FIXED-POINT REALIZATION OF LATTICE-REDUCTION AIDED MIMO RECEIVERS WITH COMPLEX K-BEST ALGORITHM

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ABSTRACT

Multiple-input multiple-output (MIMO) techniques provide high data rates but the optimal maximum likelihood (ML) detector exhibits high complexity. Recently lattice reduction (LR) aided detectors have been proposed to achieve near-ML performance with low complexity. In this paper, we develop a LR-aided complex K-best algorithm which reduces the complexity of the existing sphere decoding based K-best algorithm. Then we provide the fixed-point design of the LR-aided K-best MIMO receiver for both coded and uncoded systems. The architecture selection of each sub-module is developed and a simulation-based wordlength optimization procedure is proposed. Simulations show that the fixed-point results can keep bit error rate degradation within 0.2dB under 8×8 256-QAM MIMO systems.

Index Terms— Fixed-point, K-best algorithm, lattice reduction

1. INTRODUCTION

With the evolution of wireless communication systems, multiple-input multiple-output (MIMO) systems have been adopted to provide high data rates and high performance with maximum likelihood (ML) or near-ML algorithms, which require considerably high complexity especially with a large number of antennas and/or high order constellations [1, 2]. To reduce the complexity, suboptimal detectors, such as zero-forcing (ZF), minimum mean-square error (MMSE), and successive interference cancelation (SIC), have been deveoped. However, these schemes exhibit great bit error rate (BER) degradation compared to the ML detector due to diversity loss [3].

To approach high performance with low complexity, lattice reduction (LR) technique can be used in the MIMO detectors [4]. It is shown that LR-aided ZF, MMSE, and SIC detectors can achieve the same diversity as the ML detector [5, 6]. Furthermore, the LR-aided K-best detector can maintain near-ML performance even with small numbers of candidates, yielding much lower complexity than the conventional K-best detector [7, 8]. Meanwhile, when the K-best algorithm is implemented in complex domain instead of real domain, the complexity can be further reduced [9]. So here we focus on the LR-aided complex K-best MIMO receivers.

When considering practical systems, fixed-point design is a crucial step for hardware implementation in application-specific integrated circuits (ASICs) or field-programmable gate arrays (FPGAs). Despite the rich publications of the fixed-point design of each submodule like QR decomposition, LR, K-best, and Turbo decoder [10, 11, 12, 13], few studies focus on the fixed-point design of the overall system. In this paper, we aim to realize the fixed-point design of the overall LR-aided complex K-best MIMO receiver for both uncoded and coded systems. First, we propose a LR-aided complex K-best algorithm which exploits an on-demand child expansion and a priority queue to simplify the implementation. Then, we provide the architecture selection of each sub-module suitable for hardware. Last, we propose a simulation-based wordlength optimization scheme, which can minimize the total bit-width of fixed-point variables with relative small number of simulation iterations.

Notations: Superscript ^{T} denotes the transpose. The real and imaginary parts of a complex number are denoted as $\Re[\cdot]$ and $\Im[\cdot]$. Upper- and lower-case boldface letters indicate matrices and column vectors, respectively. $A_{i,k}$ indicates the (i, k)th entry of matrix $\mathbf{A}$, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, and $\mathbf{1}_{N\times L}$ is the $N \times L$ matrix with all entries one. $\mathbb{Z}$ is the integer set. $E\{\cdot\}$ denotes the statistical expectation. $\|\cdot\|$ denotes the 2-norm.

2. SYSTEM MODEL

Let us consider spatial multiplexing MIMO transmission with N_t transmit and N_r receive antennas as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \cdots, s_{N_t}]^T, (s_i \in \mathcal{S})$ is the complex symbol vector with $\mathcal{S}$ being a constellation set of square QAM, $\mathbf{H}$ is an $N_r \times N_t, (N_r \geq N_t)$ complex channel matrix, $\mathbf{y} = [y_1, y_2, \cdots, y_{N_r}]^T$ is the received signal vector, and $\mathbf{w} = [w_1, w_2, \cdots, w_{N_r}]^T$ is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance $N_0\mathbf{I}_{N_r}$. The real and imaginary parts of $\mathcal{S}$ are $\{-\sqrt{M}+1, -\sqrt{M}+3, \cdots, \sqrt{M}-1\}$ with M being the constellation size. We assume a quasi-static channel environment, i.e., $\mathbf{H}$ is invariant during a block and changes independently from block to block. We also assume that $\mathbf{H}$ is known at the receiver, but unknown at the transmitter.

Given the model in Eq. (1), the ML detector is

$$\hat{\mathbf{s}}^{\mathrm{ML}} = \arg\min_{\tilde{\mathbf{s}} \in \mathcal{S}^{N_t}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|^2, \tag{2}$$

which is generally non-deterministic polynomial hard (NP-hard).

2.1. LR-aided Detectors

Since the LR-aided detection only works for infinite lattice, it relaxes the boundary constraints in Eq. (2) to infinite lattice as

$$\hat{\mathbf{s}} = \arg\min_{\tilde{\mathbf{s}} \in \mathcal{U}^{N_t}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|^2, \tag{3}$$

where $\mathcal{U}$ is the unconstrained constellation set with the form $(2\mathbb{Z}+1) + (2\mathbb{Z}+1)j$.

However, since the problem in Eq. (3) loses the boundary information of QAM, it is generally not diversity-multiplexing tradeoff (DMT) optimal [14]. Instead, to achieve the DMT optimality, we adopt the MMSE-regularized problem as [15]

$$\begin{aligned}\hat{\mathbf{s}} &= \arg\min_{\tilde{\mathbf{s}} \in \mathcal{U}^{N_t}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|^2 + \frac{N_0}{\sigma_s^2}\|\tilde{\mathbf{s}}\|^2 \\ &= \arg\min_{\tilde{\mathbf{s}} \in \mathcal{U}^{N_t}} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}\tilde{\mathbf{s}}\|^2 \end{aligned} \tag{4}$$

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where $E\{\mathbf{s}\mathbf{s}^{\mathcal{H}}\} = \sigma_s^2\mathbf{I}$, $\bar{\mathbf{y}}$ is the extended received signal vector expressed as $[\mathbf{y}^T, \mathbf{0}_{1\times N_t}]^T$, and $\bar{\mathbf{H}}$ is the MMSE-extended matrix as $[\mathbf{H}^T, \sqrt{N_0/\sigma_s^2}\mathbf{I}_{N_t}]^T$.

To solve the problem in Eq. (4) with lower complexity, LR obtains a more "orthogonal" matrix $\tilde{\mathbf{H}} = \bar{\mathbf{H}}\mathbf{T}$, where $\mathbf{T}$ is a unimodular matrix, such that all the entries of $\mathbf{T}$ are Gaussian integers, and the determinant of $\mathbf{T}$ is ± 1 or $\pm j$. Given $\tilde{\mathbf{H}}$ and $\mathbf{T}$, Eq. (4) becomes

$$\hat{\mathbf{s}} = 2\mathbf{T}\arg\min_{\tilde{\mathbf{z}}\in\mathbb{Z}^{N_t}} \|\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{z}}\|^2 + (1+j)\mathbf{1}_{N_t\times 1}, \quad (5)$$

where $\tilde{\mathbf{y}}$ is the received signal vector after shifting and scaling as $(\bar{\mathbf{y}} - \bar{\mathbf{H}}\mathbf{1}_{N_t\times 1}(1+j))/2$ and $\tilde{\mathbf{s}} = 2\mathbf{T}\tilde{\mathbf{z}} + (1+j)\mathbf{1}_{N_t\times 1}$. Since $\tilde{\mathbf{H}}$ is more "orthogonal," the closest point search based on $\tilde{\mathbf{H}}$ can enjoy much lower complexity compared to that based on $\bar{\mathbf{H}}$ in Eq. (4) [16]. However, the problem in Eq. (5) is still NP-hard, which means that the closest point search is still prohibitive when N_t is large.

3. LR-AIDED COMPLEX K-BEST ALGORITHM

To achieve near-ML performance with low complexity, here we propose the LR-aided complex K-best algorithm for the problem in (5).

With QR decomposition $\tilde{\mathbf{H}} = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is an $(N_r+N_t)\times N_t$ orthonormal matrix and $\mathbf{R}$ is a $N_t \times N_t$ upper triangular matrix, the problem in (5) can be rewritten as

$$\hat{\mathbf{s}} = 2\mathbf{T}\arg\min_{\tilde{\mathbf{z}}\in\mathbb{Z}^{N_t}} \|\breve{\mathbf{y}} - \mathbf{R}\tilde{\mathbf{z}}\|^2 + (1+j)\mathbf{1}_{N_t\times 1}, \quad (6)$$

where $\breve{\mathbf{y}} = \mathbf{Q}^{\mathcal{H}}\tilde{\mathbf{y}}$.

Next, the breadth-first search from the N_tth layer to the 1st layer is performed in the LR-aided complex K-best algorithm. For each layer (e.g., the nth layer), the pseudo code of the processing procedure is summarized in Table 1 denoted as $\mathrm{Find\_Kbest\_Children}()$ subroutine, where a patrial candidate of the layer is defined as $\mathbf{z}_i^{(n)}$ as $[z_{i,n}^{(n)}, \cdots, z_{i,N_t}^{(n)}]^T$, the cost of the partial candidate is

$$cost_i^{(n)} = \sum_{j=n}^{N_t} |\breve{y}_j - \sum_{k=j}^{N_t} R_{j,k} z_{i,k}^{(n)}|^2, \quad (7)$$

and a partial candidate of the nth layer $\mathbf{z}_i^{(n)}$ is defined as a child of a partial candidate of the $(n+1)$st layer $\mathbf{z}_j^{(n+1)}$ if and only if $\mathbf{z}_i^{(n)} = [z_{i,n}^{(n)}, (\mathbf{z}_j^{(n+1)})^T]^T, z_{i,n}^{(n)} \in \mathbb{Z} + \mathbb{Z}j$ holds. Therefore, for the nth layer, the algorithm finds the K best partial candidates $[\mathbf{z}_1^{(n)}, \mathbf{z}_2^{(n)}, \cdots, \mathbf{z}_K^{(n)}]$, i.e., the K partial candidates that have the minimum costs among all the children of the K partial candidates $[\mathbf{z}_1^{(n+1)}, \mathbf{z}_2^{(n+1)}, \cdots, \mathbf{z}_K^{(n+1)}]$ in the previous $(n+1)$st layer. This process is initiated at (N_t+1)th layer as $len = 1$, $\mathbf{z}_1^{(N_t+1)} = []$, and $cost_1^{(N_t+1)} = 0$. After the 1st layer is reached, the K best candidates with minimum costs can be obtained as

$$\hat{\mathbf{s}}_k = \mathcal{Q}(2\mathbf{T}\mathbf{z}_k^{(1)} + (1+j)\mathbf{1}_{N_t\times 1}), \quad k = 1, 2, ..., K. \quad (8)$$

The main difficulty of the LR-aided K-best algorithms is that the valid children for each parent is infinite. To address the infinite children issue, similar to the LR-aided real K-best algorithm in [8], our proposed LR-aided complex K-best algorithm exploits an on-demand child expansion and a priority queue. However, different from the real case, the LR-aided complex K-best algorithm employs the complex Schnorr-Euchner (SE) strategy [17, 18], where the children of a parent in the nth layer are classified as two categories:

- Type I, where the real part of $z_{i,n}^{(n)}$ of the child $z_i^{(n)}$ is the same as the real part of $z_{k,n}^{(n)}$, where $\mathbf{z}_k^{(n)}$ is the child with the lowest cost among all the children of the same parent.
- Type II, otherwise.

Once a Type I child is chosen as one of the K-best children, both the real (lines 13-24 in Table 1) and imaginary SE (lines 28-33 in Table 1) expansions are executed to guarantee that the next smallest child of the same parent is in the priority queue, while for a Type II child, only imaginary SE expansion is used (lines 28-33 in Table 1).

Since the priority queue has, at most, $2K$ elements, the complexity of its updating (lines 24 and 33 in Table 1) is $\mathcal{O}(\log_2(2K)) \simeq \mathcal{O}(\log_2(K))$. Therefore, the overall complexity of the proposed LR-aided complex K-best for each layer is $\mathcal{O}(N_t K + K\log_2(K))$.

In uncoded systems, the hard output of the LR-aided complex K-best detector is $\hat{\mathbf{s}} = \arg\min_{\tilde{\mathbf{s}}\in\{\hat{\mathbf{s}}_k\}_{k=1}^K} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|^2$. In coded systems, the LR-aided complex K-best detector can be used in the iterative detection and decoding (IDD) structure to improve the performance [19]. In this scheme, the detector outputs the soft information of each coded bit to the decoder, which can be well approxmated by the log-likelihood ratio (LLR) as follows [19]:

$$L_E(c_i) \approx \frac{1}{2}\max_{\boldsymbol{c}\in C_s\cap S_i^{+1}} \left\{-\frac{2}{N_0}\|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|_2^2 + \boldsymbol{c}^T\boldsymbol{L}_A - L_A(c_i)\right\} - \frac{1}{2}\max_{\boldsymbol{c}\in C_s\cap S_i^{-1}} \left\{-\frac{2}{N_0}\|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|_2^2 + \boldsymbol{c}^T\boldsymbol{L}_A + L_A(c_i)\right\}, \quad (9)$$

where $\boldsymbol{c}$ is the transmitted coded bit vector mapped into the symbol vector $\tilde{\mathbf{s}}$, $\boldsymbol{L}_A$ denotes the prior information of $\boldsymbol{c}$ from the channel decoder, C_s is the candidate list of the transmitted coded bit vectors corresponding to $\{\hat{\mathbf{s}}_k\}_{k=1}^K$ from the output of the LR-aided K-best MIMO detector, S_i^{+1} represents the subset of C_s with the ith bit being $+1$, and similarly for S_i^{-1}, so that $C_s = S_i^{+1} \cap S_i^{-1}$.

4. FIXED POINT REALIZATION

The fixed-point realization contains two parts: the architecture selection and the fixed-point conversion with wordlength optimization. The former is to facilitate the hardware implementation, while the latter is to minimize the bit-width of fixed-point variables so that the hardware cost such as power, area, and delay can be reduced.

4.1. Architecture Selection of Each Sub-modules

QR Decomposition: There are three well known algorithms to perform QR decomposition [20], i.e. Householder transformation, Gram-Schmidt, and Givens rotation. Here we adopt the Givens rotation algorithm implemented by Coordinate Rotation Digital Computer (CORDIC) scheme under Triangular Systolic Array (TSA) in [21]. CORDIC is attractive due to the simple shift and add operations in hardware. And the TSA structure can be efficiently implemented by parallel and pipelining fashions to reduce the latency.

Lattice Reduction: Different LR techniques have been summarized in [4] for MIMO detection. Among them, Lenstra-Lenstra-Lovasz (LLL) [22] algorithm is a popular scheme, which can approach the optimal performance with low complexity. Here we adopt the modified complex LLL scheme proposed in [11], which is suitable for hardware realization by transforming the complicated division and the inverse square-root operations into Newton-Raphson iteration and Householder CORDIC algorithm, respectively.

Complex K-best Algorithm: The main processing procedure of the complex K-best is the $\mathrm{Find\_Kbest\_Children}()$ subroutine summa-

Input:	$\{\mathbf{z}_k^{(n+1)}\}_{k=1}^{len}, \{cost_k^{(n+1)}\}_{k=1}^{len}$
Output:	$\{\mathbf{z}_k^{(n)}\}_{k=1}^{K}, \{cost_k^{(n)}\}_{k=1}^{K}$
(1)	For $i = 1$ to len
(2)	$r_i = \breve{y}_n - \sum_{l=n+1}^{N} R_{n,l} z_{i,l}^{(n+1)}$
(3)	$z_i = \lceil r_i / R_{n,n} \rfloor$
(4)	$child_i = [z_i, (\mathbf{z}_i^{(n+1)})^T]^T$
(5)	$parent_i = i$
(6)	$childcost_i = cost_i^{(n+1)} + \lvert r_i - R_{n,n} z_i \rvert^2$
(7)	$step_i = \mathrm{sgn}(r_i / R_{n,n} - z_i)$
(8)	$type_i = 1$
(9)	End for
(10)	Initialize priority queue $\mathcal{Q}$ with $\{childcost_i\}_{i=1}^{len}$ as keys
(11)	For $k = 1$ to K
(12)	Find the index i associated with the minimum key in $\mathcal{Q}$
(13)	If $type_i = 1$ then
(14)	$len = len + 1$
(15)	$r_{len} = r_i$
(16)	$parent_{len} = parent_i$
(17)	$\Re[z_{len}] = \Re[z_i] + \Re[step_i]$
(18)	$\Im[z_{len}] = \Im[z_i]$
(19)	$child_{len} = [z_{len}, (\mathbf{z}_{parent_{len}}^{(n+1)})^T]^T$
(20)	$childcost_{len} = cost_{parent_{len}}^{(n+1)} + \lvert r_{len} - R_{n,n} z_{len} \rvert^2$
(21)	$\Re[step_{len}] = -\Re[step_i] - \mathrm{sgn}(\Re[step_i])$
(22)	$\Im[step_{len}] = \Im[step_i]$
(23)	$type_{len} = 1$
(24)	Update $\mathcal{Q}$ by adding the key $childcost_{len}$
(25)	End if
(26)	$\mathbf{z}_k^{(n)} = child_i$
(27)	$cost_k^{(n)} = childcost_i$
(28)	$\Im[z_i] = \Im[z_i] + \Im[step_i]$
(29)	$child_i = [z_i, (\mathbf{z}_{parent_i}^{(n+1)})^T]^T$
(30)	$childcost_i = cost_{parent_i}^{(n+1)} + \lvert r_i - R_{n,n} z_i \rvert^2$
(31)	$\Im[step_i] = -\Im[step_i] - \mathrm{sgn}(\Im[step_i])$
(32)	$type_i = 2$
(33)	Update $\mathcal{Q}$ using $childcost_i$ as the new key
(34)	End for
(35)	$len = K$
(36)	Output $\{\mathbf{z}_k^{(n)}\}_{k=1}^{K}, \{cost_k^{(n)}\}_{k=1}^{K}$

Table 1. The proposed $\mathrm{Find\_Kbest\_Children}()$ subroutine.

rized in Table 1. To speed up the procedure, here we avoid the division operation (lines 3 and 7 in Table 1) by normalizing the input $\breve{\mathbf{y}}$ and $\mathbf{R}$ with the corresponding diagonal element of $\mathbf{R}$, i.e. $\vec{y}_i = \breve{y}_i / R_{i,i}$, and $\vec{R}_{n,i} = R_{n,i}/R_{i,i}, \forall n$. Meanwhile, the priority queue is implemented by a binary min-heap data structure, which can efficiently implement inserting and deleting candidates by up-heap and down-heap operations, respectively.

LLR Calculation: Because of the partial candidate list in LR-aided complex K-best MIMO detector, the LLR value L_E in (9) may be large enough to prevent the decoder from correcting the error data. To avoid this problem, here we adopt LLR clipping [19] which limits the range of LLR values so that the decoder can still overcome some error data. Furthermore, it can also reduce the wordlength of the fixed-point design to decrease the hardware complexity.

Turbo Decoding: The Turbo decoder contains two elementary MAP decoders interconnected to each other by interleavers and deinterleavers in serial way [23]. Here we adopt the Max-Log-MAP scheme, which has almost the same performance as the MAP algorithm with much lower complexity [24]. For this scheme, the process of each constituent decoder consists of computing Branch Metric γ, Forward Recursion α, Backward Recursion β, and Extrinsic information L_e.

4.2. Fixed-point Conversion with Wordlength Optimization

For the fixed-point conversion, all floating-point data types and arithmetic operations are transformed into the corresponding fixed-point version. So we develop a complete fixed-point C model, which is bit-accurate with Verilog HDL source code so that it can mimic the practical data operation in hardware.

After the conversion, we optimize the wordlength to find the minimum bit-width for each fixed-point variable when the performance is kept within the tolerated error metric. The optimization can be performed by analysis or simulation [25]. The former is usually conservative and difficult to analyze in nonlinear and unsmooth operations [26]. So here we adopt the latter. However, the simulation-based scheme is an NP-hard combinatorial problem [27], which leads to exceedingly long simulation time. To avoid this problem, we propose a scheme by combining the *Heuristic procedure* and *Max-1 bit procedure* summarized in [28]. It utilizes the small number of iterations from the *Heuristic procedure* and the minimum total bit-width from *Max-1 bit procedure*. The proposed wordlength optimization procedure is depicted in Fig. 1 and summarized as follows:

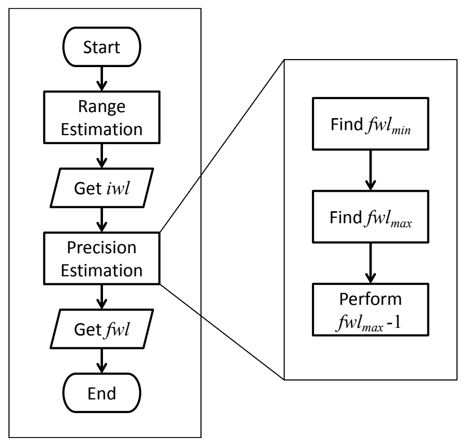

Fig. 1. The proposed wordlength optimization procedure.

- **Range Estimation**: Determine the minimum integer wordlength (iwl) to prevent overflow and underflow, which can be obtained by examining the histograms of each fixed-point variables under large data simulation.
- **Precision Estimation**: Find the minimum fraction wordlength (fwl) so that the performance degradation under quantization noise can be retained within the tolerated error metric. This step consists of the following three sub-steps:
  - § Find fwl_{min}: Obtain the smallest fwl of each variable that satisfies the tolerated error metric when the fwl of all other variables are large enough (here we use 32 bits).
  - § Find fwl_{max}: Increase fwl_{min} of all variables simultaneously with 1 bit as step size until the tolerated error metric is met. The updated values of fwl_{min} are the corresponding fwl_{max}.
  - § Perform $fwl_{max}-1$: Record the BER by reducing the fwl_{max} of each variable 1 bit while keeping the fwl_{max} of all other variables unchanged. The fwl_{max} of the variable with the best BER performance is updated as $fwl_{max}-1$. This process is repeated until the tolerated error metric is not satisfied to get the final optimized fwl.

5. NUMERICAL RESULTS

In this section, we first simulate different MIMO detectors in floating-point to demonstrate their performance. Then we provide the fixed-point results of our proposed LR-aided complex K-best algorithm. The parameters of the simulation are summarized in Table 2.

Antenna number and modulation	8×8 MIMO with 256 QAM
Channel modeling	i.i.d. Rayleigh fading channel with AWGN
MIMO detectors	Proposed LR-aided complex K-best, ML LR-aided real K-best [8], Real K-best [2]
Candidate Number K	$K = 3, 7, 15, 63, 127, 1023$
Channel coding	Turbo code with code rate: $1/2$ Codeword length: 1024 Code generator: $(1, \frac{1+D^2}{1+D+D^2})$
Iteration number of IDD receiver	4 iterations between detector and decoder 8 iterations within the turbo decoder

Table 2. Simulation parameters.

Fig. 2 shows that LR-aided K-best only needs $K{=}15$ to achieve near-ML performance in uncoded systems, while the conventional K-best needs $K{=}255$ to have similar performance. Fig. 2 also shows that both LR-aided real and complex K-best have almost the same performance given the same K.

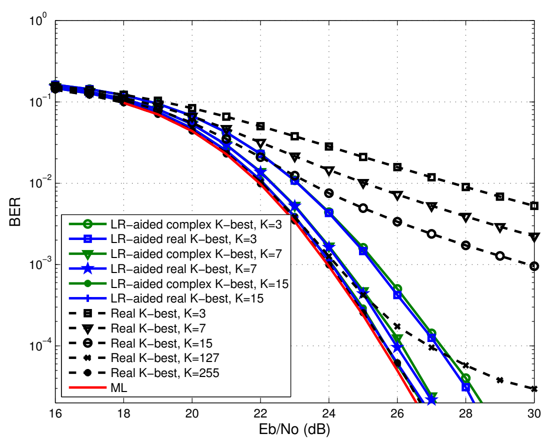

Fig. 2. Floating-point performance of uncoded systems.

Fig. 3 displays that the performance gain of LR-aided K-best is decreased compared to K-best in coded systems (here we omit the LR-aided real K-best since its results are almost the same as the LR-aided complex K-best). But we can still have 1 dB gain at 10^{-4} BER with a reasonable value $K{=}15$ in practice.

Based on the above results, we choose $K{=}15$ for the fixed-point design of the proposed LR-aided complex K-best MIMO receiver. Here we set BER as the error metric and make the the energy per bit to noise power spectral density ratio (Eb/No) degradation after fixed-point design kept within 0.2dB at 10^{-4} BER. The wordlength optimization procedure is refereed to the scheme in Fig. 1. The optimized configurations of the key fixed-point variables of each submodule are summarized in Table 3, and the corresponding performance is depicted in Fig. 4, which satisfies our predefined BER requirement.

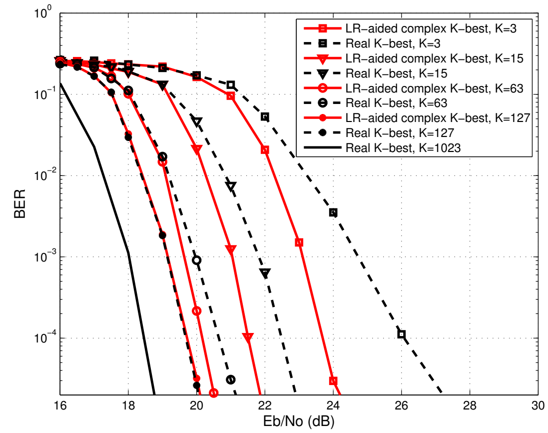

Fig. 3. Floating-point performance of coded systems.

Complex K-best				
$\vec{y}_i$	$\vec{R}_{n,i}$	r_i	z_i	$cost_i$
(8,12)	(1,14)	(8,12)	(9,0)	(8,12)
CLLL			**CORDIC in QR**	
Q	R	T	$data$	$angle$
(1,13)	(5,13)	(9,0)	(6,11)	(4,12)
Turbo Decoding				**LLR**
L_e	α	β	γ	L_E
(4,2)	(7,3)	(7,3)	(6,3)	(4,3)

Table 3. Fixed-point configurations (iwl, fwl) of key parameters.

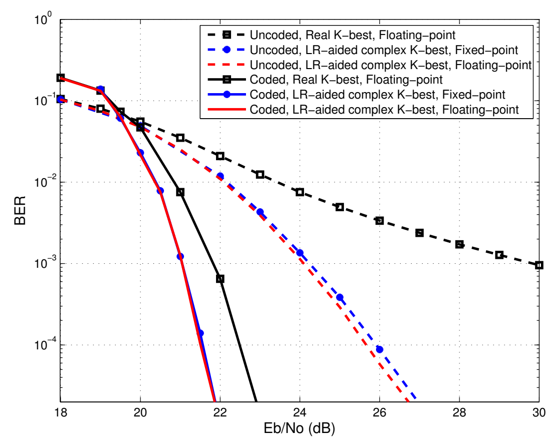

Fig. 4. Fixed-point results of LR-aided complex K-best detectors.

6. CONCLUSION

In this paper, we developed the fixed-point design of the LR-aided K-best MIMO receiver. First, we proposed a LR-aided complex K-best algorithm to facilitate the hardware realization. Then we provided the architecture selections and developed a simulation-based wordlength optimization scheme to obtain the fixed-point configurations. In future, we will implement the proposed LR-aided complex K-best algorithm in hardware to validate our fixed-point design.

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