A LOW COMPLEXITY ALGORITHM FOR COLLABORATIVE-RELAY BEAMFORMING

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ABSTRACT

We consider cooperative transmission in wireless relay networks, in which a source communicates with the destination with the help of a set of N cooperating amplify-and-forward relays. The relay weights are obtained to maximize the received signal-to-noise ratio at the destination, subject to individual power constraint. We consider two schemes that have appeared in the literature, i.e., (i) the optimal weight vector design method, which has been solved via second-order cone programming plus a bisection search, with complexity of $\mathcal{O}(N^{3.5})$, and (ii) the one-bit feedback phase control scheme, which has been formulated as a binary quadratic programming and has been solved for exact solution via a exhaustive search. We propose algorithms for these two problems that have substantially reduced complexity, i.e., $\mathcal{O}(N\log_2 N)$ or $\mathcal{O}(N)$ for the first problem, and polynomial time $\mathcal{O}(N\log_2 N)$ for the second problem.

Index Terms— Cooperative communications, relay network, collaborative-relay beamforming, binary quadratic programming (BQP).

1. INTRODUCTION

In distributed relay beamforming, a set of cooperating nodes act as a virtual antenna array and adjust their transmission weights to form a beam to the destination. This can result in diversity gains similar to those of multiple-antenna systems [1], [2]. Various effective cooperation schemes have been proposed in the literature, such as amplify-and-forward (AF), decode-and-forward (DF) [3], coded-cooperation [4], and compress-and-forward [5]. Among them, the AF relaying is of particular interest due to its simplicity [2]. In distributed relay beamforming, one of the objective is to achieve a given quality-of-service (QoS) level subject to some power constraints [6], [7], [8] (and the reference therein).

In [6], the authors considered the problem of maximizing the destination signal-to-noise ratio (SNR) by jointly optimizing the complex relay weights, subject to both individual relay power constraints and a total power constraint, under perfect channel state information (CSI). In particular, the problem for individual power constraint is solved in [6] via second-order cone programming (SOCP) plus bisection search. The SOCP in general involves a complexity $\mathcal{O}(N^{3.5})$.

In [9], the one-bit feedback phase control scheme is proposed. In this scheme, the receiver computes the binary coefficient for each relay and use a one-bit feedback per relay to send the coefficient to each relay. Then, each relay weights its received signal based on the binary coefficient, and then transmits. In [9], the received SNR maximization problem is formulated as a binary quadratic programming (BQP) and the authors simply note that, due to the difficulty of the BQP, an exhaustive search is required to get the solution.

In this paper, we investigate the aforementioned problems of [6], [9]. For the former, we proposed algorithms with substantially reduced complexity, i.e., $\mathcal{O}(N\log_2 N)$ or $\mathcal{O}(N)$. For the latter, we point out an algorithm in polynomial time $\mathcal{O}(N\log_2 N)$.

2. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a system model consisting of a source node, a destination node and N relay nodes, each node equipped with a single antenna. We assume that the direct link between the source and destination is very weak and thus ignored. The channel gains from the source to the ith relay, and from the ith relay to the destination, are denoted respectively by f_i and g_i.

Communication between the transmitter and the receiver occurs in two stages. During the first stage, the transmitter broadcasts its signal, $\sqrt{P_s}\ s$, to the relays. We assume that $\mathbb{E}(|s|^2) = 1$. P_s is the source power. The received signal at the ith relay is given by $x_i = \sqrt{P_s}\, f_i s + v_i$ where v_i represents the noise at the ith relay having zero mean and variance σ^2. During the second stage, the relays, working in AF fashion, transmit a weighted version of the signal that they received during the first stage. Let P_i be the maximum average power available at the ith relay. The ith relay weights the received signal and transmits $z_i = \alpha_i w_i x_i$, where w_i with $|w_i|^2 \le 1$ is the relay weight for the ith relay, and $\alpha_i = \sqrt{\frac{P_i}{\mathbb{E}\{|f_i|^2\}P_s+\sigma^2}}$. We assume that α_i is known by the ith relay. The received signal at the receiver equals $y = \sum_{i=1}^{N} g_i z_i + \nu$ where ν is the noise at the destination having zero mean and variance σ^2. We address two schemes as follows.

1) *Optimal weight vector design*: Let $\mathbf{h} \triangleq [h_1, \cdots, h_N]^T$

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with $h_i = \alpha_i f_i^* g_i^*$. Let $\mathbf{w} \triangleq [w_1, \cdots, w_N]^T$. The problem of maximizing the SNR at the receiver with respect to the relay weights can be expressed as [6]:

$$\max_{\mathbf{w}} \frac{P_s}{\sigma^2} \frac{|\mathbf{h}^\dagger \mathbf{w}|^2}{1 + \sum_{i=1}^N \alpha_i^2 |g_i|^2 |w_i|^2} \quad (1)$$
$$\text{s.t.} \quad |w_i|^2 \leq 1, \ i = 1, \cdots, N.$$

The problem of (1) is studied in [6] in which second-order cone programming plus bisection search is used. The SOCP can be solved by interior point methods with complexity $\mathcal{O}(N^{3.5})$ [10].

2) *One-bit feedback phase control scheme*: In this scheme, the receiver computes the coefficient $b_i \in \{-1, 1\}$ for the ith relay, $i = 1, \cdots, N$, and uses a one-bit feedback to send b_i to ith relay. Then, each relay weights its received signal based on the coefficient b_i, and then transmit. The scheme corresponds to the case in which the weights in (1) are restricted to $w_i = b_i$, $i = 1, \cdots, N$. This scheme selects $\mathbf{b} \in \{-1, 1\}^N$ such that the received SNR is maximized. This leads to the following optimization problem (letting $\mathbf{Q} = \mathbf{h}\mathbf{h}^\dagger$) [9]:

$$\max_{\mathbf{b}} \ \mathbf{b}^T \mathbf{Q} \mathbf{b} \quad (2)$$
$$\text{s.t.} \quad b_i \in \{-1, 1\}, \ i = 1, \cdots, N.$$

This belongs to binary quadratic programming (BQP). The BQP is in general (i.e., general $\mathbf{Q}$) difficult. In [9], the authors simply said that an exhaustive search is required for the exact solution of (2).

3. LOW COMPLEXITY ALGORITHMS FOR THE TWO SCHEMES

3.1. Optimal weight vector design

We address two algorithms for the problem of (1).

3.1.1. Water-filling algorithm

Let $\mathbf{V} = \mathrm{diag}(v_1, \cdots, v_N)$ with $v_i = \alpha_i^2 |g_i|^2$. Clearly, it holds that $\max_{1 \leq i \leq N} |w_i|^2 = 1$. The problem of (1) is equivalent to

$$\max_{\mathbf{w}} \frac{P_s}{\sigma^2} \frac{|\mathbf{h}^\dagger \mathbf{w}|^2}{\max_{1 \leq i \leq N} (|w_i|^2 + \mathbf{w}^\dagger \mathbf{V} \mathbf{w})} \quad (3)$$
$$\text{s.t.} \quad \max_{1 \leq i \leq N} |w_i|^2 = 1.$$

Since the objective function is invariant for scaling of $\mathbf{w}$, the problem of (3) is equivalent to

$$\max_{\mathbf{w} \neq 0} \frac{P_s}{\sigma^2} \frac{|\mathbf{h}^\dagger \mathbf{w}|^2}{\max_{1 \leq i \leq N} (|w_i|^2 + \mathbf{w}^\dagger \mathbf{V} \mathbf{w})} \quad (4)$$

Since $e^{j\omega}\mathbf{w}$ is also a solution for any real number ω if $\mathbf{w}$ is a solution, we may restrict $\mathbf{h}^\dagger \mathbf{w}$ to be real. The problem of (4) is equivalent to

$$\min_{\mathbf{h}^\dagger \mathbf{w} = 1} \max_{1 \leq i \leq N} (|w_i|^2 + \mathbf{w}^\dagger \mathbf{V} \mathbf{w}). \quad (5)$$

Denote $\Omega = \{\boldsymbol{\theta} = (\theta_1, \cdots, \theta_N) | \theta_k \geq 0, \forall k; \sum_{k=1}^N \theta_k = 1\}$. Note that

$$\max_{1 \leq i \leq N} (|w_i|^2 + \mathbf{w}^\dagger \mathbf{V} \mathbf{w}) = \max_{\boldsymbol{\theta} \in \Omega} \sum_{k=1}^N \theta_k (|w_k|^2 + \mathbf{w}^\dagger \mathbf{V} \mathbf{w})$$
$$= \max_{\boldsymbol{\theta} \in \Omega} \mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}. \quad (6)$$

So, the problem of (5) is equivalent to

$$\min_{\mathbf{h}^\dagger \mathbf{w} = 1} \max_{\boldsymbol{\theta} \in \Omega} \mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}. \quad (7)$$

Since $\mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}$ is convex with respect to $\mathbf{w}$, and concave (linear) with respect to $\boldsymbol{\theta}$, according to the minimax theorem, we have

$$\min_{\mathbf{h}^\dagger \mathbf{w} = 1} \max_{\boldsymbol{\theta} \in \Omega} \mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}$$
$$= \max_{\boldsymbol{\theta} \in \Omega} \min_{\mathbf{h}^\dagger \mathbf{w} = 1} \mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}. \quad (8)$$

The inner optimization $\min_{\mathbf{h}^\dagger \mathbf{w} = 1} \mathbf{w}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta})) \mathbf{w}$ has a solution at

$$\mathbf{w} = \frac{(\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta}))^{-1} \mathbf{h}}{\mathbf{h}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta}))^{-1} \mathbf{h}}. \quad (9)$$

With this, the problem of (8) is equivalent to

$$\min_{\boldsymbol{\theta} \in \Omega} \mathbf{h}^\dagger (\mathbf{V} + \mathrm{diag}(\boldsymbol{\theta}))^{-1} \mathbf{h}. \quad (10)$$

which can be rewritten as

$$\min_{\boldsymbol{\theta} \in \Omega} \sum_{i=1}^N \frac{|h_i|^2}{v_i + \theta_i}. \quad (11)$$

The problem of (11) can be solved by the water-filling algorithm. In detail, for some $\lambda \geq 0$ (Lagrange multiplier),

$$\theta_i = \left(\lambda |h_i| - v_i\right)^+, \ i = 1, \cdots, N; \quad (12)$$
$$\sum_{i=1}^N \left(\lambda |h_i| - v_i\right)^+ = 1 \quad (13)$$

where $(x)^+ = \max(x, 0)$. Once λ and hence $\boldsymbol{\theta}$ is obtained, the solution to the problem of (1) is given by

$$w_i = \beta \frac{\frac{h_i}{v_i + \theta_i}}{\sum_{i=1}^N \frac{|h_i|^2}{v_i + \theta_i}}, \ i = 1, \cdots, N \quad (14)$$

where β is chosen to satisfy $\max_{1 \leq i \leq N} |w_i|^2 = 1$.

For a tolerance $\epsilon > 0$, the computing complexity of the water-filling is $\mathcal{O}(N)$.

3.1.2. A direct (non-iterative) algorithm

Using the triangle inequality $|x+y| \leq |x| + |y|$, we have

$$|\mathbf{h}^{\dagger}\mathbf{w}| = \Big|\sum_{i=1}^{N} \alpha_i f_i g_i w_i\Big| \leq \sum_{i=1}^{N} \alpha_i |f_i g_i|\,|w_i|. \quad (15)$$

Equality holds if $\mathrm{Arg}(w_i) = -\mathrm{Arg}(f_i g_i)$, $i = 1, \cdots, N$ where $\mathrm{Arg}(z)$ is the argument of complex z. With this result, by letting $\mathbf{y} = [y_1, \cdots, y_N]^T$ with $y_i = |w_i|$, $i = 1, \cdots, N$, the problem of (1) is equivalent to

$$\begin{aligned} \max_{\mathbf{y}} \;& \frac{P_s}{\sigma^2} \frac{\left(\sum_{i=1}^{N} \alpha_i |f_i g_i|\, y_i\right)^2}{1 + \sum_{i=1}^{N} \alpha_i^2 |g_i|^2 y_i^2} \\ \text{s.t.} \;& 0 \leq y_i \leq 1,\ i = 1, \cdots, N. \end{aligned} \quad (16)$$

Lemma 1 There is $\eta > 0$ such that $y_k = \min\{\eta \frac{|f_k|}{\alpha_k |g_k|},\ 1\}$, $k = 1, \cdots, N$ is the solution for the problem (16).

The proof is given in Section 7.

Note that $y_k = \min\{\eta \frac{|f_k|}{\alpha_k |g_k|},\ 1\}$, $k = 1, \cdots, N$ are always feasible for the problem (16) for any $\eta > 0$. Thus, according to Lemma 1, the problem (16) is equivalent to

$$\begin{aligned} \max_{\eta} \; F(\eta) = & \frac{P_s}{\sigma^2} \frac{\left(\sum_{i=1}^{N} \alpha_i |f_i g_i|\ \min\{\eta \frac{|f_i|}{\alpha_i |g_i|},\ 1\}\right)^2}{1 + \sum_{i=1}^{N} \alpha_i^2 |g_i|^2 \left(\min\{\eta \frac{|f_i|}{\alpha_i |g_i|},\ 1\}\right)^2} \\ \text{s.t.} \quad & \eta > 0. \end{aligned} \quad (17)$$

In the following, we address a direct (non-iterative) algorithm for the exact solution with computing complexity $\mathcal{O}(N \log_2 N)$. Since f_k's and g_k's are (independent) channel gains, we assume that $f_k \neq 0, g_k \neq 0, \forall k$; $|f_i|/(\alpha_i |g_i|) \neq |f_j|/(\alpha_j |g_j|)$, $\forall i \neq j$. We rearrange $\frac{|f_i|}{\alpha_i |g_i|}$'s in descending order. Let π be a permutation of $\{1, \cdots, N\}$ such that $|f_{\pi(i)}|/(\alpha_{\pi(i)} |g_{\pi(i)}|)$'s are in descending order. Let us denote, for $i = 1, \cdots, N$

$$c_i = \alpha_{\pi(i)} |f_{\pi(i)} g_{\pi(i)}|, \text{ and } d_i = \alpha_{\pi(i)}^2 |g_{\pi(i)}|^2. \quad (18)$$

Let us denote $m = 1, \cdots, N$

$$B_m = \sum_{k=1}^{m} c_k, \text{ and } C_m = \sum_{k=1}^{m} d_k. \quad (19)$$

Lemma 2 Let $\eta^\star$ be the solution to the problem of (17).

1) If $1 + C_N - B_N \frac{d_N}{c_N} \geq 0$, then $\eta^\star = \frac{d_N}{c_N}$;

2) If $1 + C_N - B_N \frac{d_N}{c_N} < 0$, then there exists an unique $m \in \{1, \cdots, N-1\}$ such that $\frac{d_m}{c_m} < \frac{1+C_m}{B_m} \leq \frac{d_{m+1}}{c_{m+1}}$ and moreover it holds that $F(\eta)$ is strictly increasing over $\frac{d_1}{c_1} \leq \eta \leq \frac{1+C_m}{B_m}$, and strictly decreasing over $\frac{1+C_m}{B_m} \leq \eta \leq \frac{d_N}{c_N}$. $\eta^\star = \frac{1+C_m}{B_m}$.

The proof is not difficult but tedious. Due to limited space, the proof is omitted here.

From Lemma 2, if $1 + C_N - B_N \frac{d_N}{c_N} \geq 0$, then the solution is $\eta^\star = \frac{d_N}{c_N}$; otherwise, $F(\eta)$ is first increasing then decreasing over the domain $[\frac{d_1}{c_1}, \frac{d_N}{c_N}]$. From Theorem 2-2), it a very simple task to find $\eta^\star$: we only need to find the largest integer $m \in \{1, \cdots, N-1\}$ such that $\frac{1+C_m}{B_m} > \frac{d_m}{c_m}$. Then the solution is $\eta^\star = \frac{1+C_m}{B_m}$.

Based on the analysis above, we propose Algorithm 1. The proposed algorithm does not require iterations. Line 3 is sorting N numbers with a complexity $\mathcal{O}(N \log_2 N)$. Line 7-11 has a worst-case complexity $\mathcal{O}(N \log_2 N)$: The bisection search for $m \in \{1, \cdots, N-1\}$ requires at most $\log_2 N$ times while in each time checking $\frac{1+C_m}{B_m} > \frac{d_m}{c_m}$ requires $\mathcal{O}(N)$. So Algorithm 1 has a complexity $\mathcal{O}(N \log_2 N)$.

We should note that a similar algorithm has reported in [1] but with a different derivation.

Algorithm 1 Direct algorithm for Problem (17)

1: **Input:** $f_i, g_i, \alpha_i,\ i = 1, \cdots, N$.
2: **begin**
3: Calculate $c_i, d_i, i = 1, \cdots, N$ according to (18).
4: Calculate $B_N = \sum_{k=1}^{N} c_k$ and $C_N = \sum_{k=1}^{N} d_k$.
5: **if** $1 + C_N - B_N \frac{d_N}{c_N} \geq 0$
6: $\eta^\star = \frac{d_N}{c_N}$.
7: **else**
8: Find the largest integer $m \in \{1, \cdots, N-1\}$ such
9: that $\frac{1+C_m}{B_m} > \frac{d_m}{c_m}$.
10: $\eta^\star = \frac{1+C_m}{B_m}$.
11: **end**
12: **end**
13: **Output:** $\eta^\star$.

3.2. One-bit feedback phase control scheme

The BQP (2) can be solved in polynomial time $\mathcal{O}(N \log_2 N)$ [11].

4. NUMERICAL RESULTS

We performed simulations on a Dell XPS M1530 laptop (Intel CORE 2 Duo 2GHz Processor) to measure the run time of Algorithm 1 for a large number of relays. We considered $N = 10^6$ and performed 100 Monte Carlo simulations for #100 tests. The average run time for one test was 1.9279 (sec.). Then, we did simulations to compare to the SOCP method. In particular, we considered $N = 8$ relays and did Monte Carlo simulation for #1000 tests. The simulation showed that Algorithm 1 always achieves the same solution as the SOCP method.

5. CONCLUSION

The optimal weight vector design scheme and the one-bit feedback phase control scheme in cooperative transmission in wireless relay networks have been considered. The relay weights are determined so that the SNR at the destination is maximized subject to individual power constraint. We have proposed low complexity algorithms for these schemes.

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7. PROOF OF LEMMA 1

Let $F_1(\mathbf{y}) = \sum_{i=1}^{N} \alpha_i |f_i g_i| \, y_i$, $F_2(\mathbf{y}) = 1+\sum_{i=1}^{N} \alpha_i^2 |g_i|^2 y_i^2$. The objective function in (16) is expressed as $\frac{[F_1(\mathbf{y})]^2}{F_2(\mathbf{y})}$. S-ince all constraints in (16) are linear, the Karush-Kuhn-Tucker (KKT) conditions hold: for $k = 1, \cdots, N$

$$\frac{\partial}{\partial y_k}\left[\frac{[F_1(\mathbf{y})]^2}{F_2(\mathbf{y})}\right] = \begin{cases} 0 & \text{if } 0 < y_k < 1 \\ \theta_k & \text{if } y_k = 1 \\ -\lambda_k & \text{if } y_k = 0 \end{cases} \qquad (20)$$

where $\theta_k \geq 0$ and $\lambda_k \geq 0$ are respectively the Lagrange multipliers associated with the constraint $y_k \leq 1$, and $0 \leq y_k$, $k = 1, \cdots, N$. Obviously, the problem (16) has a positive optimal objective value, so $\mathbf{y}^\star \neq 0$ and $F_1(\mathbf{y}^\star) > 0$. We restrict to $F_1(\mathbf{y}) > 0$ and express

$$\frac{\partial}{\partial y_k}\frac{[F_1(\mathbf{y})]^2}{F_2(\mathbf{y})} = \frac{2[F_1(\mathbf{y})]^2 \alpha_k^2 |g_k|^2}{[F_2(\mathbf{y})]^2}\Big(\frac{F_2(\mathbf{y})}{F_1(\mathbf{y})}\frac{|f_k|}{\alpha_k|g_k|} - y_k\Big). \qquad (21)$$

From (20) and (21), noting $\theta_k \geq 0$ and $\lambda_k \geq 0$, we have the necessary conditions: for $k = 1, \cdots, N$

$$\frac{F_2(\mathbf{y})}{F_1(\mathbf{y})}\frac{|f_k|}{\alpha_k|g_k|} - y_k \begin{cases} = 0 & \text{if } 0 \leq y_k < 1 \\ \geq 0 & \text{if } y_k = 1 \end{cases} \qquad (22)$$

or equivalently, by letting $\eta = \frac{F_2(\mathbf{y})}{F_1(\mathbf{y})}$,

$$\begin{cases} y_k = \eta \frac{|f_k|}{\alpha_k|g_k|} & \text{if } 0 \leq y_k < 1, \\ 1 \leq \eta \frac{|f_k|}{\alpha_k|g_k|} & \text{if } y_k = 1 \end{cases} \qquad (23)$$

which can be rewritten as

$$y_k = \min\Big\{\eta \frac{|f_k|}{\alpha_k|g_k|},\, 1\Big\},\ k = 1, \cdots, N. \qquad (24)$$

Let $\mathbf{y}^\star = [y_1^\star, \cdots, y_N^\star]^T$ be a solution for the problem (16). Then $\mathbf{y}^\star$ satisfies (24) with $\eta = \frac{F_2(\mathbf{y}^\star)}{F_1(\mathbf{y}^\star)}$. Note that for given η, (24) determines $\mathbf{y}$ uniquely. Thus, for $\eta = \frac{F_2(\mathbf{y}^\star)}{F_1(\mathbf{y}^\star)}$, (24) gives $\mathbf{y}^\star$ exactly. This completes the proof.

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