LLR QUANTIZATION AND RESOURCE ALLOCATION OF CONSTRAINED BACKHAUL FOR MULTICELL PROCESSING

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ABSTRACT

We consider the uplink of a cellular system where base stations (BSs) cooperate for decoding signals transmitted by mobile terminals (MTs). Assuming that MTs transmit on orthogonal channels, each BS demodulates the signal coming from each MT, obtaining the log likelihood ratio (LLR) of each encoded bit. LLRs are quantized and quantization indices are forwarded on a backhaul to the radio network controller (RNC), where the reconstructed values are summed, and decoding takes place. BS-RNC links have a constraint on the maximum supported bit rate. We design the uniform quantizers in order to maximize the generalized mutual information (GMI) on the combined LLR over all MTs under the backhaul constraint. To this end, we derive the probability mass distribution (PMD) of the quantized LLR conditioned on the value of the transmitted bit, and we compute the GMI. We then propose a greedy scheduling algorithm for the choice of the number of bits used for the representation of each quantized signal exchanged on the backhaul.

***Index Terms*—** Decoding, Multicell processing, Quantization.

1. INTRODUCTION

In cellular systems coverage can be significantly improved by *multicell processing*, i.e., cooperation among base stations (BSs) for decoding uplink signals. While multicell processing yields the benefits of a distributed multiple input-multiple output (MIMO) system, it requires exchange of information among BSs on a backhaul network. However, backhaul capacity is typically limited, and a proper optimization of both the kind of exchanged signals and their scheduling is required. Since data are encoded with error correcting codes, in order to increase data rate it is better to first combine the signals arrived at the various BSs and then perform decoding. In this paper we propose that all BSs forward information on the backhaul to the radio network controller (RNC), where decoding takes place.

In order to reduce backhaul occupation we propose that each BS a) demodulates the received signals from each MT, b) computes the log likelihood ratio (LLR) on each encoded bit, and c) forwards a quantized version of the LLR to the RNC on the backhaul. We design the uniform quantizer for the LLR and provide a resource allocation algorithm for the backhaul, i.e., we optimize the bits used for the representation of each quantized signal on the backhaul. Both quantizer design and resource allocation algorithm aim at maximizing the generalized mutual information (GMI) [1] between the transmitted signals and the quantized LLR. In fact, GMI provides the rate that can be achieved with perfect coding, which is realistic when using efficient codes as low density parity check (LDPC) codes. Numerical results show the merits of the proposed solution in a typical cellular scenario.

2. PRIOR WORKS

Various approaches have been considered for BS cooperation. A common solution provides the exchange of the received signal and is particularly effective when MTs are interfering, since it allows to null interference by suitable beamforming techniques [2]. However, recently a number of alternative approaches have been proposed to reduce interference, e.g., based on BS coordination [3–5] and clustering of mobile terminals (MTs) [6]. When interference has been mitigated by these techniques, multicell processing is still important as it provides diversity gain. Since multicell processing is limited by backhaul characteristics, various works have focused on backhaul transmission scheduling, including BS clustering [7], genetic algorithms with sphere decoder [8], and greedy solutions that aim at guaranteeing a given outage probability [9, 10].

In [11] it has been proposed to perform a) demodulation and decoding of the messages coming from the MTs at each cooperating BS, b) share the soft information on the message among the BSs, and c) combine them and take the final hard decision on the data bits. However, in this approach MT achievable rates are limited by performing decoding before combining. Our solution improves this approach by performing demodulation at each BS, then combining and only finally decoding.

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3. SYSTEM MODEL

We consider the uplink of a cellular system where K mobile terminals (MTs) are communicating with N base stations (BSs). Data bits transmitted in packets by the MTs are encoded with error correcting codes and modulated with a M-QAM modulation. The channel between the each MT-BS couple is assumed flat and constant for the duration of the packet. Note that this may be the result of an equalization process, e.g., on a single-carrier frequency division multiple access (SC-FDMA) signal, as provided in the uplink of the long term evolution (LTE) 3GPP cellular communication standard. The signal to noise ratio (SNR) of the link between MT k and BS n is denoted $\Gamma(n,k)$. We assume that by some technique (see Section 2) interference among MTs has been mitigated, therefore we do not employ any interference cancellation among the BSs but instead we resort to soft data combining.

After equalization and demodulation, BS n generates the LLR relative to the i-th encoded bit $d_i(k)$ transmitted by MT k. Let $m(i) = (i \mod \log(M))$ be the position of encoded bit i within the bit-representation of the $\lceil \frac{i}{\log M} \rceil$ transmitted symbol, where $\log$ represents the base-2 logarithm. In particular, we consider the approximated minimum distance LLR, i.e.,

$$\Lambda_i(n,k) = -\frac{1}{\sigma^2}\left(\min_{s\in\mathcal{S}_k(1)}\left\{||r_{m(i)}(n,k) - h(n,k)s||^2\right\} - \min_{s\in\mathcal{S}_k(0)}\left\{||r_{m(i)}(n,k) - h(n,k)s||^2\right\}\right), \tag{1}$$

where $\mathcal{S}_k(u)$ is the set of constellation points having bit k equal to u, σ^2 is the noise power, $h(n,k)$ is the complex channel from MT k to BS n, and $r_m(n,k)$ is the received sample. The LLR is quantized by a uniform quantizer into $\hat{\Lambda}_i(n,k)$. The quantizer is characterized by saturation value $\tau(n,k)$ and $b(n,k)$ bits per sample, whose choice will be detailed in the following section. We also define the $N \times K$ matrix $\boldsymbol{b}$, with entries $b(n,k)$, that collects the number of bits used for quantization for each BS-MT couple.

The binary representations of the quantized LLRs are transmitted on the backhaul network to the RNC. In particular, the link from BS n to the RNC allows the transmission of up to $B(n)$ bits per sample. Thus the backhaul imposes N constraints, one for each each BS-RNC link, i.e.,

$$\sum_{k=1}^{K} b(n,k) \le B(n)\,, \quad n = 1,2,\ldots,N\,. \tag{2}$$

Note that if $b(n,k) = 0$, BS n is not cooperating for the decoding of the signal coming from MT k, as it reserves zero bits for the quantization of the associated LLRs.

The RNC reconstructs the quantized values from their binary representation and sums the quantized values to obtain the joint LLR

$$\tilde{\Lambda}_i(k) = \sum_{n=1}^{N} \hat{\Lambda}_i(n,k)\,. \tag{3}$$

These LLRs are then used to decode the message.

4. GENERALIZED MUTUAL INFORMATION

The target of the quantizer design – i.e., the choice of $\tau(n,k)$ and $b(n,k)$ – is the maximization of the data bit rate, considering the constraint on the backhaul. In particular, the achievable rate can be upper bounded by considering the GMI between the transmitted bit $d_i(k)$ and the joint LLR $\tilde{\Lambda}_i(k)$. The GMI has been already considered in the literature to assess the performance of bit interleaved and coded modulation (BICM) systems with mismatched decoders (see [1] and references therein).

4.1. GMI of Single Decoding BS

We first consider the GMI associated to the LLR generated by each BS for each MT, that will be used for the choice of the quantizer saturation level. It can be shown that the GMI for a single BS-MT couple coincides with the mutual information between the encoded bit and the quantized LLR. First note that the LLR associated to bit $d_i(k)$ has different statistics, depending on the bit index $m(i)$. For example, for a 16-QAM the most and the least significant bits that are both mapped along the same axis with Gray coding have different LLR statistics. We indicate with $p_{\hat{\Lambda}_i(n,k)|d,m}(V|d,m)$ the conditional probability mass distribution (PMD) of $\hat{\Lambda}_i(n,k)$ given $d_i(k) = d$ and $m(i) = m$. Let also $p_{\hat{\Lambda}_i(n,k)|m}(V|m)$ be conditional PMD of $\hat{\Lambda}_i(n,k)$ given $m(i) = m$. Then the GMI for the BS-MT couple (n,k) using a quantizer with $b(n,k)$ bits and saturation value $\tau(n,k)$ can be written as [12]

$$C(n,k,b(n,k),\tau(n,k)) = \frac{1}{2}\sum_{m=1}^{\log M}\sum_{V\in\mathcal{V}(n,k)}\sum_{d=0}^{1} p_{\hat{\Lambda}_i(n,k)|d,m}(V|d,m)\times \log\frac{p_{\hat{\Lambda}_i(n,k)|d,m}(V|d)}{p_{\hat{\Lambda}_i(n,k)|m}(V|m)}\,. \tag{4}$$

4.2. GMI of Joint BS Decoding

In our scenario, all BSs cooperate for the decoding of all MTs. For each transmitted bit the RNC receives N quantized LLRs, which are combined by (3). In order to compute the resulting GMI, we observe that, conditioned to the transmission of a bit, $\hat{\Lambda}_i(n,k)$ for different BSs are independent random variables since they are affected by independent noise at each BS.

We indicate with $p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m)$ the conditional probability mass distribution (PMD) of $\tilde{\Lambda}_i(k)$ given $d_i(k) =$

d and $m(i) = m$. Let also $p_{\tilde{\Lambda}_i(k)|m}(V|m)$ be conditional PMD of $\tilde{\Lambda}_i(k)$ given $m(i) = m$. The GMI (4) can be extended to the case of joint BS decoding as follows

$$C(k, \boldsymbol{b}, \boldsymbol{\tau}) = \sum_{V \in \mathcal{B}(k)} \frac{1}{2} \sum_{m=1}^{\log M} \sum_{d=0}^{1} p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m) \times \log \frac{p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m)}{p_{\tilde{\Lambda}_i(k)|m}(V|m)}, \tag{5}$$

where $\boldsymbol{b}$ and $\boldsymbol{\tau}$ are $N \times K$ matrices whose (n,k) entry are $b(n,k)$ and $\tau(n,k)$, respectively. In the Appendix we derive the expressions of both $p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m)$ and $p_{\tilde{\Lambda}_i(k)|m}(V|m)$.

5. OPTIMIZATION PROBLEM

For the design of the uniform quantizers we aim at maximizing the sum GMI over all MTs, under the constraint on the backhaul, i.e.,

$$C = \max_{\boldsymbol{b},\boldsymbol{\tau}} \sum_{k=1}^{K} C(k, \boldsymbol{b}, \boldsymbol{\tau}), \quad \text{s.t. (2)}. \tag{6}$$

Note that this is a mixed integer programming (MIP) problem, which is usually quite hard to solve due to the presence of both continuous variables $\tau(n,k)$ and discrete variables $b(n,k)$. In order to reduce its complexity, for each possible value of $b(n,k)$ we first optimize the saturation value by maximizing GMI (4), ignoring cooperation, therefore we compute

$$\bar{\tau}(n,k,b(n,k)) = \text{argmax}_{\tau(n,k)} \sum_{k=1}^{K} C(n,k,b(n,k),\tau(n,k)). \tag{7}$$

All the cases that we observed show that $C(n,k,b(n,k),\tau(n,k))$ is a concave function of $\tau(n,k)$, thus (7) can be solved by the gradient method. However we do not have at the moment a mathematical proof of this property.

Then we optimize the number of bits used for each quantizer by solving the integer programming problem

$$\max_{\boldsymbol{b}} \sum_{k=1}^{K} C(k, \boldsymbol{b}, \bar{\boldsymbol{\tau}}(\boldsymbol{b})), \quad \text{subject to (2)}. \tag{8}$$

where entry (n,k) of $\bar{\boldsymbol{\tau}}(\boldsymbol{b})$ is $\bar{\tau}(n,k,b(n,k))$. An algorithm to solve of this problem is provided in the following subsection.

Note that splitting (6) into (7) and (8) provides suboptimal solutions, as saturation values are not jointly optimized. However, this allows to solve problem (7) offline, and store in the devices a table with the optimal saturation values for a set of possible SNRs. Then only problem (8) will be solved online.

5.1. Quantization Bits Optimization

In order to solve (8), we start from any matrix $\boldsymbol{b}$ that satisfies (2). Then we proceed iteratively, where at each iteration we add one bit for a (BS, MT) couple. At the end of each iteration we still have a feasible solution (satisfying (2)), with a strictly increased sum GMI.

Before providing the details of the algorithm let us introduce some notation for iteration j:

- $\boldsymbol{b}^{(j)}$: matrix of quantization bits;
- $\boldsymbol{a}^{(j,n,k)}$: quantization bit matrix obtained from $\boldsymbol{b}^{(j)}$ by increasing $b^{(j)}(n,k)$ by one, i.e.,

$$\boldsymbol{a}^{(j,n,k)}(\ell,m) = \begin{cases} \boldsymbol{b}^{(j)}(\ell,m) & (\ell,m) \neq (n,k) \\ \boldsymbol{b}^{(j)}(n,k) + 1 & (\ell,m) = (n,k); \end{cases}$$

- $\mathcal{F}^{(j)}(n)$: set of MTs for which at least one bit is used for the quantization of the LLR, i.e.,

$$\mathcal{F}^{(j)}(n) = \{k : b^{(j)}(n,k) > 0\}. \tag{9}$$

If at iteration $j+1$ we increase by one $b^{(j)}(n,k)$, the variation of GMI is

$$\Delta^{(j)}(n,k) = C(k, \boldsymbol{a}^{(j,n,k)}) - C(k, \boldsymbol{b}^{(j)}). \tag{10}$$

Therefore at each iteration we insert a bit in the backhaul by maximizing $\Delta^{(j)}(n,k)$. In particular, at iteration $j+1$ the algorithm selects the couple (n,k) that maximizes the increase of GMI, i.e.,

$$(\bar{n}, \bar{k}) = \text{argmax}_{(n,k):k \in \mathcal{F}^{(j)}(n)} \Delta^{(j)}(n,k). \tag{11}$$

The quantization bit matrix at the next iteration will be $\boldsymbol{b}^{(j+1)}(\ell,m) = \boldsymbol{b}^{(j)}(\ell,m)$ if $(\ell,m) \neq (\bar{n},\bar{k})$, while $\boldsymbol{b}^{(j)}(\bar{n},k) + 1$ if $(\ell,m) = (\bar{n},\bar{k})$.

The process is stopped after $\sum_{n=1}^{N} B(n)$ iterations, when the backhaul is full.

6. NUMERICAL RESULTS

We consider a scenario with seven adjacent hexagonal cells ($N = 7$) with radius 500 m, having BSs positioned at the center of each cell. Channels are characterized by path loss with decaying coefficient 2, and Rayleigh fading with average SNR at the cell edge of 0 dB. A 16-QAM modulation is used for transmission by the MTs. We have first designed the quantizer for various values of SNR and quantization bits. Fig. 1 shows the value of $\bar{\tau}(n,k,b(n,k))$ as a function of $b(n,k)$ and of the SNR $\Gamma(n,k)$, as obtained by solving (7), for the least and the most significant bit of each axis of the 16-QAM constellation. We observe that the saturation values grow with the SNR and decrease with the number of bits used for quantization. Fig. 2 shows the average sum GMI $\mathrm{E}[C]$ as

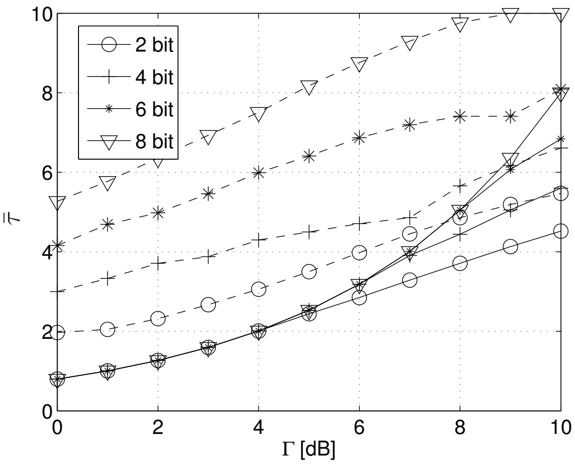

Fig. 1. Optimized saturation value of the quantizers as a function of Γ and the number of quantization bits, for the LLR of the most significant bit (solid lines) and the least significant bit (dashed lines) of the real and imaginary part of a 16-QAM constellation.

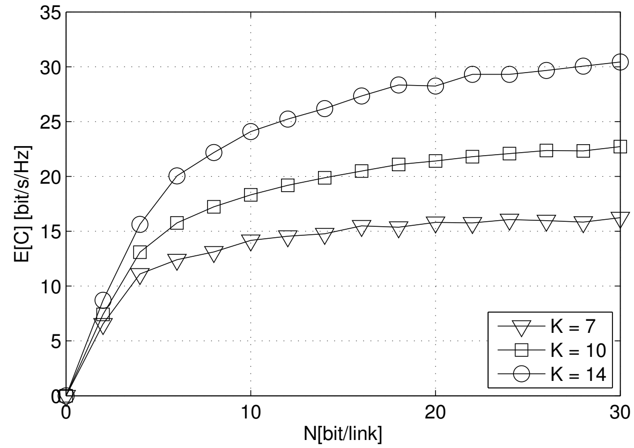

Fig. 2. Average sum GMI $\mathrm{E}[C]$ as a function of $B(n)$, the number of bits per link in the backhaul, for various values of K, the number of transmitting MTs.

a function of the maximum number of bits per BS-RNC link $B(n)$, which is assumed to be the same for all BSs. Various values of K, the number of transmitting MTs, are considered, and in all cases a large number of random MT placements is considered to obtain the average sum GMI. For all values of K we observe that the average GMI grows quickly with $B(n)$ and then saturates at a value that depends on K: for $K = 7$ (i.e., on average one MT per cell) saturation is reached for $B(n) = 7$, and the average GMI in this case is 16 bit/s/Hz. For $K = 10$ and $K = 21$ saturation is reached for about $B(n) = 12$ and $B(n) = 22$, respectively. Indeed, as the number of MTs increases, more data must be obtained from the BSs to achieve a high GMI, which is a more demanding situation for the backhaul. Note that, since decoding occurs at the RNC, if the backhaul does not allow any data exchange, no information can reach the RNC, and the GMI in this case is zero.

7. CONCLUSIONS

We have proposed a technique for the quantization of the LLR in cellular system with multicell processing, based on the maximization of the GMI. Moreover, we have provided a greedy algorithm for the resource allocation of quantized values on a constrained backhaul. Numerical results show the effectiveness of the proposed technique.

8. APPENDIX

The set of all quantized values of $\hat{\Lambda}_i(n,k)$ is

$$\mathcal{V}_i(n,k) = \{V_{i,1}(n,k), V_{i,2}(n,k), \ldots, V_{i,2^{b(n,k)}}(n,k)\}\,. \tag{12}$$

Closed form expressions of $p_{\hat{\Lambda}_i(n,k)|d,m}(V|d,m)$ and $p_{\hat{\Lambda}_i(n,k)|m}(V|m)$ have been derived in [12].

For the statistics of the combined LLR we introduce some notation:

- $\mathcal{V}_i(k)$: the set of all possible values jointly taken by the N quantized LLRs relative to bit i, i.e.,

$$\mathcal{V}_i(k) = \mathcal{V}_i(1,k) \times \mathcal{V}_i(2,k) \times \cdots \times \mathcal{V}_i(N,k); \tag{13}$$

- $\mathcal{B}_i(k)$: the set of all possible values taken by $\tilde{\Lambda}_i(k)$;
- $\mathcal{A}(V)$: the set of all quantized LLR values that sum to $V \in \mathcal{B}_i(k)$, i.e.,

$$\mathcal{A}(V) = \left\{ \boldsymbol{V} = [V_1, \ldots, V_N] \in \mathcal{V}(k) : \sum_{n=1}^{N} V_n = V \right\}.$$

The conditional PMD of $\tilde{\Lambda}_i(k)$ can now be written as

$$p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m) = \sum_{\boldsymbol{V} \in \mathcal{A}(V)} \prod_{n=1}^{N} p_{\hat{\Lambda}_i(n,k)|d,m}(V_n|d,m)\,,$$

with $V \in \mathcal{B}_i(k)$. Assuming equally probable transmitted bits, the PMD of $\tilde{\Lambda}_i(k)$ given m for $V \in \mathcal{B}_i(k)$ is

$$p_{\tilde{\Lambda}_i(k)|m}(V|m) = \sum_{d=0}^{1} \frac{1}{2} p_{\tilde{\Lambda}_i(k)|d,m}(V|d,m)\,. \tag{14}$$

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