AN EM APPROACH FOR JOINT CHANNEL ESTIMATION AND CHANNEL DECODING IN SYSTEMS EMPLOYING PHYSICAL-LAYER NETWORK CODING

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ABSTRACT

This paper applies the expectation-maximization (EM) algorithm to address the problem of joint channel estimation and channel decoding in Physical-layer Network Coding (PNC) systems. The use of PNC can significantly improve the throughput of a relay network. The throughput advantage, however, is predicated on the availability of accurate channel estimates. For channel-coded PNC systems, a major challenge is that the *maximum a posteriori probability* (MAP) channel estimation is nontrivial due to 1) the overlapping of signals from multiple users received at the relay; and 2) the correlations among data symbols introduced by channel coding. In this paper, we show that an EM algorithm implemented on a factor graph framework is well suited to tackle this problem. Through iterative message passing, the channel estimation component and the channel decoding component in the factor graph interact to improve each other's results progressively. Simulation results indicate that just one EM iteration of our algorithm can significantly improve the channel estimation accuracy as well as the BER performance of channel-coded PNC systems.

Index Terms— EM, message passing, factor graph, PNC

1. INTRODUCTION

Recently, the research community has shown growing interest in two-way relay channel (TWRC), particular TWRC that employs physical-layer network coding (PNC). In TWRC, two terminal nodes exchange information with the aid of a relay [1, 2]. PNC, originally proposed in [1], can potentially boost throughput in TWRC by 100% compared with the traditional relaying method [2].

To accurately estimate and track time-varying channels, it is desirable to obtain the *maximum a posteriori probability* (MAP) estimate. Furthermore, it is desirable that the MAP estimate is based on not just the pilots, but also the data. This is, however, a particularly challenging problem for PNC systems. PNC allows the two terminal nodes in TWRC to transmit together. The relay then transforms the received overlapped signals into a network-coded message [1, 2]. While this PNC operation can potentially boost throughput, the overlapped data also complicate the task of estimating the two channels. Adding to the complexity is the problem of channel-decoding the overlapped signals into the target network-coded message. In this paper, we argue that directly trying to solve the MAP channel estimation problem and the channel decoding problem in a separate manner is not viable; a solution is found in an expectation-maximization (EM) approach that solves the two problems jointly in an iterative manner.

We implement the EM algorithm for PNC on a factor graph [3, 4]. In the factor graph, the components for channel estimation and channel decoding are interconnected. Through iterative message passing between the two components as well as between elements within the channel decoding component, the results of channel estimation and channel decoding improve progressively toward the optimal solution. Simulation indicates that significant improvement on the channel estimation accuracy can be obtained by just one EM iteration of the proposed algorithm. As a result, the bit error rate (BER) performance of the system can be substantially enhanced without much additional computation cost.

1.1. Related Works

Ref. [5] first proposed EM as an iterative algorithm for finding the *maximum likelihood* (ML) estimates of parameters in statistical model with hidden variables that cannot be observed directly. A small extension allows the finding of the MAP estimates also [6]. Refs. [7, 8] presented a way to map EM computation to a message passing algorithm on a factor graph. It is not clear from [7, 8], however, that the factor graph representation is applicable to the specific communication problem of interest to us here. This paper provides an affirmative answer and fills in the missing details.

There have been many previous investigations on the application of EM in communication systems. Refs [9, 10, 11, 12, 13] applied EM to the problem of joint channel estimation and detection/decoding in single-user systems. PNC, howev-

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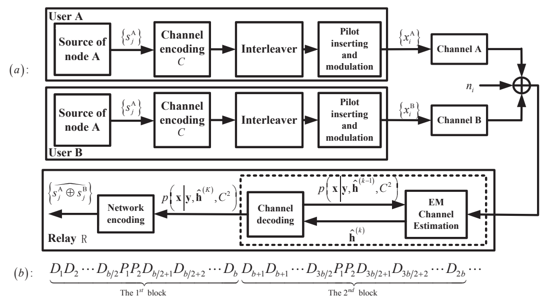

Fig. 1. (a) The system model of uplink phase in TWRC; (b) The frame structure employed by the two terminal nodes.

er, is not a single-user system. PNC is closer to multi-user systems because signals from multiple users overlap in time. Refs. [14, 15, 16] applied EM to joint channel estimation and multi-user detection for CDMA systems. Channel coding was not considered. Ref. [17] incorporated channel coding. However, the proposed method performs successive interference cancelation (SIC) and tries to decode the individual messages of different users using separate channel decoders. This is not optimal for PNC systems.

Overall, there has been little multi-user EM work that incorporates channel coding. Furthermore, PNC channel decoding [18, 19, 20] is different from the separate channel decoding [17] because the target is to channel-decode the overlapped received signals into a network-coded message [18, 19, 20] rather than the individual messages. To the best of our knowledge, ours is the first work that applies EM message passing to PNC systems (and for multi-user systems as well) for joint channel estimation and decoding.

2. SIGNAL AND CHANNEL MODELS OF PNC

We consider a two-phase PNC transmission scheme for TWRC consisting of an uplink phase and a downlink phase. In the uplink phase, two terminal nodes A and B transmit packets to a relay node R simultaneously. From the overlapped signals received from A and B, R constructs a network-coded packet and broadcasts it to A and B in the downlink phase. From the network-coded packet, A(B) then recovers the packet of B(A) using its self information [2].

This paper focuses on the uplink phase because the problem of reliably transmitting the network-coded packet in the downlink phase is similar to that in a conventional point-to-point link. We assume both A and B have one transmit antenna, and R has one receive antenna. In the uplink phase, the received signal at R in the i^{th} symbol duration can be expressed as

$$y_i = h_i^{\mathrm{A}} x_i^{\mathrm{A}} + h_i^{\mathrm{B}} x_i^{\mathrm{B}} + n_i = \mathbf{h}_i^{\mathrm{T}} \mathbf{x}_i + n_i, \qquad (1)$$

where $x_i^{\mathrm{A}}\left(x_i^{\mathrm{B}}\right)$ is the i^{th} transmitted symbol of node A(B); $h_i^{\mathrm{A}}\left(h_i^{\mathrm{B}}\right)$ is the i^{th} fading coefficient of the channel between A(B) and R; n_i is the i^{th} complex white Gaussian noise with covariance σ_n^2; $\mathbf{h}_i \triangleq [h_i^{\mathrm{A}}, h_i^{\mathrm{B}}]^{\mathrm{T}}$; and $\mathbf{x}_i \triangleq [x_i^{\mathrm{A}}, x_i^{\mathrm{B}}]^{\mathrm{T}}$. A block diagram of the system model is shown in Fig. 1 (a), where $\left\{s_j^{\mathrm{A}}\right\}$ and $\left\{s_j^{\mathrm{B}}\right\}$ are the source information bits from nodes A and B. The transmitted symbols $\left\{x_i^{\mathrm{A}}\right\}$ and $\left\{x_i^{\mathrm{B}}\right\}$ are generated after channel encoding, interleaving, constellation mapping and pilot insertions at the transmitters. We assume that A and B use the same channel encoder C and the same interleaver when mapping their source bits $\left\{s_j^{\mathrm{A}}\right\}$ and $\left\{s_j^{\mathrm{B}}\right\}$ to transmitted symbols. Pilot symbols are inserted periodically among coded data symbols. The assumed frame structure is shown in Fig. 1 (b), where P and D represent the pilot symbols and coded data symbols, respectively. Each frame consists of l data symbols, divided into l/b blocks. Each block has b data symbols and two pilot symbols. The total frame length is $L = l + 2\,(l/b)$ symbols.

We assume time-varying Rayleigh fading channels. The fading channels h_i^{A} and h_i^{B} are modeled as two independent first-order Gauss-Markov processes [21, 22]:

$$h_i^{\mathrm{A}} = \alpha h_{i-1}^{\mathrm{A}} + z_i^{\mathrm{A}} \qquad h_i^{\mathrm{B}} = \alpha h_{i-1}^{\mathrm{B}} + z_i^{\mathrm{B}}, \qquad (2)$$

where z_i^{A} and z_i^{B} are complex white Gaussian processes with zero mean and variances $\left(1-\sigma^2\right)\sigma_{\mathrm{A}}^2$ and $\left(1-\sigma^2\right)\sigma_{\mathrm{B}}^2$, and α is a correlation coefficient tied to the channel coherence time [22].

3. APPLICATION OF EM THEORY TO PNC

3.1. Objectives of EM PNC Receiver

Let $\mathbf{h}$ be the set containing all channels $\{\mathbf{h}_i\}$. Similarly, $\mathbf{x}$ is the set of all transmitted symbols $\{\mathbf{x}_i\}$, and $\mathbf{y}$ is the set of all received signal vectors $\{y_i\}$. To the relay, both $\mathbf{h}$ and $\mathbf{x}$ are unknowns to be estimated and decoded.

In a conventional receiver, $\mathbf{h}$ is first estimated, followed by the decoding of codewords $\mathbf{x}$, and network coding after that. Pilots, corresponding to known $\mathbf{x}_i$ at specific i^{th} positions, are used for the estimation of $\mathbf{h}$. This estimate of $\mathbf{h}$ is then substituted into (1) for the decoding of the unknown $\mathbf{x}_i$. This estimate of $\mathbf{h}$ makes use of only the pilot parts, and does not exploit useful information contained in the data part of $\mathbf{x}$. To fully make use of all the received symbols, including pilots and data, the receiver could perform the following:

Step 1 (channel estimation): Find MAP estimate $\widehat{\mathbf{h}}_{\mathrm{MAP}} = \arg\max_{\mathbf{h}} \left\{\log p\left(\mathbf{h}\,|\mathbf{y}\right)\right\} = \arg\max_{\mathbf{h}} \left\{\log \sum_{\mathbf{x}} p\left(\mathbf{x}, \mathbf{h}\,|\mathbf{y}\,\right)\right\}$;

Step 2 (channel decoding): Find $p(\mathbf{x}|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$

Step 3 (network coding): Compute the network-coded source message $\left\{\widehat{s_j^{\mathrm{A}} \oplus s_j^{\mathrm{B}}}\right\}$ based on the channel decoding output from Step 2 [2].

This is the PNC receiver with optimal channel estimation. If the channel coefficients were perfectly known (as assumed in previous works [18, 19, 20]), then Step 1 is not needed. The operations of Step 2 and Step 3 can be combined to form the so-called Channel-decoding-Network-Coding (CNC) process, an essence of channel-coded PNC systems [2, 18]; a subtlety of CNC as compared to conventional channel decoding is that the goal is not to decode the individual source messages from A and B, but a network-coded message that mixes the two source messages (we refer the interested readers to [2] and references therein for details on CNC). Similarly, if the MAP estimation in Step 1 could be achieved, then Step 2 and Step 3 could be implemented using the conventional CNC methods substituting $\widehat{\mathbf{h}}_{\mathrm{MAP}}$ as the channel coefficients. Unfortunately, this is not viable because the exact MAP estimate of $\mathbf{h}$ is difficult due to the complexity of the computation of $\sum_{\mathbf{x}} p\left(\mathbf{h}, \mathbf{x} \left|\mathbf{y}\right.\right)$. A difficulty, for example, is that the symbols in $\mathbf{x}$ are correlated due to channel coding; in addition, signals of the two terminal nodes are overlapped in $\mathbf{y}$.

EM tries to find $\widehat{\mathbf{h}}_{\mathrm{MAP}}$ iteratively rather than attacking the problem directly. The objective of EM is still to obtain the MAP estimate of $\mathbf{h}$ as in Step 1. However, EM combines Step 1 and Step 2 in an iterative manner to refine the estimate of $\mathbf{h}$ and the decoding of the network-coded message. In the following, we first describe the procedure of the EM algorithm and then present its implementation on the factor graph. In the terminology of EM, $\mathbf{y}$ is the observed data, $\mathbf{x}$ is the hidden data, and $\mathbf{h}$ is the unknown parameters. The k^{th} iteration of EM consists of an E-step (expectation) and an M-step (maximization) as follows [6]:

E-step: Given the previous estimate $\widehat{\mathbf{h}}^{(k-1)}$, compute the conditional expectation

$$Q\left(\mathbf{h}\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)=\sum_{\mathbf{x}} p\left(\mathbf{x}\left|\mathbf{y}, \widehat{\mathbf{h}}^{(k-1)}\right.\right)\log p\left(\mathbf{y},\mathbf{x}|\mathbf{h}\right); \quad (3)$$

M-step: Then, compute $\widehat{\mathbf{h}}^{(k)}$ by

$$\widehat{\mathbf{h}}^{(k)}=\arg\max_{\mathbf{h}}\left[Q\left(\mathbf{h}\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)+\log p(\mathbf{h})\right]. \quad (4)$$

The E-step in (3) can be broken down as follows. First, compute $p\left(\mathbf{x}\left|\mathbf{y}, \widehat{\mathbf{h}}^{(k-1)}\right.\right)$ from $\mathbf{y}$ and $\widehat{\mathbf{h}}^{(k-1)}$. This computation is similar to Step 2 above, with $\widehat{\mathbf{h}}^{(k-1)}$ replacing $\widehat{\mathbf{h}}_{\mathrm{MAP}}$. If the algorithm were to stop at iteration k, we could simply go to Step 3 to obtain the network-coded message based on $p\left(\mathbf{x}\left|\mathbf{y}, \widehat{\mathbf{h}}^{(k-1)}\right.\right)$. Otherwise, the E-step continues and uses $p\left(\mathbf{x}\left|\mathbf{y}, \widehat{\mathbf{h}}^{(k-1)}\right.\right)$ to compute $Q\left(\mathbf{h}\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)$ as in (3). After that, the M-step finds a new estimate of $\mathbf{h}$ as in (4). The schematic of this EM PNC receiver is shown in Fig. 1 (a).

3.2. Implementation of EM PNC on Factor Graph

We next consider the factor graph implementation of the EM PNC receiver. Ref. [8] explained how to transform EM computation to a factor graph implementation. It is not clear, however, that the assumptions in [8] on the functional forms of the parameter and variable probabilities are valid for our specific problem. Here, we give a derivation tailored to the channel-coded communication systems.

For factor graph implementation, we modify Step 2 in Section 3.1. Instead of finding $p(\mathbf{x}|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$ for channel decoding, we find $p(\mathbf{x}_i|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$ for each and every i. There are two reasons for this. First, for many advanced channel codes (e.g., LDPC, Turbo code), the decoding process finds $p(\mathbf{x}_i|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$ rather than $p(\mathbf{x}|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$, because finding $p(\mathbf{x}|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$ for all possible codewords $\mathbf{x}$ is generally a difficult computation-intensive problem. Second, and very importantly, our EM procedure only requires $p(\mathbf{x}_i|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$ and not $p(\mathbf{x}|\widehat{\mathbf{h}}_{\mathrm{MAP}}, \mathbf{y})$, as detailed below.

A key to factor graph implementation is to factorize $p\left(\mathbf{y},\mathbf{x}\left|\mathbf{h}\right.\right)$ in (3) and $p\left(\mathbf{h}\right)$ in (4). For $p\left(\mathbf{y},\mathbf{x}\left|\mathbf{h}\right.\right)$, we write

$$p\left(\mathbf{y},\mathbf{x}\left|\mathbf{h}\right.\right)=p\left(\mathbf{y}\left|\mathbf{x},\mathbf{h}\right.\right)p\left(\mathbf{x}\right)=\frac{I_{C^2}\left(\mathbf{x}\right)\prod_i p\left(y_i\left|\mathbf{x}_i,\mathbf{h}_i\right.\right)}{\left|C^2\right|}, \quad (5)$$

where C^2 is the valid set of $\mathbf{x}$ (we assume all codewords are equally likely) and $I_{C^2}\left(\mathbf{x}\right)$ is a indicator function defined as: $I_{C^2}\left(\mathbf{x}\right)=1$ if $\mathbf{x}\in C^2$; $I_{C^2}\left(\mathbf{x}\right)=0$ if $\mathbf{x}\notin C^2$. Note that we have used (1) in (5) in the factorization of $p\left(\mathbf{y},\mathbf{x}\left|\mathbf{h}\right.\right)$. Substituting (5) into the Q function defined in (3) and dropping the term $-\log\left|C^2\right|$, which is independent of $\mathbf{h}$ and therefore does not matter as far as the M-step is concerned, we have

$$Q\left(\mathbf{h}\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)=\sum_i\sum_{\mathbf{x}_i}\log p\left(y_i\left|\mathbf{x}_i,\mathbf{h}_i\right.\right)\overbrace{\sum_{\substack{\mathbf{x}_1\cdots\mathbf{x}_{i-1}\\ \mathbf{x}_{i+1}\cdots\mathbf{x}_L}} p\left(\mathbf{x}\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right)}^{p\left(\mathbf{x}_i\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right)}, \quad (6)$$

where $p\left(\mathbf{x}_i\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right)$ is the *a posteriori probability* (APP) that can be computed using the sum-product rule based message passing decoding algorithm [4] on the factor graph of the given channel code C^2. A subtlety here for the PNC system is that the channel encoder C^2 is a "virtual channel encoder" which takes the original information source symbols from nodes A and B $\left\{s_j^{\mathrm{A}}, s_j^{\mathrm{B}}\right\}$ as inputs, and output $\left\{\mathbf{x}_i\right\}$ as coded symbols (see [2] for details). We define the symbol-wise Q function as

$$Q_i\left(\mathbf{h}_i\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)\triangleq\sum_{\mathbf{x}_i}\log p\left(y_i\left|\mathbf{x}_i,\mathbf{h}_i\right.\right)p\left(\mathbf{x}_i\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right). \quad (7)$$

With complex white Gaussian noise, the above $\log p\left(\mathbf{y}_i\left|\mathbf{x}_i,\mathbf{h}_i\right.\right)$ as a function of the variables $\mathbf{x}_i$ and $\mathbf{h}_i$ can be obtained in closed form. We see that once $p\left(\mathbf{x}_i\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right)$ is computed, $Q_i\left(\mathbf{h}_i\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)$ as a function of $\mathbf{h}_i$ can be obtained by the weighted sum of $p\left(\mathbf{x}_i\left|\mathbf{y},\widehat{\mathbf{h}}^{(k-1)},C^2\right.\right)$ over different possible values of $\mathbf{x}_i$. The overall Q function is the sum of symbol-wise Q functions:

$$Q\left(\mathbf{h}\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)=\sum_i Q_i\left(\mathbf{h}_i\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right). \quad (8)$$

Using (8), the M-step in (4) is equivalent to

$$\widehat{\mathbf{h}}^{(k)}=\arg\max_{\mathbf{h}}\left(p\left(\mathbf{h}\right)\cdot\prod_i e^{Q_i\left(\mathbf{h}_i\left|\widehat{\mathbf{h}}^{(k-1)}\right.\right)}\right). \quad (9)$$

To see what will happen in the M-step, let us assume the Gauss-Markov channel model (2) and factorize $p(\mathbf{h})$ as

$$p(\mathbf{h}) = p(\mathbf{h}_1) \sum\nolimits_{i=2} p(\mathbf{h}_i \,|\mathbf{h}_{i-1}). \qquad (10)$$

Substituting (10) into (9), we find that the M-step can be implemented by (i) constructing the factor graph according to (10); (ii) regarding $e^{Q_i(\cdot)}$ as an input message to the variable node $\mathbf{h}_i$ for each i on the constructed factor graph; and (iii) performing the max-product rule based message passing algorithm [4] over the factor graph to solves (9). Since the input messages $\left\{e^{Q_i(\cdot)}\right\}$ and the check functions of the nodes on the factor graph representing (10) are all of exponential form, the max-product algorithm that solves (9) can be implemented using Gaussian message passing [4].

3.3. Initialization and Termination of EM Iteration

EM iteration needs to be bootstrapped with a good initial point; otherwise there is no guarantee that the algorithm will converge to the global maximum [23]. For each block, we obtain the initial $\widehat{\mathbf{h}}^{(0)}$ by minimum mean square error (MMSE) estimation [24] using the pilot symbols. Then, the channel coefficients within each data block are simply set to the estimated channel coefficient of the closest pilot.

We repeat the E-step and M-step iteratively. When iteration k exceeds a maximum limit K, we terminate EM and use the virtual channel decoder in the factor graph to compute the final decoding results $p\left(\mathbf{x}_i \,\middle|\mathbf{y}, \widehat{\mathbf{h}}^{(K)}, C^2\right)$ and $p\left(s_j^{\mathrm{A}}, s_j^{\mathrm{B}} \,\middle|\mathbf{y}, \widehat{\mathbf{h}}^{(K)}, C^2\right)$. Then, the network-coded source message is obtained by

$$\widehat{s_j^{\mathrm{A}} \oplus s_j^{\mathrm{B}}} = \arg\max_s \sum_{s_j^{\mathrm{A}}, s_j^{\mathrm{B}} : s_j^{\mathrm{A}} \oplus s_j^{\mathrm{B}} = s} p\left(s_j^{\mathrm{A}}, s_j^{\mathrm{B}} \,\middle|\mathbf{y}, \widehat{\mathbf{h}}^{(K)}, C^2\right) \qquad (11)$$

for all j. After that, the relay channel-encodes the network-coded source message and broadcasts the channel-coded message to nodes A and B in the downlink phase.

4. SIMULATION RESULTS

For performance evaluation, we define three different settings: (i) our proposed EM PNC receiver; (ii) PNC receiver with one-shot MMSE channel estimation using pilots only (this is equivalent to our EM PNC receiver with $K = 0$); (iii) an EM receiver modified from the receiver in [17] (specifically, after decoding the two individual source messages $\left\{\widehat{s}_j^{\mathrm{A}}\right\}$, $\left\{\widehat{s}_j^{\mathrm{B}}\right\}$ using the EM SIC in [17], we perform network coding as $\left\{\widehat{s}_j^{\mathrm{A}} \oplus \widehat{s}_j^{\mathrm{B}}\right\}$). We assume the channels of both terminal nodes have the same average power $\sigma_{\mathrm{A}}^2 = \sigma_{\mathrm{B}}^2$ and channel correlation coefficient $\alpha = 0.99$. BPSK modulation is used by node A and B. The regular Repeat Accumulate (RA) code with coding rate $1/3$ is employed. Each frame has 1024 information bits (thus 3072 channel-coded BPSK data symbols). We insert two pilots for every block with block size $b = 16$.

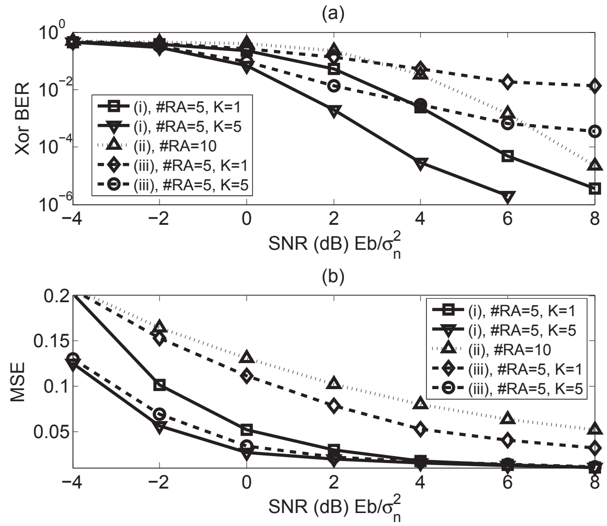

Fig. 2. Simulation results: (a) BER; (b) MSE.

This corresponds to a pilot load of $2/18 = 11.1\%$. We adopt orthogonal pilot patterns for the two terminal nodes wherein $P_1 = 1$ $P_2 = 1$ for node A and $P_1 = 1$ $P_2 = -1$ for node B. All presented simulation results are obtained by averaging over 10^4 pairs of frames.

We evaluate the BER of the network-coded messages and the mean square error (MSE) of the estimated channels. Fig. 2 (a) presents the BER of the three receivers. $\mathrm{E_b}$ is the energy per information bit. $\#\mathrm{RA}$ is the number of RA channel decoding iterations for each channel estimation iteration. For fair comparison, let us focus on the performance of receiver (i) when $\#\mathrm{RA} = 5, K = 1$ and the performance of receiver (ii) when $\#\mathrm{RA} = 10$: i.e., the total numbers of channel decoding iterations are the same in the two cases. We see that for BER performance, receiver (i) has about 2 dB gain over receiver (ii). For receiver (iii), satisfactory performance cannot be obtained. Fig. 2 (b) presents the MSE of the estimated channels. We can clearly see that (i) does give more accurate channel estimation than (ii) and (iii). We can also observe that the first EM iteration in (i) can already extract most of the gain in MSE.

5. CONCLUSION

We have proposed an EM factor-graph framework for solving the joint problem of channel estimation and channel decoding in PNC systems iteratively. In general, performance improves with the number of iterations. Our simulation, however, indicates that just one EM iteration may be enough to extract most of the gain in MSE performance. In terms of BER performance, in a setting where the pilot load is 11.1%, a 2dB gain is obtained with just one iteration. The need for just one iteration makes the approach more viable for a practical system in which the computation must be performed in real-time.

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