TWO-WAY RELAYING VIA MODULO-AND-FORWARD FOR MIMO SIGNAL TRANSMISSION

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ABSTRACT

Two-way relaying via the amplify-and-forward technique has received much recent attention. A modulo-and-forward (MF) variant of it has recently been proposed for single-antenna transmission, which can significantly reduce the peak and average relay transmission powers. The present work considers two-way relaying for multi-input multi-output (MIMO) signal transmission. We propose an MF technique for Alamouti space-time coding and spatial multiplexing. Simulation results confirm the advantage of the proposed technique in enhancing the peak and average power efficiency of the relay.

Index Terms— MIMO, modulo-and-forward, physical-layer network coding, two-way relay.

1. INTRODUCTION

Relays are considered important components in the evolving wireless communication networks. A typical half-duplex relay receives and transmits in the same frequency band. Moreover, a conventional relay receives or transmits only in one direction at any time, thus needing four time slots to complete one relay-assisted bidirectional exchange of signals between two communicating terminals. By two-way relaying, that is, letting a relay receive from or transmit to both terminals simultaneously, the number of time slots can be reduced to two, significantly improving the spectral efficiency. Some related information-theoretic results can be found in [1–4].

As to how to actually carry out two-way relaying, a variety of methods have been proposed, with different processing complexities at the relay [3,5–9]. The simplest merely amplifies and forwards (AF) the received sum signal from the two terminals. The most complicated performs a joint decoding of the two terminals' signals before combining them for transmission. In between, there are a number of methods bearing various names such as, to name a few, compress-and-forward, denoise-and-forward, and estimate-and-forward.

The present work considers the AF type of two-way relaying. It is well-recognized that in simple AF, the relay employs a "constellation" that is greater than necessary. For example, assume both terminals employ BPSK with signal values ± 1. Let the two terminal-to-relay channels both have unity gain. Then the received signal y_R at the relay may take one of three values: 0 and ± 2. To maintain the same noise performance, the average transmission power needed for this ternary signal is twice that for BPSK. However, a 3 dB reduction in the average relay transmission power (and 6 dB reduction in the peak power) can be effected by having the relay transmit a binary signal computed from y_R, namely, $y_R \% 4 - 1$, where $\%$ denotes the modulo operation. The above is essentially what XOR-based network coding does. The question is whether such a measure can be extended to the situation where each terminal employs a two-dimensional (2D) constellation of an arbitrary modulation order and is subject to an arbitrary complex terminal-to-relay channel coefficient.

In this regard, Cui *et al.* [9] propose to let the relay transmit the absolute value of the received sum signal. The study focusses mainly on low-order one-dimensional modulations, but 2D and higher-order modulations are also briefly discussed. Larsson *et al.* [10] propose to have the relay transmit a modulo version of the received sum signal. But various detailed aspects regarding the modulo operation are not addressed. Recently, the present authors have proposed a complex modulo-and-forward (MF) technique that can deal with arbitrary 2D modulations [11]. For square QAM, it is shown that, without affecting the noise performance, the technique can reduce the average relay transmission power by the order of 2.5 to 3 dB in high signal-to-noise ratio (SNR), when the two terminal-to-relay channel coefficients have an equal magnitude. The corresponding reduction in the peak relay transmission power is roughly a factor of 4.

In this work, we consider employing the MF approach in multi-input multi-output (MIMO) transmission that may use 2D modulations. Although the technique can be applied more generally, for simplicity we focus here on two particular scenarios. In the first (dubbed "A11"), both terminals employ Alamouti space-time coding (STC) [12], the relay has a single antenna, and each terminal uses only one antenna for signal reception; and in the second (dubbed "222"), the terminals

This work was supported in part by the National Science Council of R.O.C. under Grant NSC 101-2219-E-009-021.

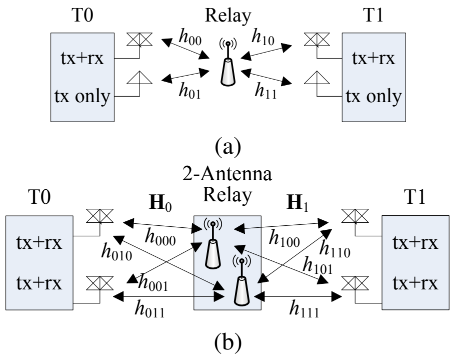

Fig. 1. MIMO transmission scenarios. (a) A11. (b) 222.

and the relay each has two antennas and the terminals employ spatial multiplexing (SM) in signal transmission. These scenarios are illustrated in Fig. 1.

In what follows, Sec. 2 describes the proposed MF scheme for the A11 scenario with Alamouti STC and presents some simulation results that confirm its power advantage. Sec. 3 extends the scheme to the 222 SM scenario. And Sec. 4 concludes the paper.

2. MF TWO-WAY RELAY FOR ALAMOUTI STC

2.1. Problem Formulation and Design Approach

Consider the scenario illustrated in Fig. 1(a). For simplicity, assume that full channel state information (CSI) is available at both terminals and the relay. Let s_{ij} denote the jth modulated data symbol transmitted from terminal Ti where $i = 0, 1$. Let $j = 0, 1$ be the indices of two data symbols over which Alamouti STC is performed. Let x_{ijk} denote the signal transmitted at time k from antenna j of terminal Ti. Then we have

$$x_{ij0} = s_{ij}, \;\; x_{ij1} = (-1)^{\bar{j}} s^*_{i\bar{j}}, \tag{1}$$

where overbar indicates binary complementation, i.e., $\bar{0} = 1$ and $\bar{1} = 0$. Assume that the channel coefficients remain unchanged in the time period considered. Then the received signal at the relay at time k ($k = 0, 1$) is given by

$$y_{Rk} = \underbrace{\sum_{j=0}^{1} h_{0j} x_{0jk}}_{\triangleq y_{R0k}} + \underbrace{\sum_{j=0}^{1} h_{1j} x_{1jk}}_{\triangleq y_{R1k}} + z_{Rk} \tag{2}$$

where z_{Rk} denotes the relay noise, assumed to be additive circularly symmetric complex white Gaussian, i.e., AWGN.

Let x_{Rk} denote the signal transmitted by the relay at time k. The received signal at terminal Ti at time k is given by

$$y_{ik} = h_{i0} x_{Rk} + z_{ik} \tag{3}$$

where z_{ik} is the noise at Ti, also assumed AWGN. In conventional AF, $x_{Rk} = \alpha_A y_{Rk}$ where the scaling factor α_A is set to meet the transmit power constraint of the relay. The receiver at Ti may subtract the "self-signal" y_{Rik} from the scaled received signal $y_{ik}/(h_{i0}\alpha_A)$ to obtain a noisy version of $y_{R\bar{i}k}$. Normal Alamouti detection can then be invoked to recover the data symbols $s_{\bar{i}j}$ transmitted by the opposite terminal T$\bar{i}$.

Note that at Ti, the Alamouti detector only needs to have (the noisy) $y_{R\bar{i}k}$, which is confined in a smaller region than (the noisy) $y_{R\bar{i}k} + y_{Rik}$ that is forwarded by conventional AF. Conventional AF thus transmits at a higher peak and a higher average power than necessary for either y_{Rik} ($i = 0, 1$) alone. The question is whether such "wastes" can be reduced by a simple modification of the relay forwarding function. For this, we see that a 2D modulo operation with a suitably chosen modulus provides an answer.

In short, we propose an MF scheme in which the relay amplifies and forwards a modulo version of y_{Rk} as [11]

$$x_{Rk} = \alpha_M \,\mathrm{cmod}(\beta y_{Rk} + C, B) \tag{4}$$

where α_M is a scaling factor to satisfy the relay power constraint, β is a phase rotation factor (i.e., $\beta = e^{j\theta}$ for some θ), C is a complex offset, B is the modulus (real and postive), and "cmod" denotes complex modulo defined as

$$\mathrm{cmod}(x, B) = \mathrm{mod}(\Re\{x\}, B) + j\,\mathrm{mod}(\Im\{x\}, B) \tag{5}$$

for an arbitrary complex value x, with

$$\mathrm{mod}(y, B) = [(y + B/2)\,\%\,B] - B/2 \tag{6}$$

for an arbitrary real value y, where $a\%b = a - \lfloor a/b \rfloor b$. The reason for β will be explained later. For the offset C, in single-antenna transmission it is found that a proper setting of C can help reducing the relay transmission power [11]. In the present work, we let $C = 0$, for reason that will not be explained due to space limit. Nevertheless, we retain it in the formulation of the MF function. Concerning the modulus, note that if B is large enough such that each y_{Rik} ($i = 0, 1$) can be contained in a square of area B^2 with enough margin to accommodate almost all z_{Rk} values, then in high terminal SNR, we have, with high probability,

$$\begin{aligned} &\mathrm{cmod}(y_{ik}/[h_{i0}\alpha_M] - C - \beta y_{Rik}, B)/\beta \\ &= \;\; y_{R\bar{i}k} + z_{Rk} + z_{ik}/(h_{i0}\alpha_M \beta), \end{aligned} \tag{7}$$

for terminal Ti. This "unmoduloed" signal can then be Alamouti detected as usual.

We now consider the design of the MF parameters β and B for maximum peak and average power efficiency of relay transmission with proper signal detection at the terminals.

2.2. Design of MF Parameters

Assume a square QAM modulation with minimum distance 2 between signal points. Fig. 2 shows an outline of the typical

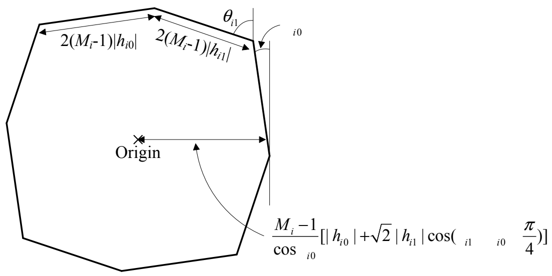

Fig. 2. Octagon shows the outline of the typical footprint of the signal component y_{Rik} at the relay.

footprint of the signal component y_{Rik} at the relay, which is a convolution of the constellations of $h_{ij}x_{ijk}$, $j = 0, 1$, where M_i is the square root of the modulation order and $\theta_{ij} = \angle h_{ij}$. The complex modulo operation defined above may be viewed as a concatenation of two operations: 1) repetition of the signal to be "moduloed" on an orthogonal lattice defined by the modulus, and 2) retaining only the result in the "fundamental region" of area B^2 centered at the origin. Too large a modulus will cause power inefficiency, whereas too small a modulus will degrade the transmission performance due to a reduced minimum distance between signal points.

In this regard, let $h_{im} = \min\{|h_{i1}|, |h_{i0}|\}$ and let the modulus be such that the lattice repetition said above maintains a minimum distance no less than $2h_{im}$ between signal points. Some trigonometry shows that a sufficient condition is given by

$$\begin{aligned} B &\geq 2\sec\theta_{i0}[(M_i - 1)|h_{i0}| \\ &\quad + \sqrt{2}(M_i - 1)|h_{i1}|\cos(\theta_{i1} - \theta_{i0} - \pi/4) + h_{im}] \\ &\triangleq b_i, \end{aligned} \tag{8}$$

where $\theta_{ij} = \angle h_{ij}$, $j = 0, 1$. The condition applies to both $i = 0, 1$. Hence a simple choice is to let $B = \max_i b_i$.

Note that the bound given in (8) depends not only on the magnitudes of the channel coefficients h_{ij}, where $i, j \in \{0, 1\}$, but also on their phase angles. For example, a different θ_{i0} may yield a different multiplicative factor in front of the brackets, even if M_i, $|h_{i0}|$, $|h_{i1}|$, and $\theta_{i1} - \theta_{i0}$ remain the same. Put alternatively, a rotation of the footprint shown in Fig. 2 may affect the value of b_i, and by letting the angle of rotation be one that minimizes $\max_i b_i$ we may minimize the above-chosen B. This is where the previously introduced factor $\beta = e^{j\theta}$ comes into play. Its use is to rotate y_{Rk} in the complex plane so that y_{Rik}, $i = 0, 1$, are rotated correspondingly (see (2)). We now specify the β that minimizes $\max_i b_i$, but omit the proof due to space limit. Before it, note that due to the assumed use of square QAM, both b_i vary periodically with θ with period $\pi/2$. Note also that a rotation of y_{Rk} by θ in angle may equivalently be viewed as rotating the orthogonal lattice by $-\theta$ in angle (with y_{Rk} unrotated).

To specify the optimal β, let

$$\theta'_{ij} = \text{mod}(\theta_{ij} + \theta - \pi/4, \pi/2) + \pi/4. \tag{9}$$

Then we have $0 \leq \theta'_{ij} < \pi/2$. Let

$$m_i = \arg\min_j \theta'_{ij}. \tag{10}$$

and, along with it,

$$\theta''_{i0} = \theta'_{im_i}, \;\; \theta''_{i1} = \theta'_{i\bar{m}_i}, \;\; h''_{i0} = h_{im_i}, \;\; h''_{i1} = h_{i\bar{m}_i}. \tag{11}$$

Define

$$\theta_t = \arctan\frac{|h''_{i0}| - \sqrt{2}|h''_{i1}|\sin(\theta''_{i1} - \theta''_{i0} - \frac{\pi}{4})}{|h''_{i0}| + \sqrt{2}|h''_{i1}|\cos(\theta''_{i1} - \theta''_{i0} - \frac{\pi}{4})}. \tag{12}$$

If $\theta''_{i0} > \theta_t$, then redefine θ''_{ij} and h''_{ij} as

$$\theta''_{i1} = \theta'_{im_i}, \;\; \theta''_{i0} = \theta'_{i\bar{m}_i} - \frac{\pi}{2}, \;\; h''_{i1} = h'_{im_i}, \;\; h''_{i0} = h'_{i\bar{m}_i}. \tag{13}$$

Now let

$$\begin{aligned} b''_i(\theta) &= 2\sec\theta''_{i0}[(M_i - 1)|h''_{i0}| \\ &\quad + \sqrt{2}(M_i - 1)|h''_{i1}|\cos(\theta''_{i1} - \theta''_{i0} - \pi/4) + h_{im}]. \end{aligned} \tag{14}$$

The optimal rotation angle and the corresponding phase rotation factor are then given by

$$\theta_{opt} = \arg\min_{0 \leq \theta < \pi/2}\max_i b''_i(\theta), \;\; \beta_{opt} = e^{j\theta_{opt}}. \tag{15}$$

To recapitulate, the MF operation at the relay is as described in (4), with four design parameters α_M, β, C, and B. We let $C = 0$ in the present work. The phase rotation angle β may be determined in the way described in the last paragraph. Then let $B = \max_i b''_i(\theta_{opt})$. The scaling factor α_M follows according to the peak or average power constraint of the relay. Computation of the design parameters requires knowing the modulation orders and the CSI.

2.3. Simulation Results for STC

In the simulations, let both terminals employ square QAM of the same order, which may be 4, 16, or 64. The two terminal-relay channels are either AWGN or Rayleigh fading.

Consider first the impact on the average relay transmission power of the proposed scheme. Fig. 3 shows the power reduction in comparison to conventional AF in both AWGN and Rayleigh fading at various relay SNR levels, when $\alpha_A = \alpha_M$. (In the Rayleigh fading case, the SNR shown are average values.) We see that on the order of 0.5–1 dB of power reduction is achieved in different conditions at high relay SNR.

Consider now the instantaneous relay transmission power. Fig. 4 shows the complementary cumulative distributaion function (CCDF) under 16-QAM in AWGN. A greater-than-2 dB reduction in peak power is effected.

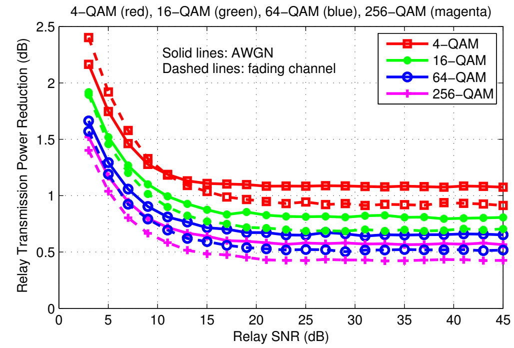

Fig. 3. Reduction in average relay transmission power by the proposed MF scheme.

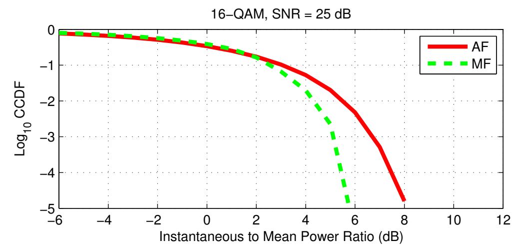

Fig. 4. CCDFs of instantaneous relay transmission power with 16-QAM in AWGN.

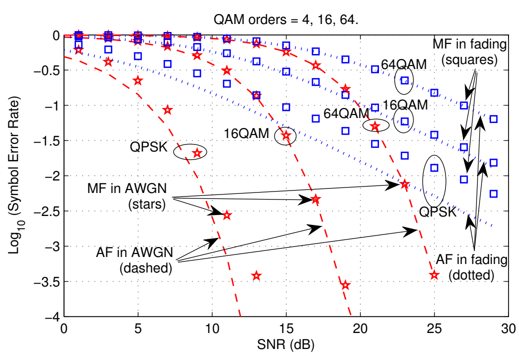

Fig. 5. SER performance of different relaying methods in AWGN and fading channels.

Finally, consider the symbol-error-rate (SER) performance. Fig. 5 shows some results for both MF and AF. We see that their performance is similar except for QPSK. Indeed, when $\alpha_A = \alpha_M$, we expect MF to perform somewhat worse than AF because the former may increase the number of nearest neighbors of a signal point at constellation boundary to a value similar to that of an interior point [11], and this increase is most detrimental to QPSK. The above degradation can be compensated by increasing α_M relative to α_A, that is, by cutting down on the power advantage of MF. The overall effect is a reduced relay transmission power for the same SER performance.

3. MF TWO-WAY RELAY FOR SM

We now extend the MF two-way relaying technique to the 222 SM scenario illustrated in Fig. 1(b). Due to the use of two antennas at the relay, the working is somewhat different than the A22 scenario. We only give a brief outline of the proposed technique here. In the case of 222 SM, the received signal vector at the relay is given by

$$\mathbf{y}_{Rk} = \mathbf{H}_0\mathbf{x}_{0k} + \mathbf{H}_1\mathbf{x}_{1k} + \mathbf{z}_{Rk} \tag{16}$$

where $\mathbf{x}_{ik}$ ($i \in \{0, 1\}$) is the signal vector transmitted by terminal Ti at time k, $\mathbf{H}_i$ is the channel matrix between Ti and the relay, and $\mathbf{z}_{Rk}$ is the additive relay noise vector at time k. For simplicity, we do MF on each element of $\mathbf{y}_{Rk}$ separately, rather than considering a multi-dimensional lattice that deals with all the elements of $\mathbf{y}_{Rk}$ simultaneously. The MF parameters may be determined using the same method as in the A11 scenario. The transmitted signal vector of the relay is then given by

$$\mathbf{x}_{Rk} = \boldsymbol{\alpha}_M \mathrm{cmod}(\boldsymbol{\beta}\mathbf{y}_{Rk} + \mathbf{C}, \mathbf{B}) \tag{17}$$

where $\boldsymbol{\alpha}_M$ is a diagonal matrix of scaling factors to meet the relay power constraint, $\boldsymbol{\beta}$ is a diagonal matrix of phase rotation factors, $\mathbf{C}$ is an offset vector (which may be set to null), $\mathbf{B}$ is the modulus vector, and the cmod function operates on the elements of its vector arguments separately. The receiver at each Ti first does "unmoduloing" by computing

$$\mathrm{cmod}(\mathbf{y}_{ik}\mathbf{H}_i^{-1}\boldsymbol{\alpha}_M^{-1} - \mathbf{C} - \boldsymbol{\beta}\mathbf{H}_i\mathbf{x}_{ik})\boldsymbol{\beta}^{-1} \triangleq \mathbf{y}'_{R\bar{i}k} \tag{18}$$

where $\mathbf{y}_{ik}$ is the received relay-forwarded signal at Ti at time k. This receiver can then employ any suitable SM signal detection method to demodulate $\mathbf{y}'_{R\bar{i}k}$.

4. CONCLUDING REMARKS

We considered AF-type two-way relay and developed a simple technique, dubbed modulo-and-forward (MF), to enhance the transmission power efficiency of the relay in MIMO signaling. Simulation results confirmed its utility.

We note that the attainable relay power saving in the MIMO cases dealt with in this work is lower than that in the single-input single-output (SISO) case treated in [11]. This is because, in MIMO, the distribution of signal values at a relay antenna as received from either terminal is a convolution of signal constellations associated with multiple transmitter antennas (see the discussion near the beginning of Sec. 2.2) and, as a result, this signal distribution is not uniform (as in typical QAM) but is "heavier" in the center. Hence the proposed modulo operation yields less relay power reduction than in the case of SISO with uniform QAM. Possible ways to address this issue are being studied.

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