# ADAPTIVE QUANTIZATION ON THE GRASSMANN-MANIFOLD FOR LIMITED FEEDBACK MULTI-USER MIMO SYSTEMS

*Stefan Schwarz*[†], *Robert W. Heath Jr.*[‡] *and Markus Rupp*[†]

[†] Institute of Telecommunications, Vienna University of Technology, Austria

[‡] Department of Electrical and Computer Engineering, The University of Texas at Austin, USA

Email: {sschwarz,mrupp}@nt.tuwien.ac.at, rheath@ece.utexas.edu

**ABSTRACT**

We propose an adaptive quantization algorithm for subspace tracking on the Grassmann-manifold of p-dimensional subspaces in the n-dimensional Euclidean space. This quantization problem arises naturally in limited feedback based wireless communication systems, which apply precoding for interference cancellation and alignment. The proposed algorithm exploits the differential geometry associated with the Grassmann-manifold for efficient differential and predictive quantization. The algorithm is applied to channel state information quantization in a multi-user block-diagonalization based wireless communication system, demonstrating large throughput gains compared to memoryless quantization.

***Index Terms***— Grassmann-manifold, differential quantization, limited feedback, block diagonalization, LTE.

## 1. INTRODUCTION

Channel state information (CSI) is useful for achieving the highest performance in multiple antenna wireless communications. Single user (SU-) MIMO systems employ CSI to match the spatial signature of the transmit signal to the channel and separate the multiplexed streams at the receiver [1]. In multi-user (MU-) MIMO broadcast systems, CSI is employed to send independent data streams to multiple users simultaneously [2]. In interference channels, CSI allows multiple interfering user pairs to communicate with less interference [3, 4]. In general, obtaining the gains from multiple antennas requires accurate channel knowledge at both, the transmitter and the receiver.

Obtaining CSI at the transmitter (CSIT) is made practical through the existence of dedicated finite-rate feedback links from the users. Prior work has thus focused on the design of limited feedback algorithms for efficient CSI quantization [5]. Accurate CSIT is central in interference limited multi-user systems, because CSI imperfections increase the multi-user interference and strongly deteriorate the throughput. It has been shown that a linear increase in the number of feedback bits with the signal to noise ratio (SNR) (in [dB]) is required to maintain a constant rate-gap to perfect CSIT [6–9]. Several multi-user precoding techniques require knowledge about the linear vector spaces spanned by the channel matrices to calculate the precoders [7, 8]. These spaces can be represented as points on a Grassmann-manifold. Grassmannian quantization has thus gained significant interest in CSI feedback, e.g., [10]. To reduce the CSI feedback overhead, the temporal correlation of the channel between consecutive quantization instants can be exploited. Several authors have considered differential and predictive quantization of one dimensional subspaces, e.g., [11–14], but the literature on quantizers for higher dimensional subspaces in the context of MU-MIMO is limited, e.g., [15]. In [16, 17], we proposed a predictive quantizer for the Grassmann manifold, but that work was limited to one dimensional subspaces.

In this paper, we propose a predictive quantization algorithm for tracking of channel subspaces of arbitrary dimension. The algorithm exploits the differential geometry associated with the Grassmann-manifold for efficient prediction and quantization. We investigate the accuracy of the proposed algorithm in terms of the chordal distance quantization error. The quantizer is applied for CSI feedback in a block-diagonalization (BD)-based MU-MIMO system, demonstrating close to optimal performance in a low-mobility scenario over a wide SNR range.

## 2. BLOCK-DIAGONALIZATION

Consider a frequency flat MU-MIMO broadcast system with input-output relationship of user $u$ at time instant $k$ given by

$$\mathbf{y}_u[k] = \mathbf{H}_u[k]^{\mathrm{H}}\mathbf{x}_u[k] + \mathbf{H}_u[k]^{\mathrm{H}} \sum_{\ell=1,\ell\neq u}^{U} \mathbf{x}_\ell[k] + \mathbf{n}_u[k], \quad (1)$$

where $\mathbf{y}_u[k] \in \mathbb{C}^{N_r}$ denotes the received symbol vector of user $u$, $\mathbf{H}_u[k] \in \mathbb{C}^{N_t \times N_r}$ is its channel matrix, and $(\cdot)^{\mathrm{H}}$ denotes conjugate-transposition. To simplify notation, we suppose that all $U$ users have the same number $N_r$ of receive antennas and that $N_r \leq N_t$. The transmit symbol vector $\mathbf{x}_u[k] \in \mathbb{C}^{N_t}$ intended for user $u$ is obtained by linearly precoding the information symbol vector $\mathbf{s}_u[k] \in \mathbb{C}^{d_u}$ with a precoding matrix $\mathbf{F}_u[k] \in \mathbb{C}^{N_t \times d_u}$

$$\mathbf{x}_u[k] = \mathbf{F}_u[k]\mathbf{s}_u[k], \quad (2)$$

with $d_u$ being the number of parallel data streams of user $u$.

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BD precoding targets zero multi-user interference. This is achieved by choosing the precoder $\mathbf{F}_u[k]$ in the null-space of the other users' channels [18]

$$\mathbf{F}_u[k] \in \operatorname{null}\left(\overline{\mathbf{H}}_u[k]\right), \quad (3)$$

$$\overline{\mathbf{H}}_u[k]=[\mathbf{H}_1[k],\ldots,\mathbf{H}_{u-1}[k],\mathbf{H}_{u+1}[k],\ldots,\mathbf{H}_U[k]]^{\mathrm{H}}. \quad (4)$$

Several conditions on the number of streams $d_u$ per user and the total number of streams must be satisfied to guarantee the existence of a valid solution; see [18]. Consider the compact singular value decomposition (SVD) of the channel

$$\mathbf{H}_\ell[k] = \mathbf{U}_\ell[k]\mathbf{\Sigma}_\ell[k]\mathbf{V}_\ell[k]^{\mathrm{H}} = \tilde{\mathbf{H}}_\ell[k]\mathbf{D}_\ell[k], \quad (5)$$

$$\tilde{\mathbf{H}}_\ell[k]=\mathbf{U}_\ell[k]\in\mathbb{C}^{N_t\times N_r},\ \mathbf{D}_\ell[k]=\mathbf{\Sigma}_\ell[k]\mathbf{V}_\ell[k]^{\mathrm{H}}\in\mathbb{C}^{N_r\times N_r}.$$

To calculate the null-space of $\overline{\mathbf{H}}_u[k]$ it is sufficient to know the semi-unitary matrices $\tilde{\mathbf{H}}_\ell[k], \forall \ell \neq u$, because

$$\operatorname{null}\left(\mathbf{H}_\ell[k]^{\mathrm{H}}\right)=\operatorname{null}\left(\tilde{\mathbf{H}}_\ell[k]^{\mathrm{H}}\right), \quad (6)$$

$$\operatorname{null}\left(\overline{\mathbf{H}}_u[k]\right) = \bigcap_{\ell\neq u} \operatorname{null}\left(\mathbf{H}_\ell[k]^{\mathrm{H}}\right). \quad (7)$$

Also, $\operatorname{null}\left(\tilde{\mathbf{H}}_\ell[k]^{\mathrm{H}}\right) = \operatorname{null}\left((\tilde{\mathbf{H}}_\ell[k]\mathbf{Q})^{\mathrm{H}}\right)$ for any unitary $\mathbf{Q}$. Hence, knowledge of the space spanned by the columns of $\mathbf{H}_\ell[k], \forall \ell$ is sufficient for the calculation of the BD precoders.

## 3. ADAPTIVE QUANTIZATION ALGORITHM

In this paper, we consider adaptive quantization to take advantage of the temporal correlation in the channel to provide higher resolution compared to memoryless techniques [7]. We propose that each user $u$ quantizes and feeds back the unitary matrix $\tilde{\mathbf{H}}_u[k]$ defined in (5). The space spanned by $\tilde{\mathbf{H}}_u[k]$ can be interpreted as point on the Grassmann-manifold $\mathcal{G}_{N_t,N_r}(\mathbb{C})$ of all $N_r$-dimensional subspaces in the $N_t$-dimensional complex Euclidean space [19, 20]. In our approach, a Grassmannian quantization codebook is adapted to the temporal evolution of the channel subspace.

To derive an efficient quantization codebook, we consider a general model for the temporal evolution of the channel matrix

$$\mathbf{H}_u[k] = D\left(\mathbf{H}_u[k-1], \mathbf{H}_u[k-2], \ldots\right) + \mathbf{J}_u[k], \quad (8)$$

where $D\left(\mathbf{H}_u[k-1], \mathbf{H}_u[k-2], \ldots\right)$ describes the deterministic dependence of the current channel on the past, and the matrix $\mathbf{J}_u[k] : \operatorname{vec}(\mathbf{J}_u[k]) \sim \mathcal{N}\left(\mathbf{0}, \sigma_j^2[k]\mathbf{I}\right)$ denotes zero-mean complex Gaussian innovation noise. We do not make any specific assumptions about the nature of the deterministic function $D(\cdot)$, because the behavior of wireless channels can vary strongly with the surrounding environment [21]. We propose a predictive quantizer that predicts the deterministic evolution of the channel and quantizes the prediction error. Assuming that a prediction $\mathbf{H}_u^{(p)}[k]$ is available, we write the channel as

$$\mathbf{H}_u[k] = \mathbf{H}_u^{(p)}[k] + \mathbf{E}_u^{(p)}[k] + \mathbf{J}_u[k] = \mathbf{H}_u^{(p)}[k] + \mathbf{E}_u[k], \quad (9)$$

with $\mathbf{E}_u^{(p)}[k]$ denoting the prediction error. We assume that $\mathbf{E}_u^{(p)}[k]$ is independent of the innovation $\mathbf{J}_u[k]$ and contains i.i.d. zero-mean Gaussian elements of variance $\sigma_p^2[k]$. Hence, the total error matrix $\mathbf{E}_u[k]$ is distributed as: $\operatorname{vec}(\mathbf{E}_u[k]) \sim \mathcal{N}\left(\mathbf{0}, \sigma_e^2[k]\mathbf{I}\right)$, with $\sigma_e^2[k] = \sigma_p^2[k] + \sigma_j^2[k]$. Using the decomposition of (5) and applying a similar decomposition to $\mathbf{H}_u^{(p)}[k]$, we rewrite the channel as

$$\tilde{\mathbf{H}}_u[k]\mathbf{D}_u[k] = \tilde{\mathbf{H}}_u^{(p)}[k]\mathbf{D}_u^{(p)}[k] + \mathbf{E}_u[k], \quad (10)$$

$$\to \tilde{\mathbf{H}}_u[k] = \left(\tilde{\mathbf{H}}_u^{(p)}[k]\mathbf{D}_u^{(p)}[k] + \mathbf{E}_u[k]\right)\mathbf{D}_u[k]^{-1}. \quad (11)$$

The differential geometry associated with the Grassmann-manifold allows to describe the error between $\tilde{\mathbf{H}}_u^{(p)}[k]$ and the observed subspace $\tilde{\mathbf{H}}_u[k]$ by means of a tangent matrix [22]

$$\mathbf{H}_u^{(t)}[k] = \left(\mathbf{I} - \tilde{\mathbf{H}}_u^{(p)}[k]\tilde{\mathbf{H}}_u^{(p)}[k]^{\mathrm{H}}\right)\tilde{\mathbf{H}}_u[k]\left(\tilde{\mathbf{H}}_u^{(p)}[k]^{\mathrm{H}}\tilde{\mathbf{H}}_u[k]\right)^{-1}$$

$$= \mathbf{P}_u[k]\tilde{\mathbf{H}}_u[k]\left(\tilde{\mathbf{H}}_u^{(p)}[k]^{\mathrm{H}}\tilde{\mathbf{H}}_u[k]\right)^{-1} = T\left(\tilde{\mathbf{H}}_u^{(p)}[k], \tilde{\mathbf{H}}_u[k]\right). \quad (12)$$

With knowledge of $\tilde{\mathbf{H}}_u^{(p)}[k]$ and $\mathbf{H}_u^{(t)}[k]$, the space spanned by $\tilde{\mathbf{H}}_u[k]$ is obtained from the geodesic $G(\tilde{\mathbf{H}}_u^{(p)}[k], \mathbf{H}_u^{(t)}[k])$; see [22, Eq. (4)]. Thus, an efficient Grassmannian codebook can be obtained from knowledge of the statistics of the tangent matrix. With (10) and (11) the tangent (12) can be reformulated as

$$\mathbf{H}_u^{(t)}[k] = \mathbf{P}_u[k]\mathbf{E}_u[k]\left(\tilde{\mathbf{H}}_u^{(p)}[k]^{\mathrm{H}}\mathbf{H}_u[k]\right)^{-1}. \quad (13)$$

Applying an SVD to $\tilde{\mathbf{H}}_u^{(p)}[k]^{\mathrm{H}}\mathbf{H}_u[k] = \mathbf{Q}_u[k]\mathbf{\Lambda}_u[k]\mathbf{W}_u[k]^{\mathrm{H}}$, (13) can be written as

$$\mathbf{H}_u^{(t)}[k] = \mathbf{P}_u[k]\mathbf{E}_u[k]\left(\mathbf{W}_u[k]\mathbf{\Lambda}_u[k]^{-1}\mathbf{Q}_u[k]^{\mathrm{H}}\right). \quad (14)$$

The tangent matrix is thus obtained as the product of the null-space component of the error matrix with respect to the prediction, $\mathbf{P}_u[k]\mathbf{E}_u[k]$, and a term that depends on the range-space component of the error. Assuming the range-space component as fixed/observed, the distribution of the tangent is given by

$$\operatorname{vec}(\mathbf{H}_u^{(t)}[k]) \sim \mathcal{N}\left(\mathbf{0}, \mathbf{C}_u^{(t)}[k]\right), \quad (15)$$

$$\mathbf{C}_u^{(t)}[k] = \sigma_e^2[k]\left(\mathbf{Q}_u[k]\mathbf{\Lambda}_u[k]^{-2}\mathbf{Q}_u[k]^{\mathrm{H}} \otimes \mathbf{P}_u[k]\right). \quad (16)$$

The range-space component of the error impacts the matrix $\mathbf{\Lambda}_u[k]$, and leads to a correlation of the elements of the tangent matrix. If $\mathbf{\Lambda}_u[k]$ were known a-priori, it could be used to generate a matching correlated codebook. Unfortunately, it depends on the current channel and is unknown to the transmitter. Thus, we cannot exploit this correlation during quantization, but approximate $\mathbf{\Lambda}_u[k]$ with a scaled identity matrix (simulations verified this approximation for small prediction errors)

$$\mathbf{C}_u^{(t)}[k] \approx \left(\frac{\sigma_e[k]}{\lambda_u[k]}\right)^2 \left(\mathbf{I} \otimes \mathbf{P}_u[k]\right). \quad (17)$$

With this approximation, the tangent matrix is obtained as the projection of an i.i.d. Gaussian matrix onto the null-space of the predicted subspace. We observe that a prediction of the subspace is sufficient for the calculation of the tangent matrix.

Based on these observations, we customize our codebook construction from [16, 17] for $\mathcal{G}_{N_t,N_r}$. At each quantization instant $k$ the following calculations are performed:

1. Predict the current channel subspace $\tilde{\mathbf{H}}_u^{(p)}[k]$ and calculate the projection matrix $\mathbf{P}_u[k]$.
2. Generate a tangent codebook

$$\mathcal{Q}_u^{(t)}[k] = \left\{ \mathbf{P}_u[k]\mathbf{O}_i | \mathbf{O}_i \in \mathbb{C}^{N_t \times N_r} \right\}, \tag{18}$$

$$\mathrm{vec}(\mathbf{O}_i) \sim \mathcal{N}\big(\mathbf{0}, g^{s[k]}\, \mathbf{I}\big). \tag{19}$$

3. Calculate the Grassmannian codebook $\mathcal{Q}_u[k]$, by projecting the tangent codebook onto the manifold using the geodesic [22, Eq. (4)], with $\tilde{\mathbf{H}}_u^{(p)}[k]$ as starting point.

The scale parameter $s[k]$ in (19) tracks the time dependent variance of the tangent matrix, $(\sigma_e[k]/\lambda_u[k])^2$, employing the same 1 bit tracking algorithm as in [16, 17].

The quantized channel subspace is obtained by minimizing the chordal distance quantization error

$$\tilde{\mathbf{H}}_u^{(q)}[k] = \underset{\mathbf{Q}_i \in \mathcal{Q}_u[k]}{\arg\min}\ N_r - \mathrm{tr}\left(\tilde{\mathbf{H}}_u[k]^{\mathrm{H}} \mathbf{Q}_i \mathbf{Q}_i^{\mathrm{H}} \tilde{\mathbf{H}}_u[k]\right). \tag{20}$$

## 4. PREDICTION ALGORITHM

In our previous work [16, 17], we use adaptive finite impulse response (FIR) filters for prediction on $\mathcal{G}_{N_t,1}$. The large number of filter coefficients involved with prediction of unitary matrices on $\mathcal{G}_{N_t,N_r}$, and the corresponding slow convergence speed, forced us to take an alternative approach. We hence extend the work [23] from $\mathcal{G}_{N_t,1}$ to $\mathcal{G}_{N_t,N_r}$.

The basic idea of [23] is to apply a linear least squares fit to $N_p$ past observed tangent matrices and, based on this fit, predict the current channel subspace that is to be quantized. The first step of the algorithm involves finding a center in $\mathcal{G}_{N_t,N_r}$ of the $N_p$ past quantized subspaces

$$\mathcal{H}_u = \left\{ \tilde{\mathbf{H}}_u^{(q)}[k-1], \ldots, \tilde{\mathbf{H}}_u^{(q)}[k-N_p] \right\}. \tag{21}$$

This is achieved with the iterative algorithm detailed in [23, Fig. 2], which can be generalized to $\mathcal{G}_{N_t,N_r}$ by replacing the tangent and geodesic mappings with Eq. (12) and [22, Eq. (4)], respectively. We denote the obtained center as $\tilde{\mathbf{H}}_u^{(c)}$. Next, the tangent matrices from $\tilde{\mathbf{H}}_u^{(c)}$ to the elements of $\mathcal{H}_u$ are calculated (for brevity we assume $N_p$ to be odd)

$$\mathbf{T}_u[i] = T\big(\tilde{\mathbf{H}}_u^{(c)}, \tilde{\mathbf{H}}_u^{(q)}[k - \lceil N_p/2 \rceil + i]\big), \tag{22}$$

$$i \in \mathcal{I} = \{ -\lfloor N_p/2 \rfloor, \ldots, \lfloor N_p/2 \rfloor \}.$$

The time dependence of the tangents is then estimated by fitting a polynomial model. In [23], a linear model for tangents to $\mathcal{G}_{N_t,1}$ was considered, which we extend to a second-order model for tangents to $\mathcal{G}_{N_t,N_r}$. An extension to higher-order models did not provide an additional gain in the simulations. The estimated tangents are then obtained from the equation

$$\hat{\mathbf{T}}_u[i] = \mathbf{T}^{(1)}\, i + \mathbf{T}^{(2)}\, i^2. \tag{23}$$

The optimal coefficient matrices $\mathbf{T}_u^{(1)}$ and $\mathbf{T}_u^{(2)}$ for (23) are obtained by minimizing the mean squared error (MSE)[1]

$$\left\{ \mathbf{T}_u^{(1)}, \mathbf{T}_u^{(2)} \right\} = \underset{\mathbf{T}^{(1)},\mathbf{T}^{(2)}}{\arg\min} \sum_{i \in \mathcal{I}} \left\| \mathbf{T}_u[i] - \hat{\mathbf{T}}_u[i] \right\|_F^2. \tag{24}$$

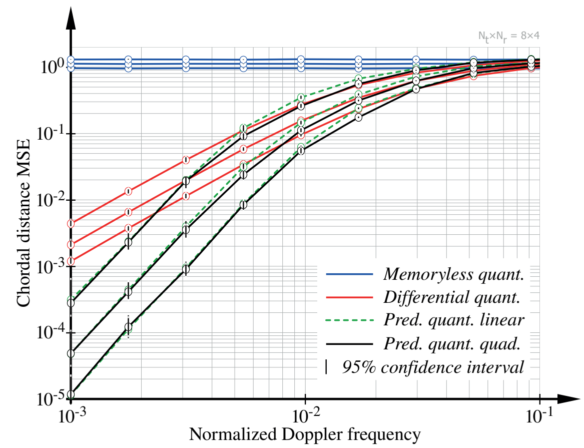


**Fig. 1**: Quantization error versus maximum channel Doppler frequency for an $N_t \times N_r = 8 \times 4$ system with different quantization codebook sizes.

The rational behind this choice of an objective function comes from a local approximation of manifold geodesic distances with tangent Frobenius norms; see [23]. Solving this optimization problem leads to the optimal coefficient matrices

$$\mathbf{T}_u^{(1)} = \frac{\sum_{i \in \mathcal{I}} \mathbf{T}_u[i]\, i}{\sum_{i \in \mathcal{I}} i^2}, \quad \mathbf{T}_u^{(2)} = \frac{\sum_{i \in \mathcal{I}} \mathbf{T}_u[i]\, i^2}{\sum_{i \in \mathcal{I}} i^4}. \tag{25}$$

Note that this solution is only valid for odd $N_p$, for even $N_p$ additional terms appear. Based on this fit, the tangent to the current channel subspace $\tilde{\mathbf{H}}_u[k]$ is predicted as

$$T(\tilde{\mathbf{H}}_u^{(c)}, \tilde{\mathbf{H}}_u[k]) \approx \hat{\mathbf{T}}_u[i+1]. \tag{26}$$

The predicted channel subspace $\tilde{\mathbf{H}}_u^{(p)}[k]$ is finally obtained from the geodesic

$$\tilde{\mathbf{H}}_u^{(p)}[k] = G(\tilde{\mathbf{H}}_u^{(c)}, \hat{\mathbf{T}}_u[i+1]). \tag{27}$$

## 5. SIMULATION RESULTS

### 5.1. Quantization Error

In this section we investigate the quantization accuracy of our algorithm in terms of the chordal distance MSE. In [7–9] it has been shown that this error is directly related to the rate loss of the limited feedback system compared to perfect CSIT.

We consider the quantization MSE as a function of the channel Doppler frequency $f_d$. The temporal correlation of the channel matrix is determined by the Doppler frequency according to Clarke's model [24]. Realistic channel realizations are generated with the sum-of-sinusoids channel model of [25]. The MSE is approximated by Monte-Carlo simulations. The normalized Doppler frequency is defined as $\nu_d = T_s f_d$, with $T_s$ being the sampling rate.

[1] In general it is necessary to constrain $\mathbf{T}_u^{(1)}$ and $\mathbf{T}_u^{(2)}$ to the tangent space of $\tilde{\mathbf{H}}_u^{(c)}$. Here, the MSE metric automatically assures a valid solution.

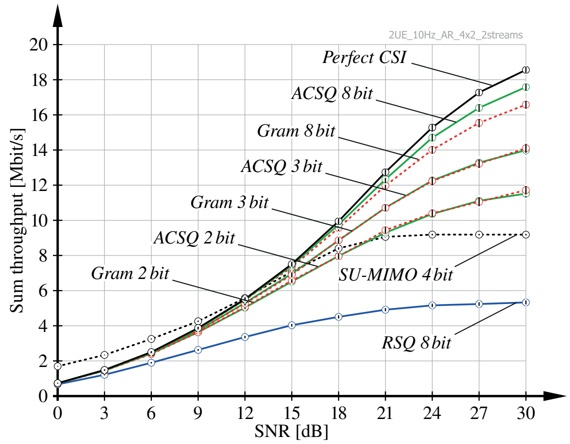


**Fig. 2**: Throughput of an $N_t \times N_r = 4 \times 2$ BD system using the proposed quantizer, the quantizer of [29] and perfect CSIT, compared to SU-MIMO.

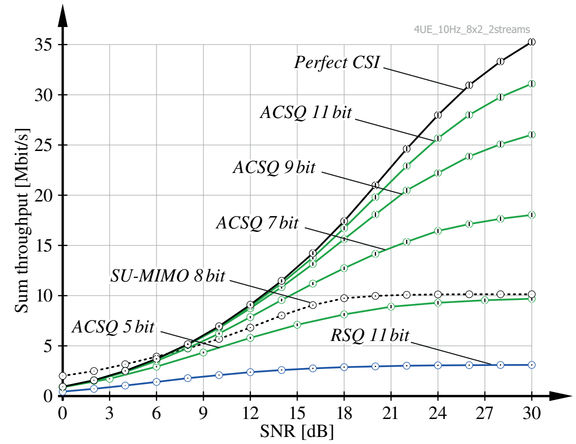


**Fig. 3**: Throughput of an $N_t \times N_r = 8 \times 2$ BD system using the proposed quantizer and perfect CSIT, compared to SU-MIMO.

In Fig. 1 we plot the MSE of an $N_t \times N_r = 8 \times 4$ system for four different quantization approaches. The memoryless quantizer employs random isotropically distributed unitary matrices for quantization [7]. The differential quantizer refers to the proposed algorithm using a trivial predictor $\tilde{\mathbf{H}}_u^{(p)}[k] = \tilde{\mathbf{H}}_u^{(q)}[k-1]$. The predictive quantizer employs the proposed predictor using a linear fit and a quadratic fit for the tangents. The feedback rate is 7, 9 and 11 bits per quantization instant. The figure shows that the predictive quantizers achieve a larger slope of the MSE curve versus Doppler frequency as the differential quantizer. At large Doppler frequencies, i.e., for small channel correlation, the performance of the proposed quantizers reduces to that of memoryless quantization.

### 5.2. Block-Diagonalization Throughput

We apply the proposed quantizer for CSI feedback in a BD-based MU-MIMO system. The results are obtained with a 3GPP LTE-A compliant link-level simulator [26, 27]. We consider an $N_t \times N_r = 4 \times 2$ and $8 \times 2$ system, serving two respectively four users in parallel, with two streams per user. The base station calculates the BD precoders based on quantized channel knowledge. The users apply the interference-averaged MMSE receivers of [28]. A 1.4 MHz frequency-flat system is simulated with a normalized Doppler frequency of $\nu_d = 10^{-2}$.

In the first simulation, we consider an autoregressive channel model of order one, according to [29]. Channel prediction is not effective for such a channel model [13], hence we consider differential quantization. For BD, an alternative limited feedback approach to subspace quantization is to quantize the channel Gram matrix. In [29], a differential quantizer for channel Gram matrices was proposed, which we compare to our quantizer in Fig. 2. The codebook size of [29] is limited to $\leq 2N_t^2$, but it can be furhter increased by considering non-orthogonal geodesics in the codebook construction of [29]. We observe that this quantizer (denoted as *Gram*) achieves the same performance as our proposal (denoted as *adaptive channel subspace quantization (ACSQ)*) as long as the codebook size is $\leq 2N_t^2$. With the considered codebook size-extension, the *Gram 8 bit* algorithm is slightly less efficient than *ACSQ 8 bit*. We also plot the performance of BD with memoryless quantization (*RSQ*) and SU-MIMO in Fig. 2. Memoryless quantization performs significantly worse than differential quantization at the considered Doppler frequency, in conformance to Section 5.1. At low to moderate SNR, SU-MIMO (based on the LTE codebook) outperforms MU-MIMO. This is because we do not consider scheduling for MU-MIMO, but always serve two users over two streams. At high SNR, MU-MIMO achieves a larger multiplexing gain than SU-MIMO, provided the CSIT is sufficiently accurate.

In our second simulation, we consider the channel model of [25] which is based on Clarke's model. In this case, channel prediction is effective and employed with our quantizer. We were not able to achieve a performance gain compared to memoryless quantization with the differential quantizer of [29] for this channel model and $N_t \times N_r = 8 \times 2$ (a small gain was observed for $N_t \times N_r = 4 \times 2$), hence it is not shown in Fig. 3. With our predictive algorithm we achieve similar behavior as in the previous simulation. If the quantization codebook size is less than 6 bits, SU-MIMO outperforms MU-MIMO over the entire SNR range at the considered Doppler frequency, due to the residual multi-user interference. Hence, it is necessary to either increase the codebook size or temporal feedback rate, or to reduce the number of users served in parallel.

## 6. CONCLUSION

We derive an efficient predictive subspace quantization algorithm, by exploiting the differential geometry associated with the Grassmann-manifold. We show that the tangent matrix, describing the error between the predicted subspace and the observation, is approximately i.i.d. Gaussian distributed in the null-space of the predicted subspace. Based on these results, we propose a Grassmannian quantization codebook construction. We investigate the quantization MSE of the proposed algorithm, demonstrating large gains in accuracy compared to memoryless quantization if the channel correlation is sufficiently large. The algorithm is applied to CSI quantization in a BD-based MU-MIMO system, where we obtain large throughput gains compared to memoryless quantization.

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