EFFICIENT SINR FAIRNESS ALGORITHM FOR LARGE DISTRIBUTED MULTIPLE-ANTENNA NETWORKS

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ABSTRACT

This paper studies the joint beamforming and power control in a multiuser distributed antenna uplink network, wherein the number of users and the number of separately located antennas grow large with ratio being bounded. We consider the SINR fairness problem under individual power constraint and present a distributed iterative algorithm. This algorithm, though converging to the instantaneous optimal solution, requires instantaneous power update. In order to design a low complexity algorithm that achieves optimality in the asymptotic sense, we leverage the large system structure to derive an asymptotic solution requiring only channel statistics. In this algorithm, the asymptotic power is slowly updated and the asymptotic beamformer can be obtained in a non-iterative manner. The convergence property of the proposed algorithm is also studied.

***Index Terms*—** Power control, beamforming, large distributed multiple-antenna network, random matrix theory, nonlinear Perron-Frobenius theory

1. INTRODUCTION

Small-cell network [1] has demonstrated the potential to improve the spectral efficiency of current wireless networks and increase the flexibility of network operation. Such economical network structure involves dense deployment of distributed low-power, low-cost base stations that can operate in a fully self-organizing manner, or form a cooperative cluster [2] to enable joint processing. In this work, we are interested in a large distributed structure of base stations that operate in a cooperative manner to their serving users, in order to jointly satisfy the system metric of interest. Due to practical constraints such as the finite capacity of the backhaul [2] and limited feedback [3], low complexity efficient algorithms are often favored to help scale the system performance. Herein, we consider a joint beamforming and power control problem [4] in a distributed antenna uplink network to enforce max-min fairness across users, and leverage the large system structure to design an efficient algorithm.

Relation to Prior Work and Contributions: The max-min fairness, also known as SINR balancing problem is one canonical problem in optimization [5–10]. It was tackled using mainly three methodologies. The first approach [5, 6] relies on an extended coupling matrix, and the second approach [7] employs conic programming. These two approaches are centralized, and can not characterize the convergence rate of the algorithm. The recent third approach [8–10] uses nonlinear Perron-Frobenius theory to prove the convergence rate of the proposed algorithm. Herein, we are interested in a multiuser uplink with individual power constraint, and we employ the power control algorithm in [10] as the building block to present a distributed algorithm to compute the optimal power and beamformer. The algorithm uses an iterative approach to compute the instantaneous optimal power, whose complexity increases with the system dimension. In order to obtain an efficient algorithm that can leverage the large system structure [11], we employ random matrix theory [12] to examine the asymptotic behavior of the system. The concept of designing a low complexity algorithm to achieve max-min fairness was initially considered in [13] for a generic multiuser downlink. Similar strategy is employed in [14] for a power minimization problem. In [15], asymptotic analysis is utilized to compare several cooperative schemes. In [16], we consider algorithmic issues for a coordinated multicell downlink with network duality [17]. Leveraging random matrix theory also provides insights into other important system design problems, e.g., see [18–20] and the reference therein. Herein, we consider a large distributed antenna uplink network, whose distributed geometry further complicates the transition from asymptotic analysis to algorithm design. Moreover, we study the convergence property of the proposed algorithm and draw relationship with the co-located antenna case in [16]. The effectiveness of the algorithm is further demonstrated with simulation results.

The work of Y. Huang and B. D. Rao was supported by Ericsson endowed chair funds, the Center for Wireless Communications, UC Discovery grant com09R-156561 and NSF grant CCF-1115645. The work of C. W. Tan was supported by grants from the Research Grants Council of Hong Kong, Project No. RGC CityU 125212, and Qualcomm Inc.

2. SYSTEM MODEL

Consider a distributed multiple-antenna uplink network with K single-antenna users transmitting simultaneously to N distributed base stations, which are connected to a central processing unit through dedicated backhaul, i.e., see Fig. 1 for illustration. Each distributed base station is assumed to be equipped with a single antenna, and the analysis in this work can be directly extended to the scenario with multiple antennas per distributed base station. The central processing unit is assumed to process the signals from the N base stations, wherein this virtual cell assumption corresponds to a cooperative small cell network. The received vector $\mathbf{y} \in \mathbb{C}^{N\times 1}$ at the central processing unit is written as

$$\mathbf{y} = \sum_{k=1}^{K} \sqrt{\frac{p_k}{K}} \mathbf{h}_k x_k + \mathbf{z}, \tag{1}$$

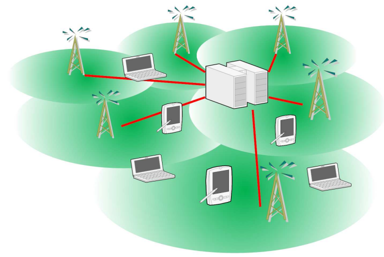

Fig. 1. Illustration of a large distributed multiple-antenna network.

where $\frac{p_k}{K}$ denotes the transmit power of user k, x_k is an information symbol with unit power, $\mathbf{h}_k \triangleq (h_{1,k}, \cdots, h_{N,k})^\mathsf{T} \in \mathbb{C}^{N\times 1}$ represents the channel vector from user k to the N distributed base stations, and $\mathbf{z}$ characterizes the additive white noise effect with zero mean and covariance matrix $\sigma\mathbf{I}$.

Linear beamforming strategy is assumed at the central processing unit, and thus a set of beamforming vectors is employed to map the received signal to independent scalar decoders for recovering the transmitted symbol. Denote $\mathbf{u}_k \in \mathbb{C}^{N\times 1}$ as the normalized beamformer for user k, i.e., $||\mathbf{u}_k||^2 = 1$, then the SINR for user k is expressed as

$$\Gamma_k(\mathbf{p}, \mathbb{U}) = \frac{\frac{p_k}{K}|\mathbf{u}_k^\dagger \mathbf{h}_k|^2}{\sum_{l\neq k}\frac{p_l}{K}|\mathbf{u}_k^\dagger \mathbf{h}_l|^2 + \sigma}, \tag{2}$$

where $\mathbf{p} = (p_1, \cdots, p_K)^\mathsf{T}$, $\mathbb{U} = (\mathbf{u}_1, \cdots, \mathbf{u}_K)$.

Now consider a joint beamforming and power control problem, where the objective is to maximize the minimum weighted SINR among users under individual power constraint, in order to enforce egalitarian fairness across users. Let β_k represent the priority for user k illustrating different service priorities with $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]^\mathsf{T}$. The weighted SINR balancing problem subject to individual power constraint can be written as

$$\begin{array}{ll} \text{maximize} & \min\limits_k \frac{\Gamma_k(\mathbf{p},\mathbb{U})}{\beta_k} \\ \text{subject to} & 0 \leq p_k \leq \bar{p},\ \forall k \\ & \|\mathbf{u}_k\|^2 = 1,\ \forall k \\ \text{variables :} & \mathbf{p}, \mathbb{U}. \end{array} \tag{3}$$

3. FINITE SYSTEM ANALYSIS

The optimization problem (3) appears non-convex, but can be solved in an optimal manner by using geometric transformation. However, employing standard convex optimization methods to find the optimal solution typically requires centralized computation and incurs a fair amount of parameter tuning. In this section, we present a distributed algorithm for computing the optimal solution of (3) by using nonlinear Perron-Frobenius theory and techniques developed in [8, 10].

For any given beamforming matrix $\mathbb{U}$, a simpler optimization problem for (3) can be formulated by only optimizing the power vector $\mathbf{p}(\mathbb{U})$. It is known that at optimality, the weighted SINR for different users are the same, and there exists at least one user that achieves the maximum power $\bar{p}$ [8, 10]. Furthermore, given the power vector $\mathbf{p}$, the optimal beamforming matrix $\mathbb{U}^*(\mathbf{p})$ (up to a scaling factor) can be shown to be the minimum variance distortionless response (MVDR) beamformer, namely:

$$\mathbf{u}_k^*(\mathbf{p}) = \frac{(\sum_{l\neq k}\frac{p_l}{K}\mathbf{h}_l\mathbf{h}_l^\dagger + \sigma\mathbf{I})^{-1}\mathbf{h}_k}{\|(\sum_{l\neq k}\frac{p_l}{K}\mathbf{h}_l\mathbf{h}_l^\dagger + \sigma\mathbf{I})^{-1}\mathbf{h}_k\|}. \tag{4}$$

In [10], a distributed algorithm is proposed to solve the power control problem. Herein, we present the following distributed algorithm (**Algorithm A**) to compute the optimal solution for the joint beamforming and power control problem (3).

Algorithm A:

- Initialize arbitrary $\mathbf{p}[0] \in \mathbb{R}_{++}^{K\times 1}$, and $\mathbf{u}_k[0] \in \mathbb{C}^{N\times 1}$ for $k = 1, \ldots, K$ such that $\|\mathbf{u}_k[0]\|^2 = 1$, and $p_k[0] \leq \bar{p}, \forall k$.

1. Update power $\mathbf{p}[\imath+1]$:

$$p_k[\imath+1] = \left(\frac{\beta_k}{\Gamma_k(\mathbf{p}[\imath], \mathbb{U}[\imath])}\right) p_k[\imath] \quad \forall k. \tag{5}$$

2. Normalize $\mathbf{p}[\imath+1]$:

$$p_k[\imath+1] \leftarrow \frac{p_k[\imath+1]\bar{p}}{\max_l p_l[\imath+1]} \quad \forall k. \tag{6}$$

3. Update beamforming matrix $\mathbb{U}[\imath+1]$:

$$\mathbf{u}_k[\imath+1] = \frac{(\sum_{l\neq k}\frac{p_l[\imath+1]}{K}\mathbf{h}_l\mathbf{h}_l^\dagger + \sigma\mathbf{I})^{-1}\mathbf{h}_k}{\|(\sum_{l\neq k}\frac{p_l[\imath+1]}{K}\mathbf{h}_l\mathbf{h}_l^\dagger + \sigma\mathbf{I})^{-1}\mathbf{h}_k\|} \quad \forall k. \tag{7}$$

The convergence property of **Algorithm A** is presented in the following theorem.

Theorem 1. Starting from any initial point $\mathbf{p}[0]$, and $\mathbb{U}[0]$, the $\mathbf{p}[\imath]$, and $\mathbb{U}[\imath]$ in **Algorithm A** converge geometrically fast to the optimal solution $\mathbf{p}^*$, and $\mathbb{U}^*$.

Proof. (Sketch) The key step to the proof is to establish the convergence property of the power $\mathbf{p}$ via nonlinear Perron-Frobenius theory [21]. Define the mapping $f^{(1)} : \mathbb{R}_+^{K\times 1} \to \mathbb{R}_+^{K\times 1}$ as

$$f_k^{(1)}(\mathbf{p}) = \frac{\beta_k}{\mathbf{h}_k^\dagger(\sum_{l\neq k}\frac{p_l}{K}\mathbf{h}_l\mathbf{h}_l^\dagger + \sigma\mathbf{I})^{-1}\mathbf{h}_k}. \tag{8}$$

It can be shown that $f_k^{(1)}(\mathbf{p})$ is a concave self-mapping of $\mathbf{p}$. Note that the individual power constraint in (3) induces a norm on $\mathbb{R}_+^{K\times 1}$: $\|\mathbf{p}\|_\infty \triangleq \max_l(p_l/\bar{p})$, which is different from the induced norm in [16] with a weighted sum power constraint. The rest of the proof can be carried out using a technique similar to that in [16, Theorem 1]. □

Even though the algorithm can compute the optimal beamformer and power for (3), and is computationally fast, it is iterative in nature. The instantaneous power has to be iteratively updated to compute the optimal beamformer. This distributed algorithm is suitable for a finite system when K and N are not sufficiently large. In order to leverage the large system structure to design a low complexity algorithm that computes the power vector only depending on channel statistics, we perform a large system analysis next.

4. LARGE SYSTEM ANALYSIS AND ALGORITHM DESIGN

This section is devoted to the large system analysis when K and N grows sufficiently large while the ratio $\lim \frac{K}{N}$ remains bounded. Intuitively, we aim to obtain the asymptotically optimal power that only depends on channel statistics and so can be updated in a slower time scale. Once the asymptotic power is obtained, the beamformer can be non-iteratively computed using (4). Also, the asymptotic power for each user can be used for transmission.

4.1. Channel Model

Now we present the channel model employed in this section. Firstly, different users may possess different large scale channel effects including path loss and shadowing. Secondly, since the base stations are located in a distributed manner, the large scale channel effects between different base stations and a given user can be different. The following channel model is assumed for further analysis:

$$\mathbf{h}_k = \mathbf{D}_k^{\frac{1}{2}} \tilde{\mathbf{h}}_k, \tag{9}$$

where $\mathbf{D}_k \triangleq \mathrm{diag}(d_{1,k}, \cdots, d_{N,k})$ and $d_{n,k}$ represents the large scale channel effect between base station n and user k. The $\tilde{\mathbf{h}}_k$ denotes the normalized CSI whose elements are independent and identically distributed as $\mathcal{CN}(0, 1)$. This assumption corresponds to the scenario when the distributed antennas are located far apart and there is no spatial correlation across the distributed antennas. Herein, the channel model (9) is employed for further analysis and algorithm design. Dealing with some practical limitations such as channel estimation error, line of sight, and more general channel models can be found in recent advances related to large random matrix theory, e.g., see [19, 20] and the reference therein.

4.2. Large System Result

From (5), we know that in order to compute the asymptotic power, the asymptotic analysis for Γ_k is needed. Substituting the MVDR beamformer in (4), we have

$$\Gamma_k(\mathbf{p}) = \frac{p_k}{K} \mathbf{h}_k^\dagger \left(\sum_{l \neq k} \frac{p_l}{K} \mathbf{h}_l \mathbf{h}_l^\dagger + \sigma \mathbf{I} \right)^{-1} \mathbf{h}_k. \tag{10}$$

The instantaneous $\Gamma_k(\mathbf{p})$ is a random variable in quadratic form. Our aim is to examine the deterministic quantity that tightly approximates $\Gamma_k(\mathbf{p})$ in the asymptotic sense. Such asymptotic approximation, or deterministic equivalent, can be obtained by extensively referring to the Stieltjes transform method (or Bai-Silverstein method) [22–24]. The asymptotic approximation for $\Gamma_k(\mathbf{p})$ under the assumed channel model is given in the following lemma.

Lemma 1. The instantaneous random variable $\Gamma_k(\mathbf{p})$ can be approximated by a deterministic quantity[1] $\gamma_k(\mathbf{p})$ such that $\Gamma_k(\mathbf{p}) - \gamma_k(\mathbf{p}) \xrightarrow{a.s.} 0$ as the system dimension $N \to \infty$. Also, $\gamma_k(\mathbf{p}) = p_k \phi_k(\mathbf{p})$, where $\phi_k(\mathbf{p})$ is described by the following implicit K system equations:

$$\phi_k(\mathbf{p}) = \frac{1}{K} \sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K} \sum_{l \neq k} \frac{p_l}{1 + p_l \phi_l(\mathbf{p})}} \quad \forall k. \tag{11}$$

Proof. The main proof relies on [24, Theorem 1] and [24, Theorem 2] and thus is omitted here. □

Before performing algorithm design, we examine two special cases for the distributed multiple-antenna network. In the first special case, we have $d_{n,k} = d_k$, which corresponds to the co-located antenna scenario, i.e., one base station with N antennas. In the second special case, we have $d_{n,k} = d_n$, which corresponds to the co-located user scenario, i.e., users having the same set of large scale channel effects to the distributed base stations. For these two special cases, the asymptotic expression and calculation for $\Gamma_k(\mathbf{p})$ can be further simplified since they correspond to the separable variance profile defined in [23]. Herein, we present the asymptotic approximations for these two cases in the following corollaries.

Corollary 1. For the first case with co-located antennas, $\Gamma_k^{\mathsf{CA}}(\mathbf{p})$ can be approximated by a deterministic quantity $\gamma_k^{\mathsf{CA}}(\mathbf{p})$ such that $\Gamma_k^{\mathsf{CA}}(\mathbf{p}) - \gamma_k^{\mathsf{CA}}(\mathbf{p}) \xrightarrow{a.s.} 0$ as the system dimension $N \to \infty$. Also, $\gamma_k^{\mathsf{CA}}(\mathbf{p}) = p_k \phi_k^{\mathsf{CA}}(\mathbf{p})$, where $\phi_k^{\mathsf{CA}}(\mathbf{p})$ is described by the following fixed-point equation:

$$\phi_k^{\mathsf{CA}}(\mathbf{p}) = \frac{N}{K} \frac{d_k}{\sigma + \frac{d_l}{K} \sum_{l \neq k} \frac{p_l}{1 + p_l \phi_k^{\mathsf{CA}}(\mathbf{p})}} \quad \forall k. \tag{12}$$

Corollary 2. For the second case with co-located users, $\Gamma_k^{\mathsf{CU}}(\mathbf{p})$ can be approximated by a deterministic quantity $\gamma_k^{\mathsf{CU}}(\mathbf{p})$ such that $\Gamma_k^{\mathsf{CU}}(\mathbf{p}) - \gamma_k^{\mathsf{CU}}(\mathbf{p}) \xrightarrow{a.s.} 0$ as the system dimension $N \to \infty$. Also, $\gamma_k^{\mathsf{CU}}(\mathbf{p}) = p_k \phi_k^{\mathsf{CU}}(\mathbf{p})$, where $\phi_k^{\mathsf{CU}}(\mathbf{p})$ is described by the following fixed-point equation:

$$\phi_k^{\mathsf{CU}}(\mathbf{p}) = \frac{1}{K} \sum_{n=1}^{N} \frac{d_n}{\sigma + \frac{d_n}{K} \sum_{l \neq k} \frac{p_l}{1 + p_l \phi_k^{\mathsf{CU}}(\mathbf{p})}} \quad \forall k. \tag{13}$$

Remark: Comparing (12) and (13) with (11), it can be seen that the inter-dependence of the vector $\boldsymbol{\phi} \triangleq (\phi_1, \cdots, \phi_K)^\mathsf{T}$ is much simplified in the two special cases. For the general case (11), $\boldsymbol{\phi}$ is obtained by solving the K dependent system equations. However, for the two special cases, each ϕ_k can be solved separately by the fixed-point equation.

4.3. Algorithm Design and Convergence Analysis

In this part, the obtained large system result is employed for algorithm design to compute the asymptotic power. From (11), it can be observed that $\mathbf{p}$ and $\boldsymbol{\phi}$ are coupled and their relationship only depends on channel statistics reflected in the $\mathbf{D}_k$. Herein, in order to design efficient algorithm to compute $\mathbf{p}$ and $\boldsymbol{\phi}$, we need to examine the conditional convergence property of these two vectors separately.

The convergence property of $\boldsymbol{\phi}$ given $\mathbf{p}$ is provided in the following theorem.

Theorem 2. For a given $\mathbf{p}$, starting from any initial $\boldsymbol{\phi}[0]$, the following iterative computation of $\boldsymbol{\phi}[\jmath]$:

$$\phi_k[\jmath + 1] = \frac{1}{K} \sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K} \sum_{l \neq k} \frac{p_l}{1 + p_l \phi_l[\jmath]}} \quad \forall k \tag{14}$$

converges to the unique solution of the implicit system equations expressed in (11).

Proof. (Sketch) Define the mapping $f^{(2)} : \mathbb{R}_+^{K \times 1} \to \mathbb{R}_+^{K \times 1}$ as

$$f_k^{(2)}(\boldsymbol{\phi}) = \frac{1}{K} \sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K} \sum_{l \neq k} \frac{p_l}{1 + p_l \phi_l}}. \tag{15}$$

The key step establishing the proof is to show that $f_k^{(2)}(\boldsymbol{\phi})$ is a standard interference function [25] that satisfies monotonicity and scalability. The detailed proof is omitted. After proving that the mapping is a standard interference function, the convergence property follows from [25]. □

[1] Note that we present the asymptotic behavior of $\Gamma_k(\mathbf{p})$ with a given power vector $\mathbf{p}$, not with the instantaneous optimal power vector $\mathbf{p}^*$. The instantaneous optimal power vector is a function of channel and thus complicates standard large system analysis. Herein, iterative method is used to compute the asymptotic power.

The convergence property of $\mathbf{p}$ given $\boldsymbol{\phi}$ is provided in the following theorem.

Theorem 3. For a given $\boldsymbol{\phi}$, starting from any initial $\mathbf{p}[0]$, the following iterative computation of $\mathbf{p}[\jmath]$:

$$p_k[\jmath+1] = \min\left\{\beta_k K \left(\sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K}\sum\limits_{l\neq k}\frac{p_l[\jmath]}{1+p_l[\jmath]\phi_l}}\right)^{-1}, \bar{p}\right\} \forall k \tag{16}$$

converges to the optimal solution of the following power control problem:

$$\begin{array}{ll} \text{maximize} & \min\limits_{k} \frac{\gamma_k(\mathbf{p})}{\beta_k} \\ \text{subject to} & 0 \leq p_k \leq \bar{p},\ \forall k \\ \text{variables}: & \mathbf{p}. \end{array} \tag{17}$$

Proof. (Sketch) Define the mapping $f^{(3)}: \mathbb{R}_{+}^{K\times 1} \rightarrow \mathbb{R}_{+}^{K\times 1}$ as

$$f_k^{(3)}(\mathbf{p}) = \beta_k K \left(\sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K}\sum_{l\neq k}\frac{p_l}{1+p_l\phi_l}}\right)^{-1}. \tag{18}$$

It can be proved that $f_k^{(3)}(\mathbf{p})$ is a standard interference function. From [25, Theorem 7], the mapping $\min\{f_k^{(3)}(\mathbf{p}), \bar{p}\}$ is standard. Then, since at optimality of (17), the weighted γ_k are the same and at least one user achieves $\bar{p}$, the aforementioned mapping can be linked to the optimal solution of (17). The detailed proof is omitted. Note that another proof is to use the nonlinear Perron-Frobenius theory to prove that $f_k^{(3)}(\mathbf{p})$ is a concave self-mapping under the induced norm defined in Theorem 1. □

Now, by combining the results from Theorem 2 and Theorem 3 that treat $\boldsymbol{\phi}$ and $\mathbf{p}$ separately, we present a single timescale algorithm (**Algorithm B**) to compute the asymptotic power, as follows. Note that the joint convergence property of the algorithm remains to be proved, and is observed empirically in Section 5.

Algorithm B:

- Initialize arbitrary $\mathbf{p}[0] \in \mathbb{R}_{++}^{K\times 1}$ for $k = 1, \ldots, K$ such that $p_k[0] \leq \bar{p}, \forall k$.

1. Update power $\mathbf{p}[\jmath+1]$: $\forall k$

$$p_k[\jmath+1] = \min\left\{\beta_k K \left(\sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K}\sum\limits_{l\neq k}\frac{p_l[\jmath]}{1+p_l[\jmath]\phi_l[\jmath]}}\right)^{-1}, \bar{p}\right\}. \tag{19}$$

2. Update $\boldsymbol{\phi}[\jmath+1]$:

$$\phi_k[\jmath+1] = \frac{1}{K}\sum_{n=1}^{N} \frac{d_{n,k}}{\sigma + \frac{d_{n,l}}{K}\sum_{l\neq k}\frac{p_l[\jmath+1]}{1+p_l[\jmath+1]\phi_l[\jmath]}} \quad \forall k. \tag{20}$$

Remark: The time scales for **Algorithm A** and **Algorithm B** are vastly different. **Algorithm A** iteratively computes the instantaneous optimal power, and **Algorithm B** computes the asymptotic power depending only on channel statistics. This algorithm design not only leverages the large system structure to reduce the system design complexity, but also has the potential to achieve the optimality in the asymptotic sense.

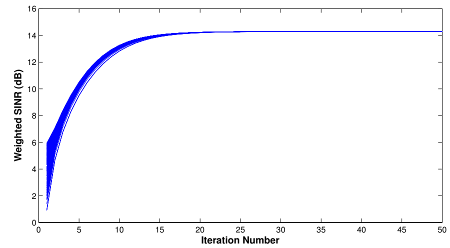

Fig. 2. Convergence plot of the weighted SINR employing Algorithm B.

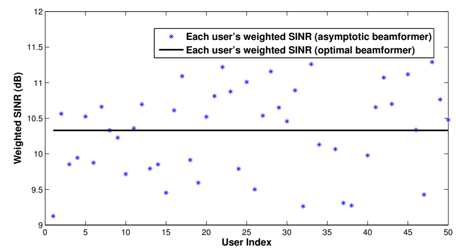

Fig. 3. Comparison of the achieved weighted SINR for each individual user using asymptotic beamformer and the optimal beamformer for one channel realization.

5. NUMERICAL RESULTS

In this section, a numerical study is conducted. We consider $N = 50$ distributed antennas randomly placed in a 500m×500m area with $K = 50$ randomly dropped users. The path loss (in dB) model is assumed to be $15.3 + 37.6\log_{10} d$ for distance d in meters and a log-normal shadowing with standard deviation of 8 dB is employed. The noise power spectral density is set to be -162 dBm/Hz. Each user has the same priority of service ($\boldsymbol{\beta} = 1$), and the maximum power $\bar{p} = 100$ mW.

For a given geometry, we demonstrate the convergence behavior of the weighted SINR using **Algorithm B** in Fig. 2. The SINR of all users are not differentiated, and use the same type of line for illustration. From Fig. 2, the joint convergence of the proposed algorithm is observed empirically (the conditional convergence is proved in Section 4.3). The convergence plot as well as the converged value depend on both the placement of the antennas and the user geometry. Our empirical results observe that the convergence happens within 100 runs of iteration. Fig. 3 shows the comparison of the weighted SINR for each individual user using optimal beamformer and the asymptotic beamformer, for one channel realization. For the optimal beamformer case, the instantaneous optimal power and beamformer are computed using **Algorithm A**. For the asymptotic beamformer case, the asymptotic power is computed in advance using channel statistics to non-iteratively determine the instantaneous beamformer. We can see from Fig. 3 that the achieved SINRs for the latter case fluctuate around the optimal one, which means the SINR fairness can be achieved in an asymptotic sense using low complexity algorithm. These two figures demonstrate the empirical results; proving the joint convergence property as well as the asymptotic optimality using **Algorithm B** are interesting directions for future work.

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