_{t,k} + R_{r,k}}{\lambda}\right] \cdot \exp\left[j m_t g_{t,k} + j m_r g_{r,k}\right] \tag{4}$$

where

$$g_{t,k} \triangleq 2\pi \tfrac{D_t}{\lambda} \sin\left(\theta_{t,k} - \phi_t\right), \quad g_{r,k} \triangleq 2\pi \tfrac{D_r}{\lambda} \sin\left(\theta_{r,k} - \phi_r\right). \tag{5}$$

Assuming that $D_t = D_r = \lambda/2$ to avoid aliasing and under the assumption that all targets have the same bistatic range, (1) can be restated as

$$\mathbf{y}(n) = \exp\left[-j2\pi \frac{2\alpha}{\lambda}\right] \cdot \left(\sum_{k=1}^{K} \beta_k \mathbf{r}_k^* \mathbf{t}_k^H \right) \mathbf{x}(n) + \mathbf{w}(n). \tag{6}$$

We define by $\mathbf{M} = \sum_{k=1}^{K} \beta_k \mathbf{r}_k^* \mathbf{t}_k^H$ the target matrix which, under the constraints $K \leq M_t$ and $K \leq M_r$, can be rearranged in the form

$$\mathbf{M} = \mathbf{R}^* \mathbf{B} \mathbf{T}^H = \left[\mathbf{r}_1^* \;\; \mathbf{r}_2^* \;\; \cdots \;\; \mathbf{r}_K^*\right] \cdot \begin{bmatrix} \beta_1 & 0 & \cdots & \cdots \\ 0 & \beta_2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \beta_K \end{bmatrix} \cdot \begin{bmatrix} \mathbf{t}_1^H \\ \mathbf{t}_2^H \\ \vdots \\ \mathbf{t}_K^H \end{bmatrix} \tag{7}$$

where

$$\mathbf{r}_k = \begin{bmatrix} e^{-j(M_r-k+1)g_{r,k}} \\ \vdots \\ e^{-j(M_r-1)g_{r,k}} \\ 1 \\ e^{-jg_{r,k}} \\ \vdots \\ e^{-j(M_r-k)g_{r,k}} \end{bmatrix}, \quad \mathbf{t}_k = \begin{bmatrix} e^{-j(M_t-k+1)g_{t,k}} \\ \vdots \\ e^{-j(M_t-1)g_{t,k}} \\ 1 \\ e^{-jg_{t,k}} \\ \vdots \\ e^{-j(M_t-k)g_{t,k}} \end{bmatrix}. \quad \begin{matrix} 1 \\ \vdots \\ k-1 \\ k \\ k+1 \\ \vdots \\ M_t \end{matrix}$$

Thus,

$$\frac{1}{M_t} \mathbf{T}^H \mathbf{T} = \frac{1}{M_t} \begin{bmatrix} \mathbf{t}_1^H \\ \mathbf{t}_2^H \\ \vdots \\ \mathbf{t}_K^H \end{bmatrix} \begin{bmatrix} \mathbf{t}_1 & \mathbf{t}_2 & \cdots & \mathbf{t}_K \end{bmatrix}. \tag{8}$$

The elements of the main diagonal of $\frac{1}{M_t}\mathbf{T}^H\mathbf{T}$ are equal to 1. Consider indices k and l, where $k < l$, $l - k = d$ and $k, l \leq K$. Then, the off-diagonal elements of $\mathbf{T}^H\mathbf{T}$ can be formed as

$$\left(\mathbf{T}^H\mathbf{T}\right)_{(k,l)} = e^{jdg_{t,l}} \sum_{m_t=M_t-k+1}^{M_t-1} e^{jm_t(g_{t,k}-g_{t,l})}+ \\ +e^{-j(M_t-d)g_{t,l}} \sum_{m_t=0}^{d-1} e^{jm_t(g_{t,k}-g_{t,l})}+e^{jdg_{t,l}} \sum_{m_t=0}^{M_t-l} e^{jm_t(g_{t,k}-g_{t,l})}. \tag{9}$$

Based on (9), matrix $\frac{1}{M_t}\mathbf{T}^H\mathbf{T}$ is a $K \times K$ matrix whose elements represent the mutual coupling between targets as "observed" from the transmit array. The matrix is hermitian symmetric, thus diagonalizable. All the elements on the main diagonal equal to 1 and the off-diagonal elements tend to 0 for increasing M_t, being the weighted sums of complex sinusoids. For increasing M_t, $\frac{1}{M_t}\mathbf{T}^H\mathbf{T} \approx \mathbf{I}_K$. From singular value decomposition (SVD), $\mathbf{M} = \mathbf{U\Sigma V}^H$ where $\mathbf{U}$ contains the left singular vectors, $\mathbf{V}$ contains the right singular vectors, and $\mathbf{\Sigma}$ is a diagonal $M_r \times M_t$ matrix whose elements are the positive singular values, thus

$$\frac{1}{M_t}\mathbf{M}\mathbf{M}^H = \mathbf{R}^*\mathbf{B}\left(\frac{1}{M_t}\mathbf{T}^H\mathbf{T}\right)\mathbf{B}^H\mathbf{R}^T = \\ = \mathbf{U\Sigma}\left(\frac{1}{M_t}\mathbf{V}^H\mathbf{V}\right)\mathbf{\Sigma}^H\mathbf{U}^H \approx \mathbf{R}^*\mathbf{B}\mathbf{B}^H\mathbf{R}^T \tag{10}$$

where $\mathbf{B}\mathbf{B}^H = \mathbf{B}^H\mathbf{B}$ is a diagonal $K \times K$ matrix whose elements on the main diagonal are the square magnitudes of the target strengths ($|\beta_k|^2, k = 1, \ldots, K$). If we follow the same reasoning, then $\frac{1}{M_r}\mathbf{R}^T\mathbf{R}^* \approx \mathbf{I}_K$. Thus,

$$\frac{1}{M_r}\mathbf{M}^H\mathbf{M} = \mathbf{T}\mathbf{B}^H\left(\frac{1}{M_t}\mathbf{R}^T\mathbf{R}^*\right)\mathbf{B}\mathbf{T}^H = \\ \mathbf{V\Sigma}^H\left(\frac{1}{M_r}\mathbf{U}^H\mathbf{U}\right)\mathbf{\Sigma V}^H \approx \mathbf{T}\mathbf{B}^H\mathbf{B}\mathbf{T}^H \tag{11}$$

Equations (10) and (11) indicate that, even if $\frac{1}{M_t}\mathbf{T}^H\mathbf{T}$ does not approximate the identity matrix $\mathbf{I}_K$ properly, estimating θ_r is independent of the knowledge of θ_t and vice versa. This is in accordance with the results of [25], where $\text{CRLB}_{\theta_r,\theta_t} = \mathbf{0}$.

3. TARGET ESTIMATION

To extend our analysis to more than one time sample, the transmit array broadcasts a pulse train composed of N vector subpulses simultaneously emitted from all array elements. An attenuated replica of the pulse train arrives at the receive array from K targets at the same range, thus

$$\mathbf{Y}(n) = \mathbf{MX} + \mathbf{W}(n) \tag{12}$$

where

$$\mathbf{Y}(n) = \begin{bmatrix}\mathbf{y}(n-N+1) & \mathbf{y}(n-N+2) & \cdots & \mathbf{y}(n)\end{bmatrix}$$

and

$$\mathbf{X} = \begin{bmatrix}\mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(n) & \cdots & \mathbf{x}(N)\end{bmatrix}.$$

We define $\hat{\mathbf{R}}_{xx} \triangleq \mathbf{X}\mathbf{X}^H = \sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) = \sigma_x^2\mathbf{I}_{M_t}$ and

$$\mathbf{W}(n) = \begin{bmatrix}\mathbf{w}(n-N+1) & \mathbf{w}(n-N+2) & \cdots & \mathbf{w}(n)\end{bmatrix}$$

where $\mathbf{W}$ represents spatio-temporal white noise with covariance matrix with $E\left[\mathbf{W}(n)\mathbf{W}^H(n)\right] = \sum_{i=n-N+1}^{n} E\left[\mathbf{w}(i)\mathbf{w}^H(i)\right] = N\sigma_w^2\mathbf{I}_{M_r}$, and $E\left[\mathbf{W}^H(n)\mathbf{W}(n)\right] = M_r\sigma_w^2\mathbf{I}_N$. We also define

$$\begin{aligned}\mathbf{R}_{yyr} &= E\left[\mathbf{Y}(n)\mathbf{X}^H \cdot (\mathbf{Y}(n)\mathbf{X}^H)^H\right] = \\ &= E\left[\left(\mathbf{MXX}^H + \mathbf{W}(n)\mathbf{X}^H\right)\left(\mathbf{XX}^H\mathbf{M}^H + \mathbf{XW}^H(n)\right)\right] \\ &= \sigma_x^4\mathbf{M}\mathbf{M}^H + E\left[\mathbf{W}(n)\mathbf{X}^H\mathbf{X}\mathbf{W}^H(n)\right] \\ &\approx M_t\sigma_x^4\mathbf{R}^*|\mathbf{B}|^2\mathbf{R}^T + M_t\sigma_x^2\sigma_w^2\mathbf{I}_{M_r}\end{aligned} \tag{13}$$

where

$$\begin{aligned}&E\left[\mathbf{W}(n)\mathbf{X}^H\mathbf{X}\mathbf{W}^H(n)\right] \\ &= E\left[\sum_{i=1}^{N}\mathbf{w}(n-N+i)\mathbf{x}^H(i)\sum_{j=1}^{N}\mathbf{x}(j)\mathbf{w}^H(n-N+j)\right] \\ &= E\left[\sum_{i=1}^{N}\mathbf{w}(n-N+i)\mathbf{x}^H(i)\mathbf{x}(i)\mathbf{w}^H(n-N+i)\right] + \mathbf{0} \\ &= \sum_{i=1}^{N}\mathbf{x}^H(i)\mathbf{x}(i)E\left[\mathbf{w}(n-N+i)\mathbf{w}^H(n-N+i)\right] \\ &= \sigma_w^2\mathbf{I}_{M_r}\text{trace}(\mathbf{X}^H\mathbf{X}) = M_t\sigma_x^2\sigma_w^2\mathbf{I}_{M_r}\end{aligned} \tag{14}$$

and

$$\begin{aligned}\mathbf{R}_{yyt} &= E\left[(\mathbf{Y}(n)\mathbf{X}^H)^H \cdot \mathbf{Y}(n)\mathbf{X}^H\right] \\ &= E\left[(\mathbf{X}\mathbf{Y}^H(n)\mathbf{Y}(n)\mathbf{X}^H\right] \\ &= \sigma_x^4\mathbf{M}^H\mathbf{M} + \mathbf{X}E\left[\mathbf{W}^H\mathbf{W}\right]\mathbf{X}^H \\ &= \sigma_x^4\mathbf{M}^H\mathbf{M} + M_r\sigma_w^2\mathbf{X}\mathbf{I}_N\mathbf{X}^H \\ &\approx M_r\sigma_x^4\mathbf{T}|\mathbf{B}|^2\mathbf{T}^H + M_r\sigma_w^2\sigma_x^2\mathbf{I}_{M_t}.\end{aligned} \tag{15}$$

At this point, we can perform target estimation, based on the covariance matrices of (13) and (15). A conventional method is the use of Capon estimators, however the form of the obtained covariance matrices permits the use of other nonparametric target estimation methods. Due to the fact that that we have limited data available, we typically form the sample covariance matrices, $\hat{\mathbf{R}}_{yyr} = \mathbf{YX}^H\mathbf{XY}^H$ and $\mathbf{R}_{yyt} = \mathbf{XY}^H\mathbf{YX}^H$. For DOD estimation, the Capon spatial design problem is

$$\min_{\mathbf{v}}(\mathbf{v}^H\mathbf{R}_{yyr}\mathbf{v}) \quad \text{subject to} \quad \mathbf{v}^H\mathbf{r}(\theta_r) = 1. \tag{16}$$

The solution to (16) is given by (see [26])

$$\mathbf{v} = \frac{1}{\mathbf{r}^H(\theta_r)\mathbf{R}_{yyr}^{-1}\mathbf{r}(\theta_r)}\mathbf{R}_{yyr}^{-1}\mathbf{r}(\theta_r). \tag{17}$$

The Capon DOD estimates are obtained as the locations of the K largest peaks of the function in (17). Equivalently, for the DOA,

$$\min_{\mathbf{u}}(\mathbf{u}^H\mathbf{R}_{yyt}\mathbf{u}) \quad \text{subject to} \quad \mathbf{u}^H\mathbf{t}(\theta_t) = 1 \tag{18}$$

with

$$\mathbf{u} = \frac{1}{\mathbf{t}^H(\theta_t)\mathbf{R}_{yyt}^{-1}\mathbf{t}(\theta_t)}\mathbf{R}_{yyt}^{-1}\mathbf{t}(\theta_t). \tag{19}$$

From (17) and (19), we can estimate β_k, $g_{r,k}$ and $g_{t,k}$ for all K targets. However, the goal of the previous procedure is to estimate the target positions, or $R_{r,k}$ and $R_{t,k}$. There are three options.

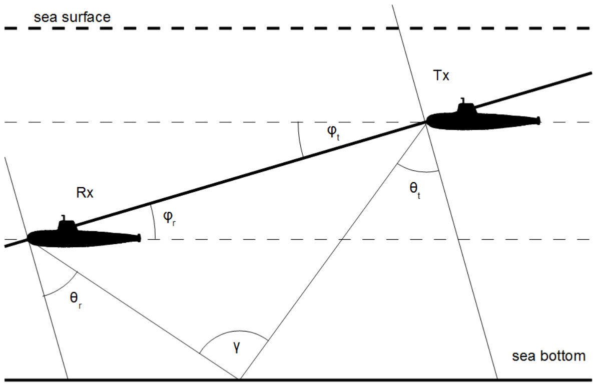

Fig. 2. Bistatic sonar by separate AUVs.

- The position and orientation of both the transmitter and the receiver are known, or L, ϕ_t, and ϕ_r are known. Then, the estimation of the target positions is straightforward, since $R_{t,k}/L = \cos\theta_{r,k}/\sin(\theta_{t,k} - \theta_{r,k})$ and $R_{r,k}/L = \cos\theta_{t,k}/\sin(\theta_{t,k} - \theta_{r,k})$ ([11], Ch. 3).
- The position and orientation of both transmit and receive ULA are not known and the target positions cannot be estimated directly.
- The relative orientation between transmit and receive ULA are known, or L remains unknown but $\phi_t - \phi_r = \phi$ with ϕ known, which we will extend shortly.

With some rearrangement of the bistatic radar theory equations ([11], Ch. 3), each target on the same isorange ellipse should comply with

$$\frac{R_t + R_r}{L} = \frac{1}{e} = \frac{\cos\theta_t + \cos\theta_r}{\sin(\theta_t - \theta_r)} = \frac{2\cos\left(\frac{\theta_t+\theta_r}{2}\right)\cos\left(\frac{\theta_t-\theta_r}{2}\right)}{\sin(\theta_t - \theta_r)}. \tag{20}$$

In (20), the quantity $h = \theta_t - \theta_r = \arcsin(g_t/\pi) - \arcsin(g_r/\pi) + \phi$ is estimated, while $x_1 = \theta_t + \theta_r$ and $x_2 = 1/e$ are unknown. We can use the series expansion of $\arccos x = \pi/2 - \sum_{n=0}^{\infty} x^{2n+1}\left((2n)!/2^{2n}(n!)^2(2n+1)\right) = \pi/2 - x - x^3/6 - 3x^5/40 - \ldots \approx \pi/2 - x$ for $|x| < 1$ to obtain

$$\frac{2\cos\left(\frac{\theta_t+\theta_r}{2}\right)\cos\left(\frac{h}{2}\right)}{\sin h} = \frac{1}{e} \Rightarrow x_1 = \arccos\left(x_2 \frac{\sin h}{2\cos\left(\frac{h}{2}\right)}\right) \Rightarrow$$

$$x_1 \approx \pi/2 - \frac{\sin h}{2\cos\left(\frac{h}{2}\right)} x_2 \Rightarrow x_1 + \frac{\sin h}{2\cos\left(\frac{h}{2}\right)} x_2 \approx \pi/2 \tag{21}$$

Considering all K targets, we can form an overdetermined system of equations to produce a least-squares solution for the target angular orientation $(\theta_t + \theta_r)$ and range bin eccentricity $(L/(R_t + R_r))$, with free parameter the baseline L, that can be estimated from the time difference of arrival (TDOA) between different range bins. A possible scenario is illustrated in Fig. 2, where there are two AUV's with ULA attached on their keels, moving horizontally with zero trim (or with pre-defined trim) and performing bottom mapping.

4. SIMULATION RESULTS

To illustrate the proposed algorithm, we perform a comparison with the algorithm from [15], since Yan et al. use the Capon estimator. Our objective is to illustrate that the covariance matrix formulation of the proposed algorithm is superior to the formulation used in [15] and other works. We set the number of elements M_t of the transmit ULA to 3, the number M_r of elements of the receive ULA to 3, and the number of pulses of the transmitted pulse train N to 8. The pulses were produced at random. We also set the baseline $L = 50m$, the sound frequency $f_0 = 20kHz$, the sound speed $v = 1500m/sec$, the total travel time $R_t + R_r = 100m$, $\phi_t = \phi_r = 0$, and one target at $\theta_r = -\pi/4$. The target strength β of the target was set to 1, the range from -1 to 1 was divided to $N_{grd} = 90$ sectors, and the simulation was repeated 1000 times for each SNR in the range from -15 to 20 dB.

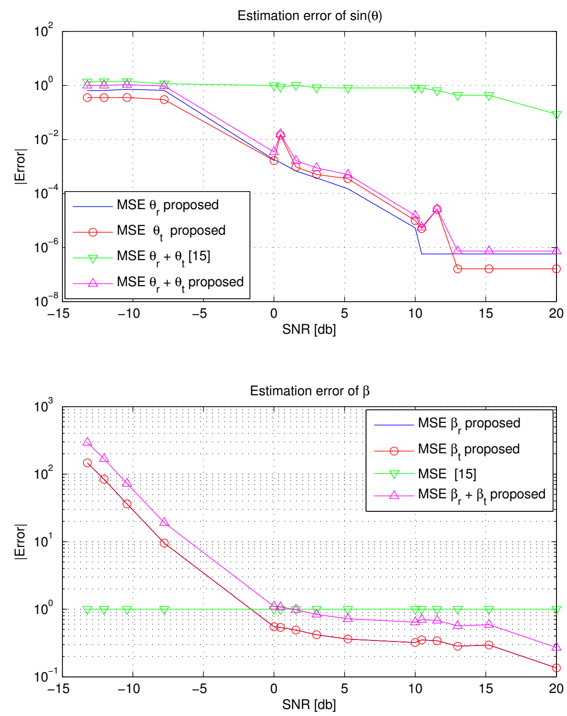

Fig. 3. Simulation results.

We can see in Fig. 3 that the algorithm by Yan et al. lacks the ability to estimate β, producing small estimates for all SNRs, due to the fact that the covariance matrix is close to singular. Apart from the improved performance of the proposed algorithm in terms of DOD, DOA, and target strength estimation, the complexity is significantly reduced as well. Instead of $O\left(N_{grd}^2 \times (M_t \times M_r)^3\right)$ operations, only $O\left(N_{grd} \times (M_t^3 + M_r^3)\right)$ operations are required.

5. REFERENCES

[1] G. Roberts and R. Sutton, *Advances in Unmanned Marine Vehicles*. Stevenage, UK: The Institution of Engineering and Technology, 2011.

[2] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics, 2nd Edition*. Springer, 2011.

[3] A. D. Waite, *Sonar for Practising Engineers, 3rd Edition*. West Sussex, UK: Wiley, 2002.

[4] M. A. Ainslie, *Principles of Sonar Performance Modeling*. Springer, 2010.

[5] R. P. Hodges, *Underwater Acoustics: Analysis, Design, and Performance of Sonar*. West Sussex, UK: Wiley, 2010.

[6] P. C. Etter, *Underwater Acoustic Modeling and Simulation, 3rd Edition*. London, UK: Spon Press, 2003.

[7] H. L. V. Trees, *Detection, Estimation and Modulation Theory: Radar-Sonar Processing and Gaussian Signals in Noise*. Wiley, 2001.

[8] M. I. Skolnik, *Radar Handbook, 3rd Edition*. McGraw-Hill, 2008.

[9] F. E. Nathanson, J. P. Reilly, and M. N. Cohen, *Radar Design Principles*. Mendham, NJ: SciTech Publishing, 1999.

[10] P. Blondel, *The Handbook of Sidescan Sonar*. Springer, 2009.

[11] N. J. Willis, *Bistatic Radar*. Raleigh, NC: SciTech Publishing, 2005.

[12] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Hoboken, NJ: Wiley, 2009.

[13] P. Vaidyanathan and P. Pal, "MIMO radar, SIMO radar, and IFIR radar: A comparison," in *Proc. 43rd Asilomar Conf. Signals, Systems and Comput.*, Pacific Grove, CA, Nov. 2009, pp. 160–167.

[14] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Trans. Signal Process.*, vol. 54, pp. 3873–3883, Oct. 2006.

[15] H. Yan, J. Li, and G. Liao, "Multitarget identification and localization using bistatic MIMO radar systems," *EURASIP J. Advances Signal Process.*, vol. 2008, pp. 1–8, Dec. 2007.

[16] H. Jiang, D.-F. Wang, and C. Liu, "Estimation of DOD and 2D-DOA and polarizations for bistatic MIMO radar," in *Proc. 19th Annu. Wireless Optical Commun. Conf. (IEEE WOCC 2010)*, Shanghai, China, May 2010, pp. 1–5.

[17] M. Jin, G. Liao, and J. Li, "Joint DOD and DOA estimation for bistatic MIMO radar," *Signal Processing*, vol. 89, pp. 244–251, Feb. 2009.

[18] J. Li, G. Liao, K. Ma, and C. Zeng, "Waveform decorrelation for multitarget localization in bistatic MIMO radar systems," in *Proc. 2010 IEEE Radar Conference*, Arlington, VA, May 2010, pp. 21–24.

[19] J. Chen, H. Gu, and W. Su, "Direction finding with automatic pairing for bistatic MIMO radar," *Signal Processing*, vol. 92, pp. 198–203, Jan. 2012.

[20] B. Dang, J. Li, and G. Liao, "Taylor polynomial expansion based waveform correlation cancellation for bistatic MIMO radar localization," *Signal Processing*, vol. 92, pp. 1404–1410, Jun. 2012.

[21] X. Liu and G. Liao, "Direction finding and mutual coupling estimation for bistatic MIMO radar," *Signal Processing*, vol. 92, pp. 517–522, Feb. 2012.

[22] D. Nion and N. D. Sidiropoulos, "Tensor algebra and multidimensional harmonic retrieval in signal processing for MIMO radar," *IEEE Trans. Signal Process.*, vol. 58, pp. 5693–5705, Nov. 2010.

[23] Y. Cheng, H. Gu, and W. Su, "Joint 4-D angle and Doppler shift estimation via tensor decomposition for MIMO array," *IEEE Commun. Lett.*, vol. 16, pp. 917–920, Jun. 2012.

[24] A. Hassanien, S. Vorobyov, and A. B. Gershman, "Moving target parameters estimation in non-coherent MIMO radar systems," *IEEE Trans. Signal Process.*, vol. 60, pp. 2354–2361, May 2012.

[25] H. Chen, X. Li, and Z. Zhuang, "Antenna geometry conditions for MIMO radar with uncoupled direction estimation," *IEEE Trans. Antennas Propag.*, vol. 60, pp. 3455–3465, Jul. 2012.

[26] P. Stoica and R. Moses, *Spectral Analysis of Signals*. Upper Saddle River, NJ: Pearson Education, 2005.