ZERO-FORCING PRE-EQUALIZATION WITH TRANSMIT ANTENNA SELECTION IN MIMO SYSTEMS

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ABSTRACT

In this paper, we jointly solve the problem of transmit antenna selection and zero-forcing (ZF) precoding in a multiple input multiple output (MIMO) system. A new problem formulation is proposed which enables efficient semi-definite programming (SDP) to solve the originally non-convex problem of antenna selection. This has been accomplished by imposing the Group Lasso sparsity promoting term in the precoding design criterium as a convex relaxation of the ℓ_0-norm operation. For the selected set of antennas, we then minimize the overall transmit power, subject to a constraint on the maximum achievable throughput. Simulation results reveal the power saving advantage of the proposed algorithm compared to a randomly selected subset of antennas.

Index Terms— Multiple input multiple output (MIMO) , linear precoding, convex optimization, antenna selection, Group Lasso.

1. INTRODUCTION

A major critical factor in increasing the number of antennas in a multiple input multiple output (MIMO) system is the cost of the radio frequency (RF) chain consisting of low noise amplifiers, mixers and analog to digital converters (ADCs). Antenna selection at the transmitter and/or receiver is a promising way to reduce the hardware costs yet capture the benefits of the capacity increase in MIMO channels. Particularly, it has been shown that antenna selection retains the diversity degree of the complete antenna array [1].

Assuming a MIMO system with M_t transmit antennas and M_r receive antennas; in order to maximize the throughput by selecting the optimal subset of transmit antennas, the channel capacity has to be computed for $\binom{M_t}{L_t}$ combinations of antennas where L_t is the number of available RF chains (selected transmit antennas). This is computationally impractical, especially for a large number of antennas. In general, this is a mathematically challenging optimization problem which is known to be non-convex and NP-hard [2] .

Suboptimal selection techniques both for the transmitter and receiver side have been studied intensively in the past decade, see [1, 2, 3] and references therein. A differentiable and convex problem formulation was introduced in [2] for receive antenna selection by proposing a semi-definite relaxation of the original problem where the discrete selection parameter holding the values 0 and 1 is transformed into a continuous interval of $[0, 1]$ with a cut-off threshold (rounding-off scheme).

Recently, MIMO systems with a very large number of antennas (in the order of a hundred) are proposed in [4] for very aggressive spatial multiplexing adopting a very low transmission power. The antenna selection approach could play a very important role in such mass-MIMO systems to reduce the hardware cost, since assigning a separate RF chain to each antenna is extremely costly. The question which then comes to mind is how to find the best trade-off between capacity, power and complexity.

In this paper, we approach the antenna selection problem by designing a sparse precoder via jointly selecting the optimal subset of transmit antennas and removing the interference between multiple streams in a MIMO transmitter. A zero-forcing (ZF) linear precoder is designed to remove the co-channel and inter-symbol interference between the multiple transmit antennas. ZF is a widely used suboptimal precoder scheme which equips the MIMO system with multiple independent subchannels and consequently reduces the precoder design to a convex power allocation problem [5]. This is particularly suitable for communication systems with more processing power available at the transmitter or multi-user MIMO systems where only the base station is aware of the full channel state information (CSI) [6].

ZF precoding is possible when the number of available transmitters is at least equal to the number of receivers such that the channel inverse can be obtained at the transmitter. However, possible channel nulls may lead to a significant increase of the transmit power which violates the total transmit power constraint and may push the amplifier to a non-linear regime. To overcome this issue, more transmit antennas than receive antennas are utilized. This way the ZF linear equations form an underdetermined linear system such that the extra degrees of freedom can facilitate the design of a ZF precoder with a limited power.

Very recently a standard semi-definite programming formulation is proposed in [7] for the sparse beamforming problem in the context of multi-cast transmission subject to a required quality of service for each user. A similar approach is used in [8] to design a sparse multi-cell receive filter for cooperative base stations. In this paper, a similar idea is applied to joint precoder and antenna selection. The idea basically boils down to relaxing the integer valued solution set present in the conventional selection problem. Differently, we consider the antenna selection problem at the transmitter for a general MIMO system with more antennas available at the transmitter than the receiver side.

Assuming a limited number of available RF chains at the transmitter (still larger than the number of receive antennas though), we try to find out the optimal subset of antennas that can provide a certain desired data rate while keeping the transmit power at an acceptable level and all this in the setting of a ZF precoding scenario.

The remainder of the paper is organized as follows. In Sec. 2 we introduce the system model and the design criterium for the linear ZF precoder. The proposed problem formulation for jointly designing the beamformer and antenna subset selection is given in Sec. 3 followed by a 3-step SDP algorithm to solve the optimization problem. Matlab simulation proof for the proposed algorithm is presented in Sec. 4. The important remarks of the paper is provided in Sec. 5.

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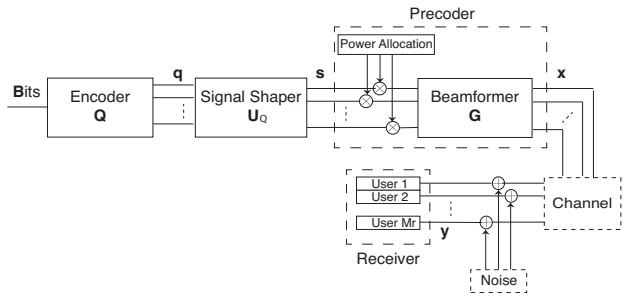

Fig. 1. Block diagram of MIMO link with linear precoding.

2. SYSTEM MODEL

We consider a standard MIMO or equivalently a MISO multi-user system with M_t transmit antennas and M_r antennas/users at the receiver [1]. The number of transmit antennas is assumed to be larger than the number of receivers, $M_t \geq M_r$. The received data vector $\mathbf{y}$ is expressed as a linear combination of the MIMO channel $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ and the transmit vector $\mathbf{x}$,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}\,. \tag{1}$$

The noise is considered to be a zero-mean Gaussian vector $\mathbf{n}$ of length M_r. The variance of the noise is assumed to be $\sigma_n^2 = 1$ for normalization purposes.

2.1. Linear Precoding

Fig.1 shows the different blocks in a MIMO link. The encoder unit determines the covariance matrix of the output signal vector $\mathbf{q} \in \mathbb{C}^{M_r \times 1}$. In turn the signal shaper matrix is given by the eigenvectors of the codeword covariance matrix $\mathbf{U}_Q$, where the transmit sequence of length M_r is given by $\mathbf{s} = \mathbf{U}_Q\mathbf{q}$. This is the optimal choice for all precoder design criteria [5]. As a result, the covariance matrix of the input signal $\mathbf{s}$ to the precoder block is an identity matrix; $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{M_r}$, which is an important assumption for the transmitter design. Free design parameters for a linear precoder $\mathbf{W} \in \mathbb{C}^{M_t \times M_r}$ are shown in the precoder block in Fig. 1 including the power allocation unit and the beamformer.

In general, a linear precoder $\mathbf{W}$ can be considered as a beamforming matrix $\mathbf{G}$ and a diagonal matrix $\mathbf{\Sigma}$, which is related to power allocation in the subchannels:

$$\mathbf{W} = \mathbf{G}\mathbf{\Sigma}\,. \tag{2}$$

Given perfect channel state information at the transmitter (CSIT), a linear precoder can be designed to pre-equalize the channel at the transmitter side. In this paper, the precoder design criterium is assumed to be ZF in the sense that it forces the interference between symbols at the receiver to zero, i.e, $\mathbf{H}\mathbf{G} = \mathbf{I}$ so we have

$$\mathbf{H}\mathbf{W} = \mathbf{\Sigma} = \mathbf{P}^{1/2}\,, \tag{3}$$

where $\mathbf{P}$ is the unknown diagonal power matrix $\mathbf{P} = \text{diag}\,(p_1, p_2, ..., p_{M_r})$, and $p_j \in \mathbf{R}^+$, $j = 1, 2, ..., M_r$, is the SNR on the jth receive antenna (assuming a unit variance noise on the receiver). Note that the square root of the diagonal matrix $\mathbf{P}$ is defined by the square root of the real non-negative diagonal elements. Accordingly, the ZF precoding matrix can be expressed as

$$\mathbf{W} = (\mathbf{H}^\dagger + \mathbf{J}_\perp\mathbf{Z})\mathbf{P}^{1/2}\,, \tag{4}$$

[1]Henceforth, the term *receiver* is used for a set of collocated antennas as well as for a collection of multiple users each with a single antenna.

where the first factor in (4) is the generalized inverse of $\mathbf{H}$, and $\mathbf{H}^\dagger$ is the pseudo-inverse or right inverse of $\mathbf{H}$. The orthogonal projector onto the null space of $\mathbf{H}$ is $\mathbf{J}_\perp$, and $\mathbf{Z}$ is any arbitrary matrix.

2.2. Capacity Constrained ZF Precoding

Conventionally, the transmitter is designed considering a total transmit power constraint $P^\star$. The total transmit power is directly determined by the precoding matrix as $P = E\{||\mathbf{x}||^2\} = E\{\mathbf{W}\mathbf{s}\,\mathbf{s}^H\mathbf{W}^H\} = \text{Tr}\{\mathbf{W}\mathbf{W}^H\}$ because $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{Mr}$. This power P is related to the diagonal matrix $\mathbf{P}$, which contains the SNR values at the receiver, after the channel matrix [9].

The next step towards the design of a ZF linear precoder is to find $\mathbf{P}$ for a specific performance measure. A common figure of merit for a MIMO system to maximize is the throughput or capacity which for the system in Fig. 1 with $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{Mr}$, unit noise variance and ZF precoder is given by

$$C(\mathbf{H}, \mathbf{W}) = \log_2[\det\left(\mathbf{I}_{M_r} + \mathbf{H}\mathbf{W}\mathbf{W}^H\mathbf{H}^H\right)]\,. \tag{5}$$

Having the ZF criterium in (3), the capacity maximization problem with a total power constraint can be formulated as

$$\begin{array}{ll} \underset{\mathbf{P},\mathbf{W}}{\text{maximize}} & C(\mathbf{P}) = \log_2[\det\left(\mathbf{I}_{M_r} + \mathbf{P}\right)] \\ \text{s.t.} & \mathbf{H}\mathbf{W} = \mathbf{P}^{1/2} \\ & \text{Tr}\{\mathbf{W}\mathbf{W}^H\} \leq P^\star \end{array} \tag{6}$$

The ZF precoding design for maximizing the capacity in (6) is non-convex on $\mathbf{W}$ due to the nonlinear constraints . However, it can be easily relaxed by linearization of the quadratic variable. For more information on the convex relaxation for ZF precoding, see [9, 10] and references therein.

3. PROPOSED PROBLEM FORMULATION

So far, the extra degrees of freedom resulting from the underdetermined system of equations in (3) is exploited to minimize the total transmit power while the throughput of the system is maximized. Clearly, using less transmit antennas increases the total transmit power for the same throughput, or it will decrease the throughput for the same total transmit power. However, by carefully selecting the right transmit antennas, we can keep this loss to a minimum.

3.1. Transmit Antenna Selection

Suppose that only $M_r \leq L_t \leq M_t$ RF chains are available so at most L_t antennas can be utilized for simultaneous transmission. This means that some of the transmit antennas are not used and, consequently, the corresponding channel columns will be removed. This is the same as the famous antenna selection problem [1] but now the selection is performed by the aid of the precoding matrix. Thus, the NP-hard problem of finding an orthogonal selection matrix with zero and one entries is relaxed.

Let us first introduce some notations; the vector $\mathbf{a} \in \mathbb{R}^{n \times 1}$ related to the matrix $\mathbf{A} \in \mathbb{C}^{n \times m}$ represents a vector consisting of the ℓ_2-norms of the matrix rows, so $\mathbf{a} = [||\mathbf{a}_1||_2, ||\mathbf{a}_2||_2, ..., ||\mathbf{a}_n||_2]^T$ where $\mathbf{a}_i^T$, $i = 1, 2, ..., n$ corresponds to the rows of the matrix $\mathbf{A}$. These two notations should not be confused as we use them frequently.

In this paper, we jointly solve the antenna selection problem and ZF precoding constrained by the total transmit power. The solution of interest for $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{M_t}]^T$ needs to have some rows

$\mathbf{w}_i^T$ of all zeros in order to eliminate the corresponding transmit antennas. In other words, we need to minimize the cardinality of the rows of the beamformer.

Accordingly the optimization problem for antenna subset selection is formulated as

$$\begin{aligned} \underset{\mathbf{P},\mathbf{W}}{\text{minimize}} \quad & ||\mathbf{w}||_0 \\ \text{s.t.} \quad & \mathbf{HW} = \mathbf{P}^{1/2} \\ & \text{Tr}\{\mathbf{WW}^H\} \leq P^\star \\ & C(\mathbf{P}) \geq C^\star \end{aligned} \tag{7}$$

This is a challenging non-convex cardinality minimization problem that can not be solved efficiently even when the constraints are affine.

Here, the idea is to satisfy the constraints while minimizing a convex sparsity-promoting criterium which affects the complete rows of the beamformer. In general, joint sparsity models are used to represent an ensemble of signals being sparse. There are different approaches to induce jointly sparse solutions; Group Lasso regularization [11] is used here to relax the non-convex ℓ_0-norm operation in (7). The Group Lasso term here is defined as the ℓ_1-norm or summation of the ℓ_2-norms of the matrix rows; $||\mathbf{w}||_1 = \sum_{i=1}^{M_t} ||\mathbf{w}_i||_2$. This leads to an ℓ_1-norm relaxation on $||\mathbf{w}||_0$:

$$\begin{aligned} \underset{\mathbf{P},\mathbf{W}}{\text{minimize}} \quad & ||\mathbf{w}||_1 \\ \text{s.t.} \quad & \mathbf{HW} = \mathbf{P}^{1/2} \\ & \text{Tr}\{\mathbf{WW}^H\} \leq P^\star \\ & C(\mathbf{P}) \geq C^\star \end{aligned} \tag{8}$$

In order to solve (8), the quadratic term needs to be linearlized by defining a new semidefinite variable, $\mathbf{WW}^H = \mathbf{\Phi} \in \mathbb{C}^{M_t \times M_t}$. However, the solution for $\mathbf{\Phi}$ is required to be of rank M_r so that we can decompose it as $\mathbf{WW}^H$ but any rank constraint is non-convex. Even though it can be proved that by dropping the Group Lasso cost function, the solution to the remaining feasibility problem always satisfies the rank constraint [10], the Group Lasso minimization breaks this rule and by forcing some of the dependent rows and columns in $\mathbf{\Phi}$ to zero increases the rank of the solution. That is why we explain an SDP formulation for (8) in the following section.

3.2. Semi-Definite Program Formulation

We propose a 3-step formulation to solve (8) which yields a convex and semi-definite standard problem that can be solved using efficient interior point methods. First, we find the power allocation matrix $\mathbf{P}$ which maximizes the capacity constraint subject to the ZF criterium. In the second step, the L_t transmit antennas and the corresponding beamforming matrix are solved jointly. In the last step, the total transmit power is minimized for the selected subset of antennas. This can be summarized as follows:

1. Solve the maximization problem in (6) for $\mathbf{P}$.

   Looking at (8), we realize that the capacity constraint does not depend on $\mathbf{W}$ as long as the ZF equality is satisfied. Hence, we can first maximize the capacity for $\mathbf{P}$ without considering the choices for $\mathbf{W}$.

   It is proved in [10] that the optimal solution for the ZF equality constraint in terms of minimizing the total transmit power is $\mathbf{W}_{opt} = \mathbf{H}^\dagger \mathbf{P}^{1/2}$ for $\mathbf{Z} = \mathbf{0}$ in (4) which relaxes the ZF constraint. This important result transforms (6) to a concave maximization problem with one linear equality constraint. The total power is determined by $P = \text{Tr}\{\ \mathbf{W}_{opt}{\mathbf{W}_{opt}}^H\} = \text{Tr}\{\ \mathbf{\Phi}_{opt}\}$ so we have

   $$\begin{aligned} \underset{\mathbf{P}}{\text{maximize}} \quad & C(\mathbf{P}) = \textstyle\sum_j \log_2(1+p_j) \\ \text{s.t.} \quad & \text{Tr}\{\mathbf{PH}^\dagger {\mathbf{H}^\dagger}^H\} \leq P^\star \end{aligned} \tag{9}$$

   This is the famous water filling problem which is known to be the optimal power allocation scheme for maximizing the rate in MIMO systems. Henceforth, the solution to (9) is referred to as $\mathbf{P}_{opt}$. The resulting optimal capacity is denoted as C_{opt}, and in all the further steps, we will actually take $C^\star$ equal to C_{opt}, meaning that we will not sacrifice any capacity.

2. Given $\mathbf{P}_{opt}$ from (9), drop the capacity constraint in (8) and solve the antenna selection problem

   It is clear that the capacity reaches its maximum C_{opt} as long as the ZF constraint is satisfied with $\mathbf{P}_{opt}$. However, by dropping transmit antennas we will not be able to reach that solution, unless we allow the system to increase its transmit power. That is why we tackle the antenna selection problem in the following way:

   $$\begin{aligned} \underset{\alpha \geq 1, \mathbf{W}}{\text{minimize}} \quad & \alpha + \lambda \textstyle\sum_{i=1}^{M_t} ||\mathbf{w}_i||_2 \\ \text{s.t.} \quad & \mathbf{HW} = \mathbf{P}_{opt}^{1/2} \\ & \text{Tr}\{\mathbf{WW}^H\} \leq \alpha P^\star \end{aligned} \tag{10}$$

   The solution to (10) is referred to as $\mathbf{W}_s$. The resulting sparse $\mathbf{W}_s$ will be different from the optimal precoder $\mathbf{W}_{opt}$ but it will still satisfy $\mathbf{HW}_s = \mathbf{P}_{opt}^{1/2}$. As a result, the $\mathbf{Z}$ matrix related to $\mathbf{W}_s$ will not be zero and thus the total transmit power will obey $\text{Tr}\{\mathbf{W}_s\mathbf{W}_s\} \geq \text{Tr}\{\mathbf{W}_{opt}\mathbf{W}_{opt}\}$.

   The excess power penalty that we have to pay is parameterized by α in (10). We aim to find the sparsest solution for the ZF precoder which delivers the maximum capacity in (9) and simultaneously minimizes the α. The regularization parameter λ which is a positive scalar smaller than one, regulates the trade-off between the power penalty α and the number of transmit antennas L_t that will be adopted. Increasing λ leads to a more sparse solution where less transmit antennas will be used, but it will increase the power penalty α.

   Since we apply a Group Lasso relaxation, $\mathbf{W}_s$ is not optimal in the sense of providing minimum total power. For this reason, we should formulate a minimization problem considering the total power delivered by the solution of (10).

3. Minimize the total power for the selected subset of transmit antennas.

   Once the subset of antennas that minimizes the excess power is found, we need to find the minimum power beamformer associated with the selected set.

   Proposition 1. *Let $\mathbf{W}_s$ be the optimal solution to* (10) *where $||\mathbf{w}_s||_0 = L_t$. There exists an optimal ZF beamformer $\mathbf{W}'$ with the same sparsity pattern as $\mathbf{W}_s$ such that $||\mathbf{W}'||_F \leq ||\mathbf{W}_s||_F$.*

   Proof. From convex optimization theory, for any value of regularization parameter λ, there exists an ε such that the Group Lasso term in (10) can be appended to the constraints as $\sum_{i=1}^{M_t} ||\mathbf{w}_i||_2 \leq \varepsilon$. The resulting optimization problem then minimizes only the excess transmit power (α) subject to the ZF constraint and a sparsity constraint.

   So indeed, if the sparsity pattern is fixed and the sparsity constraint is removed, the ℓ_1-norm of the solution can go beyond

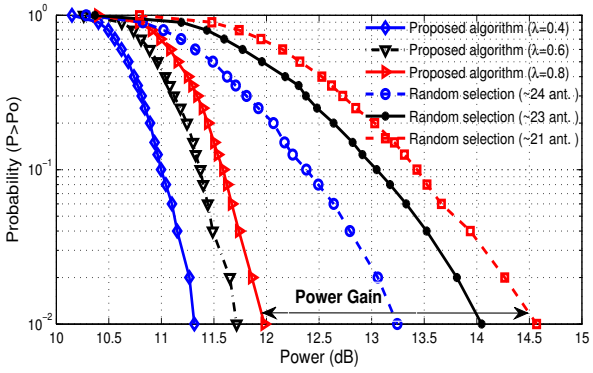

Fig. 2. Statistical performance of the proposed algorithm in terms of the CCDF. The total receive power $\mathbf{P}$ is set to 10 dB for capacity maximization with $M_t = 32$ transmit antennas which leads to a throughput of almost 13 bit/Hz by waterfilling power allocation .

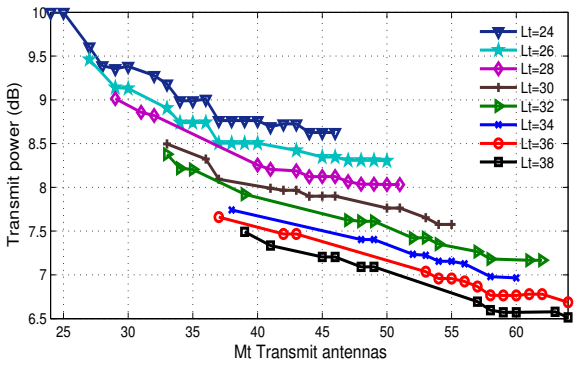

Fig. 3. Transmit power (dB) versus the number of existing transmit antennas (M_t), for the different fixed number of RF chains (L_t).

the former sparsity constraint and improve upon the solution of (10) in terms of the transmit power. □

Remember that $\mathbf{w}_s$ and $\mathbf{w}'$ are vectors containing the ℓ_2-norms of the rows of the matrices $\mathbf{W}_s$ and $\mathbf{W}'$, respectively. Thus, in order to find the minimum power solution, $\mathbf{W}'_s$ we solve the following minimization problem

$$\begin{aligned} \underset{\mathbf{W}'}{\text{minimize}} \quad & ||\mathbf{W}'||_F \\ \text{s.t.} \quad & \mathbf{H}\mathbf{W}' = \mathbf{P}_{opt}^{1/2} \\ & \mathcal{S}(\mathbf{W}') = \mathcal{S}(\mathbf{W}_s) \end{aligned} \tag{11}$$

Where the operator $\mathcal{S}(\mathbf{W}')$ gives the sparsity pattern (locations of the sparse rows) of $\mathbf{W}'$. The solution to (11) is the scaled pseudo-inverse of the reduced size channel; $\mathbf{W}'_s = \mathbf{H}_s^{\dagger}\sqrt{\mathbf{P}_{opt}}$, where $\mathbf{H}_s \in \mathbb{C}^{M_r \times L_t}$ is realized by removing the columns of $\mathbf{H}$ corresponding to the zero rows of $\mathbf{W}_s$.

4. SIMULATION RESULTS

The optimization problems in (9) , (10) and (11) can be solved efficiently. There are various developed SDP software packages including SDPT3, SeDuMi, DSDP, etc. that can be used to solve (9) and (10). We used the CVX toolbox in Matlab which calls SeDuMi for solving the defined problem [12]. In this section, we validate through simulations the performance of the proposed scheme.

In the simulation set-up we assume an independent identically distributed (i.i.d.) Rayleigh fading channel which is generated with zero mean Gaussian random variables for both real and imaginary parts of the complex channel and the noise variance is assumed to be one. The transmitter consists of M_t antennas and the receiver has a fixed number of antennas; $M_r = 6$.

For producing Fig. 2, 1000 different channel realizations are generated randomly and the proposed algorithm was carried out for different values of $\lambda = 0.4, 0.6, 0.8$. The number of transmit antennas is set to 32. The beamformer power ($||\mathbf{W}'_s||_F^2$) in dB is shown in terms of the CCDF (complementary cumulative distributive function) which indicates the probability that the power exceeds a certain value. Note that the number of selected antennas L_t could be different for each channel realization and the same value of λ. We consider the average L_t in Fig. 2 . We compare our method with the randomly selection scheme which selects the same number of antennas L_t at each channel realization.

Fig. 2 illustrates the power gain achieved by the sparse precoding algorithm, measured as the difference between the proposed algorithm's CCDF curve for a particular λ and the corresponding random selection, for example, $L_t \approx 21$ is related to the largest $\lambda = 0.8$. This is referred to as the power gain achieved by the sparse precoding algorithm.

As expected, by reducing the sparsity promoting regularization parameter λ, the number of selected antennas increases and consequently the power drops as well as the power gain. Moreover, one can see that the total transmit power for the proposed algorithm is 1.25 dB smaller with a fewer number of selected antennas (21 on average) when the proposed algorithm is performed, compared to the random selection with even more antennas (24 on average).

Fig. 3 shows the required transmit power for different numbers of transmit antennas (M_t) and selected antennas (L_t). It shows that 1 dB transmit power (or more) can be saved only by increasing the number of existing antennas at the transmitter while the number of available RF chains (L_t) remains the same. The throughput is determined as a system constraint that needs to be satisfied. These results promote the use of large arrays at the transmitter equipped with the proposed antenna selection algorithm as a power saving technique.

5. CONCLUDING REMARKS

Given the CSIT, a new formulation for transmit precoding design is proposed which combines antenna selection and beamforming and expresses them as a convex optimization problem. Accordingly, a 3-step SDP algorithm is proposed to solve for the sparse beamformer which selects the optimal set of channel columns that minimizes the total transmit power under a capacity constraint. Given a fixed number of available antennas at the transmitter, simulation results verify a notable power gain when L_t antennas are selected by performing the proposed algorithm compared to the random selection scheme.

Furthermore, the amount of saved power is shown to be sensitive to the number of available antennas at the transmitter (M_t), even though the number of RF chains is not changing. This is a promising result which enables us to transmit with a lower power and a fixed data rate only by putting extra antenna elements at the transmitter. Especially, large MIMO systems become more feasible by the development of new 60 GHz technology where more antennas can be squeezed in a small area.

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