FULL-DIVERSITY DISTRIBUTED SPACE–TIME CODES WITH AN EFFICIENT ML DECODER FOR ASYNCHRONOUS COOPERATIVE COMMUNICATIONS

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ABSTRACT

We consider an asynchronous cooperative communication system where two distributed transmitters are communicating with one destination. In this paper, a *bounded delay-tolerant time interleave reversal Alamouti code* (BDT-TIR AC) is proposed. BDT-TIR AC achieves full diversity, given that the path delay difference is within a tolerance limit. Furthermore, we design an efficient maximum-likelihood (ML) decoder. By employing a divide-and-conquer strategy, the original decoding problem is decoupled into several sub-problems in small size. In fact, these sub-problems contain either Alamouti code structure or only four symbols. By parallel decoding on the sub-problems, the proposed ML decoder exhibits low complexity advantage in large scale problems. Simulations of BDT-TIR AC confirm the full diversity gain. The bit error rate performance of BDT-TIR AC in the considered asynchronous scenarios is comparable with that of synchronized Alamouti code, and outperforms the latest delay-tolerant space-time code.

Index Terms— Asynchronous cooperative communication, space-time code, full diversity

1. INTRODUCTION

Space-time code (STC) [1–5] was originally proposed to achieve transmit diversity gain in synchronized multiple-input multiple-output (MIMO) systems. The individual nodes of a MIMO system may not be able to support multiple antennas, due to size, cost or hardware constraints. As an alternative approach, cooperative communication systems [6–8] have recently attracted considerable attention. By taking advantage of the cooperation between nodes, the alternative multi-antenna system is built in a distributed manner, where the nodes may have arbitrary geographical locations. Unlike synchronized transmission in conventional MIMO system, the transmission in cooperative communication systems is not perfectly synchronized because of the distributed nature. The lack of perfect synchronization may cause rank deficiency in STC matrices, resulting in diversity loss [9, 10]. Therefore, researchers have recently focused on asynchronous STC, which can achieve full diversity without synchronization requirements [11–27].

These asynchronous STC designs can be generally grouped into two categories: frequency domain approaches and time domain approaches. For frequency domain approaches [16, 19], orthogonal frequency-division multiplexing (OFDM) is a prerequisite to mitigate asynchronism. Thus, such approaches are not applicable for non-OFDM systems. Time domain approaches can be further classified into two branches, namely delay-tolerant STC and bounded delay-tolerant STC (BDT-STC). Delay-tolerant STC achieves full diversity under an arbitrary delay profile. The rationale behind delay-tolerant STC is to design linear independent vectors as the rows of a STC matrix, such that the full rank is maintained in asynchronous scenarios. In [14], delay-tolerant distributed threaded algebraic space-time codes (D-TAST) were proposed with linear independent vectors over the whole symbol sequence. Nevertheless, D-TAST suffers from a low code rate problem. With improvements on the code rate, distributed linear convolutive STC (DLC-STC) was presented in [18] by applying a linear independent vector on each symbol and expanding the vector over the whole sequence under the utilization of linear dispersion code structure [4, 5]. The structure of linear dispersion codes was further investigated in [21] and [24]. Recently, a family of delay-tolerant linear dispersion codes (DT-LDC) was studied in [25] with optimization over the STC matrix to minimize pairwise error probability (PEP). DT-LDC manages to exhibit similar performance to synchronized codes. However, DT-LDC requires prior knowledge of delay distribution for the optimization, and this optimization is computationally expensive, since the amount of pairs to calculate PEP increases exponentially as the number of symbols grows. For time domain approaches, another branch is known as BDT-STC, based on the foundation work in [23]. BDT-STC seeks to maintain full diversity when the delay difference is within a designed bounded range. This idea can be dated back to Time-Reversal STC (TR-STC) [11], where the cases of bounded delay difference were tackled by incorporating guard intervals. However, we find that bounded delay-tolerant Alamouti code (BDT AC) [23], a subset of BDT-STC, has diversity loss. Besides, BDT AC is decoded by an exhaustive maximum-likelihood (ML) decoder, which is computationally prohibitive for large scale problems.

In this paper, we propose a *bounded delay-tolerant time interleave reversal Alamouti code* (BDT-TIR AC). BDT-TIR AC is proven to achieve full diversity with bounded delay difference. Moreover, by parallel processing, the proposed ML decoder of BDT-TIR AC has complexity $\mathcal{O}(|\mathcal{S}|^4)$ for any symbol sequence of length as multiples of four, where $|\mathcal{S}|$ is the cardinality of the adopted constellation $\mathcal{S}$.

Our established work has distinguishing features from prior work. BDT-TIR AC can be regarded as an evolved version of BDT AC [23] with full diversity gain and an efficient ML decoder. As we will show in the paper, BDT-TIR AC in asynchronous scenarios achieves almost identical performance as Alamouti code [1] in the synchronized scenario. Compared to DT-LDC [25], BDT-TIR AC has competitive bit error rate performance with lower peak power.

The remainder of this paper is organized as follows: In Section 2, the problem of interest is formulated. In Section 3, the systematic construction of the proposed code is presented. Simulation results are shown in Section 4, and we conclude the paper in Section 5.

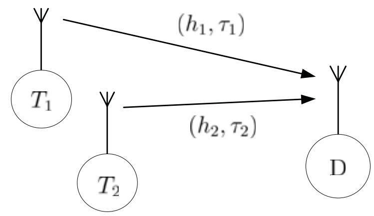

Fig. 1: Asynchronous cooperative communications for 2 transmitters and 1 destination

2. PROBLEM FORMULATION

2.1. System Model

We consider a cooperative communication system that comprises two transmitters T_1, T_2 and one destination D, as shown in Fig. 1. Each node is equipped with a single antenna. The channel fading coefficient and path delay coefficient from T_i to D are denoted by (h_i, τ_i), $i = 1, 2$. Those channel fading coefficients and path delay coefficients are only known at D. The delay difference is $\Delta\tau = \tau_2 - \tau_1$, which is bounded by $|\Delta\tau| \leq \Delta\tau_m$, where $\Delta\tau_m$ is a positive integer. τ_1 and τ_2 are integer multiples of the symbol period. The fractional parts of τ_1 and τ_2 can be regarded as multipath effects, which could be addressed by equalizers [12].

The channels between the transmitters T_1, T_2 and destination D form a 2×1 multiple-input single-output system, which has the potential for transmit diversity gain. However, the difference between τ_1 and τ_2 may cause severe diversity loss. Thus, we focus on the effect of $|\tau_2 - \tau_1|$ instead of τ_1 and τ_2.

2.2. Diversity Loss in BDT AC

In this subsection, we provide an example to illustrate that BDT AC in [23] is not a full diversity STC.

Let $\Delta\tau_m = 2$, the codeword matrix $\mathbf{A}(\mathbf{s})$ of BDT AC for transmitting sequence $\mathbf{s} = [s_1, s_2, \ldots, s_6]$ is [23]

$$\mathbf{A}(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 & s_3 & -s_4^* & -s_5^* & -s_6^* \\ s_4 & s_5 & s_6 & s_1^* & s_2^* & s_3^* \end{bmatrix},$$

where $s_i \in \mathcal{S}$, $i = 1, 2, \ldots, 6$, and $\mathcal{S}$ is the constellation adopted.

Assuming $\Delta\tau = 1$, the codeword matrix becomes

$$\mathbf{A}^{\Delta\tau}(\mathbf{s}) = \begin{bmatrix} s_1 & s_2 & s_3 & -s_4^* & -s_5^* & -s_6^* & 0 \\ 0 & s_4 & s_5 & s_6 & s_1^* & s_2^* & s_3^* \end{bmatrix}.$$

Suppose $\bar{\mathbf{s}} = [\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_6]$ is another symbol sequence transmitted in matrix $\mathbf{A}(\bar{\mathbf{s}})$, where $\mathbf{s} \neq \bar{\mathbf{s}}$. the codeword difference matrix $\mathbf{B}^{\Delta\tau} \triangleq \mathbf{A}^{\Delta\tau}(\mathbf{s}) - \mathbf{A}^{\Delta\tau}(\bar{\mathbf{s}})$ is

$$\mathbf{B}^{\Delta\tau} = \begin{bmatrix} \Delta s_1 & \Delta s_2 & \Delta s_3 & -\Delta s_4^* & -\Delta s_5^* & -\Delta s_6^* & 0 \\ 0 & \Delta s_4 & \Delta s_5 & \Delta s_6 & \Delta s_1^* & \Delta s_2^* & \Delta s_3^* \end{bmatrix},$$

where $\Delta s_i = s_i - \bar{s}_i$, $i = 1, 2, \ldots, 6$.

Suppose that the binary phase-shift keying (BPSK) constellation is adopted. Let $[\Delta s_1, \Delta s_3, \Delta s_5] = [0, 0, 0]$ and $[\Delta s_2, \Delta s_4, \Delta s_6] = [2, 2, -2]$. Then, $\mathbf{B}^{\Delta\tau}$ becomes

$$\mathbf{B}^{\Delta\tau} = \begin{bmatrix} 0 & 2 & 0 & -2 & 0 & 2 & 0 \\ 0 & 2 & 0 & -2 & 0 & 2 & 0 \end{bmatrix},$$

which is of rank one instead of full row rank.

Therefore, BDT AC [23] is not a full diversity STC.

3. NEW ASYNCHRONOUS CODE DESIGN

Unlike BDT AC in [23], we propose a *bounded delay-tolerant time interleave reversal Alamouti code* (BDT-TIR AC) to achieve full diversity. In this section, the full diversity proof and the code rate analysis will be given. Moreover, an efficient ML decoder is also devised by exploiting the benefits of the codeword structure. It will be shown that the proposed ML decoder of BDT-TIR AC has significantly lower complexity than that of BDT AC.

3.1. Code Structure

Let $l \geq \Delta\tau_m$, $l \in \mathbb{Z}_+$. To transmit a sequence $\mathbf{s}$ of $4l$ symbols, the codeword matrix $\mathbf{A}(\mathbf{s})$ of BDT-TIR AC is

$$\mathbf{A}(\mathbf{s}) = \begin{bmatrix} \mathbf{c}_0 & \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \\ -\overleftarrow{\mathbf{c}}_2^* & -\overleftarrow{\mathbf{c}}_3^* & \overleftarrow{\mathbf{c}}_0^* & \overleftarrow{\mathbf{c}}_1^* \end{bmatrix}$$

where

$$\begin{aligned} \mathbf{c}_i &= [s_{il+1}, s_{il+2}, \ldots, s_{il+l}], \quad i = 0, 1, 2, 3 \\ -\overleftarrow{\mathbf{c}}_i^* &= [-s_{il+l}^*, -s_{il+l-1}^*, \ldots, -s_{il+1}^*], \quad i = 2, 3 \\ \overleftarrow{\mathbf{c}}_i^* &= [s_{il+l}^*, s_{il+l-1}^*, \ldots, s_{il+1}^*], \quad i = 0, 1. \end{aligned}$$

Specifically, when $\Delta\tau = 0$, that is, synchronized case, we find that BDT-TIR AC can be explicitly divided into several Alamouti codes. For example, when $l = 2$, assuming $\Delta\tau = 0$, the codeword matrix of BDT-TIR AC is

$$\begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ -s_6^* & -s_5^* & -s_8^* & -s_7^* & s_2^* & s_1^* & s_4^* & s_3^* \end{bmatrix}.$$

Thus, the performance of BDT-TIR AC in synchronized case is equivalent to that of Alamouti code.

Based on the understanding of the code structure of $\mathbf{A}(\mathbf{s})$, we briefly explain the two-level meaning of time interleave reversal (TIR, for short). In the first level, "time interleave" stands for the order among $-\overleftarrow{\mathbf{c}}_2^*$, $-\overleftarrow{\mathbf{c}}_3^*$, $\overleftarrow{\mathbf{c}}_0^*$ and $\overleftarrow{\mathbf{c}}_1^*$ on the second row of $\mathbf{A}(\mathbf{s})$. By altering the order, the diversity loss problem in BDT AC is avoided, which will be proven in Section 3.2. In the second level, "reversal," represented by "←," indicates the decreasing order of the index within each $-\overleftarrow{\mathbf{c}}_2^*$, $-\overleftarrow{\mathbf{c}}_3^*$, $\overleftarrow{\mathbf{c}}_0^*$ and $\overleftarrow{\mathbf{c}}_1^*$ on the second row of $\mathbf{A}(\mathbf{s})$. This idea is originated from TR-STC [11], where Alamouti code structure is retained even in asynchronous scenarios. Aided by "reversal," the complexity of ML decoder is significantly reduced as shown in the following example and analysis in Section 3.4.

For example, when $l = 2$, assuming $\Delta\tau = 1$, the codeword matrix of BDT-TIR AC becomes

$$\begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & 0 \\ 0 & -s_6^* & -s_5^* & -s_8^* & -s_7^* & s_2^* & s_1^* & s_4^* & s_3^* \end{bmatrix},$$

where s_2 and s_6 form an Alamouti code, and meanwhile s_4 and s_8 form another Alamouti code, that is,

$$\begin{bmatrix} s_2 & s_6 \\ -s_6^* & s_2^* \end{bmatrix}, \text{ and } \begin{bmatrix} s_4 & s_8 \\ -s_8^* & s_4^* \end{bmatrix}.$$

After removing those columns with Alamouti code structure, the remaining columns are

$$\begin{bmatrix} s_1 & s_3 & s_5 & s_7 & 0 \\ 0 & -s_5^* & -s_7^* & s_1^* & s_3^* \end{bmatrix}.$$

The original eight-symbol matrix has been split into a four-symbol matrix and two Alamouti codes, thereby significantly facilitating an efficient decoding procedure, as we will explain in Section 3.4.

$$\mathbf{A}^{\Delta\tau}(\mathbf{s}) = \left[\begin{array}{cccccccccccccc} s_1 & \dots & s_{\Delta\tau} & s_{\Delta\tau+1} & \dots & s_l & s_{l+1} & \dots & s_{l+\Delta\tau} & s_{l+\Delta\tau+1} & \dots & s_{2l} & s_{2l+1} & \dots \\ 0 & \dots & 0 & -s_{3l}^* & \dots & -s_{2l+\Delta\tau+1}^* & -s_{2l+\Delta\tau}^* & \dots & -s_{2l+1}^* & -s_{4l}^* & \dots & -s_{3l+\Delta\tau+1}^* & -s_{3l+\Delta\tau}^* & \dots \end{array}\right.$$
$$\left.\begin{array}{ccccccccccccc} s_{2l+\Delta\tau} & s_{2l+\Delta\tau+1} & \dots & s_{3l} & s_{3l+1} & \dots & s_{3l+\Delta\tau} & s_{3l+\Delta\tau+1} & \dots & s_{4l} & 0 & \dots & 0 \\ -s_{3l+1}^* & s_l^* & \dots & s_{\Delta\tau+1}^* & s_{\Delta\tau}^* & \dots & s_1^* & s_{2l}^* & \dots & s_{l+\Delta\tau+1}^* & s_{l+\Delta\tau}^* & \dots & s_{l+1}^* \end{array}\right] \quad (1)$$

$$\mathbf{A}_1^{\Delta\tau} = \left[\begin{array}{cccccccccccc} s_{\Delta\tau+1} & \dots & s_l & s_{l+\Delta\tau+1} & \dots & s_{2l} & s_{2l+\Delta\tau+1} & \dots & s_{3l} & s_{3l+\Delta\tau+1} & \dots & s_{4l} \\ -s_{3l}^* & \dots & -s_{2l+\Delta\tau+1}^* & -s_{4l}^* & \dots & -s_{3l+\Delta\tau+1}^* & s_l^* & \dots & s_{\Delta\tau+1}^* & s_{2l}^* & \dots & s_{l+\Delta\tau+1}^* \end{array}\right] \quad (2)$$

$$\mathbf{A}_2^{\Delta\tau} = \left[\begin{array}{ccccccccccccccc} s_1 & \dots & s_{\Delta\tau} & s_{l+1} & \dots & s_{l+\Delta\tau} & s_{2l+1} & \dots & s_{2l+\Delta\tau} & s_{3l+1} & \dots & s_{3l+\Delta\tau} & 0 & \dots & 0 \\ 0 & \dots & 0 & -s_{2l+\Delta\tau}^* & \dots & -s_{2l+1}^* & -s_{3l+\Delta\tau}^* & \dots & -s_{3l+1}^* & s_{\Delta\tau}^* & \dots & s_1^* & s_{l+\Delta\tau}^* & \dots & s_{l+1}^* \end{array}\right] \quad (3)$$

$$\mathbf{B}_{2,i}^{\Delta\tau} = \left[\begin{array}{ccccc} \Delta s_i & \Delta s_{l+\Delta\tau+1-i} & \Delta s_{2l+i} & \Delta s_{3l+\Delta\tau+1-i} & 0 \\ 0 & -\Delta s_{2l+i}^* & -\Delta s_{3l+\Delta\tau+1-i}^* & \Delta s_i^* & \Delta s_{l+\Delta\tau+1-i}^* \end{array}\right] \quad (4)$$

3.2. Full Diversity Gain

Theorem 1. *In a 2×1 asynchronous communication system with delay difference bound $\Delta\tau_m$, BDT-TIR AC of sequence length $4l$ achieves full diversity, that is, diversity order 2, if $l \geq \Delta\tau_m$.*

Proof: Given the space limit, only the situation of $\Delta\tau > 0$ is considered. The codeword matrix of $\mathbf{s}$ is $\mathbf{A}^{\Delta\tau}(\mathbf{s})$ as shown in (1). Let $\bar{\mathbf{s}}$ be another sequence with codeword matrix $\mathbf{A}^{\Delta\tau}(\bar{\mathbf{s}})$, such that $\bar{\mathbf{s}} \neq \mathbf{s}$. The codeword difference matrix $\mathbf{B}^{\Delta\tau}$ is defined as

$$\mathbf{B}^{\Delta\tau} \triangleq \mathbf{A}^{\Delta\tau}(\mathbf{s}) - \mathbf{A}^{\Delta\tau}(\bar{\mathbf{s}}).$$

To prove the existence of full diversity gain, we have to show that $\mathbf{B}^{\Delta\tau}$ is always full rank for any $\mathbf{s} \neq \bar{\mathbf{s}}$.

Let us first analyze the structure of $\mathbf{A}^{\Delta\tau}(\mathbf{s})$. $\mathbf{A}^{\Delta\tau}(\mathbf{s})$ can be exactly separated into two parts: $\mathbf{A}_1^{\Delta\tau}$ and $\mathbf{A}_2^{\Delta\tau}$. As shown in (2), $\mathbf{A}_1^{\Delta\tau}$ includes those columns with Alamouti code structure, whereas the remaining columns are included in $\mathbf{A}_2^{\Delta\tau}$ shown by (3).

Furthermore, $\mathbf{A}_2^{\Delta\tau}$ can be separated into $\Delta\tau$ matrices $\mathbf{A}_{2,i}^{\Delta\tau}$ as

$$\mathbf{A}_{2,i}^{\Delta\tau} = \left[\begin{array}{ccccc} s_i & s_{l+\Delta\tau+1-i} & s_{2l+i} & s_{3l+\Delta\tau+1-i} & 0 \\ 0 & -s_{2l+i}^* & -s_{3l+\Delta\tau+1-i}^* & s_i^* & s_{l+\Delta\tau+1-i}^* \end{array}\right],$$

where $i = 1, 2, \dots, \Delta\tau$. Notice that $\mathbf{A}_{2,i}^{\Delta\tau}$ is a four-symbol matrix which can be efficiently decoded by exhaustive ML.

Similar to $\mathbf{B}^{\Delta\tau}$, we define difference matrix $\mathbf{B}_1^{\Delta\tau}$ and $\mathbf{B}_{2,i}^{\Delta\tau}$ as

$$\begin{aligned} \mathbf{B}_1^{\Delta\tau} &\triangleq \mathbf{A}_1^{\Delta\tau}(\mathbf{s}) - \mathbf{A}_1^{\Delta\tau}(\bar{\mathbf{s}}), \\ \mathbf{B}_{2,i}^{\Delta\tau} &\triangleq \mathbf{A}_{2,i}^{\Delta\tau}(\mathbf{s}) - \mathbf{A}_{2,i}^{\Delta\tau}(\bar{\mathbf{s}}). \end{aligned}$$

The matrix $\mathbf{B}_{2,i}^{\Delta\tau}$ is shown in (4), where $\Delta s_j \triangleq s_j - \bar{s}_j$, $j = 1, 2, \dots, 4l$.

When $\Delta s_j \neq 0$, if Δs_j falls into $\mathbf{B}_1^{\Delta\tau}$, that is, $j \in [\Delta\tau + 1, l] \bigcup [l + \Delta\tau + 1, 2l] \bigcup [2l + \Delta\tau + 1, 3l] \bigcup [3l + \Delta\tau + 1, 4l]$, $\mathbf{B}_1^{\Delta\tau}$ is full rank due to Alamouti code structure. Thus, $\mathbf{B}^{\Delta\tau}$ is full rank.

When $\Delta s_j \neq 0$, if Δs_j falls into $\mathbf{B}_{2,i}^{\Delta\tau}$, then we have the following discussion for $j \in \{i, 2l+i, l+\Delta\tau+1-i, 3l+\Delta\tau+1-i\}$, $i = 1, 2, \dots, \Delta\tau$.

If $\Delta s_i \neq 0$, the first and fourth columns of $\mathbf{B}_{2,i}^{\Delta\tau}$ in (4) form a rank two matrix as

$$\left[\begin{array}{cc} \Delta s_i & \Delta s_{3l+\Delta\tau+1-i} \\ 0 & \Delta s_i^* \end{array}\right],$$

which means that $\mathbf{B}_{2,i}^{\Delta\tau}$ is full rank. Thus, $\mathbf{B}^{\Delta\tau}$ is full rank as well.

Same rank analysis can be applied on $\mathbf{B}_{2,i}^{\Delta\tau}$ for the case when $\Delta s_i = 0$ and $\Delta s_{3l+\Delta\tau+1-i} \neq 0$, and the case when $[\Delta s_i, \Delta s_{3l+\Delta\tau+1-i}] = [0, 0]$ and $\Delta s_{2l+i} \neq 0$.

Finally, if $[\Delta s_i, \Delta s_{3l+\Delta\tau+1-i}, \Delta s_{2l+i}] = [0, 0, 0]$ and $\Delta s_{l+\Delta\tau+1-i} \neq 0$, the second and fifth columns of $\mathbf{B}_{2,i}^{\Delta\tau}$ form a rank two matrix as

$$\left[\begin{array}{cc} \Delta s_{l+\Delta\tau+1-i} & 0 \\ 0 & \Delta s_{l+\Delta\tau+1-i}^* \end{array}\right],$$

which means that $\mathbf{B}_{2,i}^{\Delta\tau}$ is full rank. Thus, $\mathbf{B}^{\Delta\tau}$ is full rank as well.

$\forall\, j \in [1, 4l]$, if $\Delta s_j \neq 0$, $\mathbf{B}^{\Delta\tau}$ is full rank. ■

3.3. Code Rate Analysis

At destination D, totally $4l$ symbols are received within $4l + |\tau_2 - \tau_1|$ symbol periods. The code rate R is

$$R = \frac{4l}{4l + |\tau_2 - \tau_1|} = \frac{1}{1 + \frac{|\tau_2 - \tau_1|}{4l}}.$$

Since R is increasing with the sequence length $4l$, to approach the rate one, we recommend that $4l$ should be as large as possible.

3.4. Decoding Complexity

In this subsection, we will briefly introduce the design of ML decoder of BDT-TIR AC. Thereafter, complexity comparisons between ML decoder of BDT-TIR AC and that of BDT AC [23] are given.

In the ML decoder design, the "divide-and-conquer" strategy is adopted to facilitate the decoding procedure. The procedure begins with "divide" on the codeword matrix $\mathbf{A}^{\Delta\tau}$ in (1). Recall that $\mathbf{A}^{\Delta\tau}$ comprises $\mathbf{A}_1^{\Delta\tau}$ in (2) and $\mathbf{A}_2^{\Delta\tau}$ in (3), where $\mathbf{A}_1^{\Delta\tau}$ has the Alamouti code structure, whereas $\mathbf{A}_2^{\Delta\tau}$ does not. $\mathbf{A}_2^{\Delta\tau}$ can be further split into $\mathbf{A}_{2,i}^{\Delta\tau}$, where $i = 1, 2, \dots, |\Delta\tau|$. After the "divide" part, the original problem of decoding $\mathbf{A}^{\Delta\tau}$ with $4l$ symbols has been decoupled into independently decoding several small matrices as $\mathbf{A}_1^{\Delta\tau}$ and $\mathbf{A}_{2,i}^{\Delta\tau}$. For $\mathbf{A}_1^{\Delta\tau}$ with Alamouti code structure, decoding such symbols can be performed with computation burden $\mathcal{O}\big(4(l - |\Delta\tau|)|\mathcal{S}|\big)$, where $|\mathcal{S}|$ is the cardinality of the constellation $\mathcal{S}$. For $\mathbf{A}_2^{\Delta\tau}$, each $\mathbf{A}_{2,i}^{\Delta\tau}$ is decoded by exhaustive ML with computation burden $\mathcal{O}(|\mathcal{S}|^4)$. The overall computation burden is $\mathcal{O}\big(4(l - |\Delta\tau|)|\mathcal{S}| + |\Delta\tau||\mathcal{S}|^4\big)$. Since $|\Delta\tau| = \Delta\tau_m$ is the worst case, the complexity of ML decoder for BDT-TIR AC is $\mathcal{O}\big(4(l - \Delta\tau_m)|\mathcal{S}| + \Delta\tau_m|\mathcal{S}|^4\big)$. Specifically, when adopting the minimum required sequence length, that is, $4l = 4\Delta\tau_m$, the ML decoder of BDT-TIR AC has complexity as low as $\mathcal{O}(\Delta\tau_m|\mathcal{S}|^4)$.

In Table 1, the minimum required sequence length and corresponding decoding complexity are compared between the ML decoder of BDT-TIR AC and that of BDT AC, with respect to different

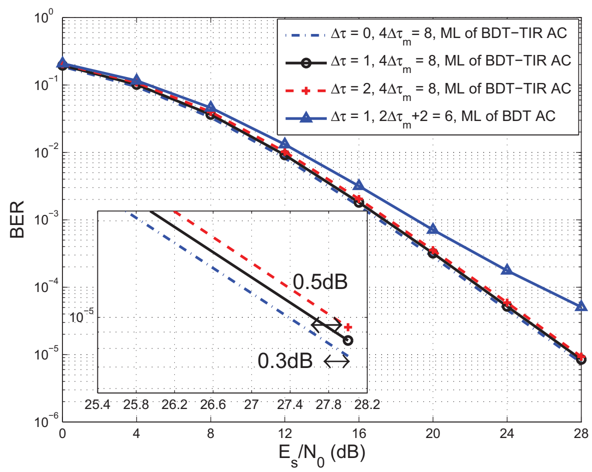

Fig. 2: Performance comparison between proposed BDT-TIR AC ($4\Delta\tau_m = 8$) and BDT AC [23] in QPSK

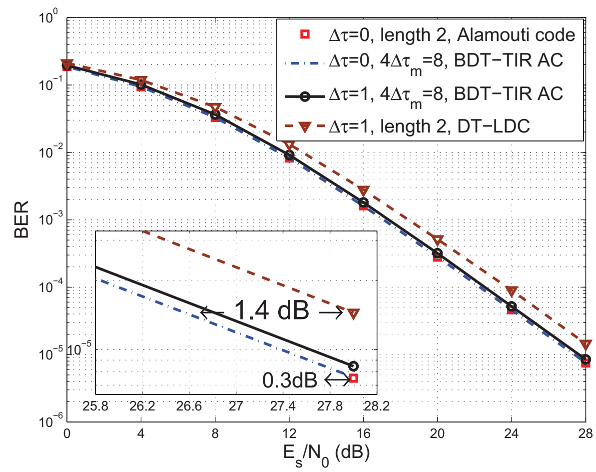

Fig. 3: Bit Error Rate comparison between proposed BDT-TIR AC, DT-LDC [25] and Alamouti code [1] in QPSK

Table 1: Minimum required sequence length (MRSL) and corresponding complexity of ML decoder for BDT AC [23] and BDT-TIR AC with respect to delay difference bound $\Delta\tau_m$

$\Delta\tau_m$	BDT AC in [23]		BDT-TIR AC	
	MRSL	Complexity	MRSL	Complexity
1	4	$\mathcal{O}(\|\mathcal{S}\|^4)$	4	$\mathcal{O}(\|\mathcal{S}\|^4)$
2	6	$\mathcal{O}(\|\mathcal{S}\|^6)$	8	$\mathcal{O}(2\|\mathcal{S}\|^4)$
3	8	$\mathcal{O}(\|\mathcal{S}\|^8)$	12	$\mathcal{O}(3\|\mathcal{S}\|^4)$
…	…	…	…	…
$\Delta\tau_m$	$2\Delta\tau_m + 2$	$\mathcal{O}(\|\mathcal{S}\|^{2\Delta\tau_m+2})$	$4\Delta\tau_m$	$\mathcal{O}(\Delta\tau_m\|\mathcal{S}\|^4)$

$\Delta\tau_m$. As shown in [23], exhaustive ML is adopted for BDT AC with complexity $\mathcal{O}(|\mathcal{S}|^{2\Delta\tau_m+2})$, where $2\Delta\tau_m + 2$ is the minimum required sequence length of BDT AC. Evidently, the ML decoder of BDT-TIR AC decodes more symbols with remarkably lower complexity than that of BDT AC, when $\Delta\tau_m > 1$. For example, when $\Delta\tau_m = 3$ and quadrature phase-shift keying (QPSK) is adopted, the ML decoder of BDT-TIR AC decodes 12 symbols with complexity $\Delta\tau_m|\mathcal{S}|^4 = 768$, whereas the ML decoder of BDT AC decodes 8 symbols with complexity $|\mathcal{S}|^{2\Delta\tau_m+2} = 65536$. Therefore, the proposed ML decoder is much more efficient than that of BDT AC.

By parallel processing, the complexity of ML decoder for BDT-TIR AC can be further reduced to $\mathcal{O}(|\mathcal{S}|^4)$. Notably, $\mathbf{A}_1^{\Delta\tau}$ and $\mathbf{A}_{2,i}^{\Delta\tau}$ are decoded independently. Thus, by parallel decoding of $\mathbf{A}_1^{\Delta\tau}$ and $\mathbf{A}_{2,i}^{\Delta\tau}$, the resultant decoding complexity is equivalent to $\mathcal{O}(|\mathcal{S}|^4)$.

4. SIMULATION RESULTS

In this section, we use simulations to examine the performance of the proposed BDT-TIR AC with ML decoder. The channel fading coefficients h_1 and h_2 are generated following i.i.d. complex Gaussian distribution with zero-mean and unit variance. The maximum delay difference $\Delta\tau_m = 2$, which means that the adopted minimum required sequence length of BDT-TIR AC and BDT AC in [23] is $4\Delta\tau_m = 8$ and $2\Delta\tau_m + 2 = 6$, respectively.

In Fig. 2, we compare the bit error rate (BER) performance between proposed BDT-TIR AC and BDT AC [23] with respect to E_s/N_0, where E_s is the average received power for each symbol. The synchronized case, that is, $\Delta\tau = 0$, can be regarded as the performance of Alamouti code, which serves as a lower bound. The performance of BDT-TIR AC with $\Delta\tau = 1$ and $\Delta\tau = 2$ is plotted. The two lines are barely distinguishable from the synchronized case. The identical slope of the three lines in high E_s/N_0 region demonstrates the full diversity gain. Moreover, as shown in the subfigure, the gaps between both cases of $\Delta\tau \neq 0$ and the synchronized one are within only 0.5 dB, when $E_s/N_0 = 28$ dB. Meanwhile, the performance of BDT AC [23] with $\Delta\tau = 1$ is also plotted. The diversity loss can be clearly seen from the difference in the slope.

Fig. 3 shows the BER comparison among proposed BDT-TIR AC, DT-LDC [25] and Alamouti code [1]. As expected, the performance of BDT-TIR AC in the synchronized case is identical to Alamouti code. For the case considered, the gap between BDT-TIR AC with $\Delta\tau = 1$, DT-LDC and Alamouti code is approximate 0.3 dB and 1.4 dB, respectively. Moreover, as shown in (1), the transmitting power of BDT-TIR AC remains constant over time. In contrast, the power of DT-LDC does not hold this property. This observation suggests that DT-LDC requires larger peak power than BDT-TIR AC so as to satisfy the same E_s/N_0. We conclude that BDT-TIR AC yields better performance than DT-LDC, the latest delay-tolerant STC, given that the delay difference is bounded.

5. CONCLUSION

In this paper, we proposed BDT-TIR AC with full diversity and an efficient ML decoder in asynchronous cooperative communication. BDT-TIR AC was proven to achieve full diversity under bounded delay difference. Moreover, we designed an efficient ML decoder to decouple the original decoding problem into several independent small sub-problems, thereby greatly facilitating the decoding procedure. Simulations demonstrated the full diversity gain and also illustrated that the proposed code is competitive on bit error rate performance compared to the latest delay-tolerant STC.

6. REFERENCES

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