ORTHOGONALLY-DISTRIBUTED SPACE-TIME BLOCK CODES FOR RELAY CHANNELS

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ABSTRACT

The direct application of orthogonal space-time block coding (OSTBC) for multiple input and multiple output (MIMO) systems to distributed cooperative relay networks makes the equivalent channel matrix for maximum likelihood (ML) detection lose its orthogonality. Hence, this paper proposes a new design that makes the channel matrix be *orthogonally distributed* (OD) for a suboptimal symbol-by-symbol detector (SBSD). Using ODSTBC, an asymptotic symbol error probability (SEP) formula with SBSD is derived, showing the optimal diversity gain function is achieved. In addition, two kinds of ODSTBC designs for the distributed relay networks are presented, which interestingly renders that SBSD is equivalent to the ML detector. Numerical results verify the diversity analysis and indicate competitive error performance to currently available orthogonal distributed STBC designs with much simpler complexity.

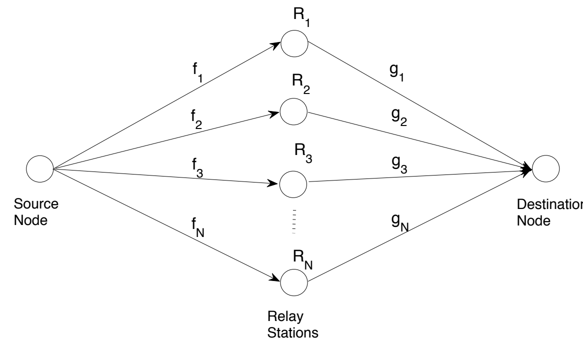

Fig. 1. System Model of Distributed Relay Channels

1. INTRODUCTION

Installing multiple antennas as in MIMO systems is often impractical in mobile communications. Therefore, cooperative diversity has recently been revived [1–11], in which the in-cell mobile users share the use of their antennas to create a virtual array through distributed transmission and signal processing. Since this arrangement forms a distributed MIMO system, the diversity techniques for the MIMO systems have been naturally extended to such relaying networks for the design of so-called distributed STBC [7,9,12]. It is known that among all STBC designs for MIMO systems, orthogonal STBC [13–16] is particularly attractive, since they can provide maximum diversity using a linear processing maximum likelihood detector. Hence, a natural question is *whether or not OSTBC can be directly extended to distributed cooperative networks?* Unfortunately, the answer to this question is negative. Unlike in a MIMO system, the channel gain in the relay system is the *product* of two Gaussian random channels. As a result, the direct application of OSTBC for MIMO systems to distributed cooperative relay networks will make the equivalent channel matrix for ML detection lose its orthogonality. This fact was first realized in [17]. Hence, the researchers in [18, 19] proposed distributed orthogonal STBC designs with the ML receiver. The work of this paper is closely related to those in [18, 19]. However, the idea here is significantly different. We require that the channel matrix is *orthogonally distributed* for the SBSD rather than the equivalent whitened channel matrix is orthogonal for the ML detector, thereby, avoiding the inverse operation of the noise covariance matrix. Another contribution of this paper is to derive an asymptotic SEP formula for all ODSTBCs with the SBSD, showing the optimal diversity gain function is achieved, which, however, were just verified by computer simulations in [18, 19] without any calculations of SEP. Very interestingly, two kinds of simple ODSTBC designs presented in this paper render that the SBSD is equivalent to the ML detector.

Notation: Column vectors and matrices are boldface lowercase and uppercase letters, respectively; the matrix transpose, the complex conjugate, the Hermitian are denoted by $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, respectively; $\mathrm{E}[\cdot]$ denotes the expected value of the expression in brackets; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; Notation $\mathbf{A} \succeq \mathbf{B}$ denotes that $\mathbf{A}$ and $\mathbf{B}$ are positive semi-definite and $\mathbf{A} - \mathbf{B}$ is also positive semi-definite; The entry of matrix $\mathbf{A}$ in the ith row and jth column is denoted by $[\mathbf{A}]_{ij}$; Notation $\otimes$ denotes the Kronecker product. Notation $f(x) = \mathcal{O}(g(x))$ with $g(x) \geq 0$ denotes that there exists a pair of constants, c_1 and c_2, independent of the variable x such that $c_1 g(x) \leq f(x) \leq c_2 g(x)$.

2. SYSTEM MODEL

In this section, we consider a distributed one-way relay network consisting of $N+2$ nodes: one source, one destination and N relays, each of which is equipped with a single antenna, as shown in Fig. 1. The channel gain between the nth relay and source is denoted by f_n, while the channel gain between the nth relay and destination is denoted by $g_n, n = 1, \dots N$. All of these coefficients are assumed to be independent, identically distributed (*i.i.d.*) and circularly symmetric complex Gaussian random variables with zero mean and unit variance. In addition, the channels are assumed to be quasi-static and flat fading. For practical application, we also assume that the relays do not have any CSI, while the destination could have CSI of channels from source to relays and from relays to destination through training or a feedback channel.

In a whole transmission process, total K symbols are transmitted from the source to the destination. More specifically, there are two communication phases, the first one of which has K time

978-1-4799-0356-6/13/$31.00

slots while the second one has $T \geq K$ slots. In the first K slots, K symbols are transmitted consecutively from the source to all relays:

$$\mathbf{r}_n = f_n\mathbf{x} + \boldsymbol{\xi}_n, \tag{1}$$

where $\mathbf{r}_n = [r_{n1}, r_{n2}, \ldots, r_{nK}]^T$, $\mathbf{x} = [x_1, x_2, \ldots, x_K]^T$ and $\boldsymbol{\xi}_n = [\xi_{n1}, \xi_{n2}, \ldots, \xi_{nK}]^T$. The symbols x_k for $k = 1, 2, \cdots, K$ are randomly, independently and equally likely chosen from the M-ary QAM constellation with $\mathrm{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}$. The noise vectors are assumed to be circularly symmetric complex Gaussian with covariance matrix $\mathrm{E}[\mathbf{n}_m\mathbf{n}_n^H] = \delta_{mn}\sigma^2\mathbf{I}_K$, where δ_{mn} denotes the delta sequence, i.e., $\delta_{mn} = 1$ if $m = n$, and otherwise, 0. Then, a linear dispersion coding scheme is adopted at the relay stations, i.e.,

$$\mathbf{z}_n = \mathbf{A}_n\mathbf{r}_n + \mathbf{B}_n\mathbf{r}_n^*, \tag{2}$$

where $\mathbf{A}_n \in \mathbb{C}^{T\times K}$ and $\mathbf{B}_n \in \mathbb{C}^{T\times K}, n = 1, 2, \ldots, N$. In the next communication phase, each relay forwards these coded signals to the destination during T consecutive time slots. The (relative) symbol rate of this system is thus defined by K/T per channel use. Therefore, the signal vector received at the destination can be written as

$$\mathbf{y} = \mathcal{A}\mathbf{x} + \mathcal{B}\mathbf{x}^* + \boldsymbol{\eta}, \tag{3}$$

where $\mathcal{A} = \sum_{n=1}^N h_n\mathbf{A}_n$, $\mathcal{B} = \sum_{n=1}^N \bar{h}_n\mathbf{B}_n$ with $h_n = f_n g_n$ and $\bar{h}_n = f_n^* g_n$, and $\boldsymbol{\eta} = [\mathbf{A}_1\boldsymbol{\xi}_1 + \mathbf{B}_1\boldsymbol{\xi}_1^*, \cdots, \mathbf{A}_N\boldsymbol{\xi}_N + \mathbf{B}_N\boldsymbol{\xi}_N^*]\mathbf{g} + \mathbf{v}$. Here, the noise vector $\mathbf{v}$ at the destination is also assumed to be circularly symmetric complex Gaussian with zero mean and covariance matrix $\mathrm{E}[\mathbf{v}\mathbf{v}^H] = \sigma^2\mathbf{I}_T$, thereby, resulting in the fact that the covariance matrix of $\boldsymbol{\eta}$ is

$$\mathbf{R} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^H] = \sigma^2\left[\mathbf{I}_T + \sum_{n=1}^N |g_n|^2(\mathbf{A}_n\mathbf{A}_n^H + \mathbf{B}_n\mathbf{B}_n^H)\right]. \tag{4}$$

It is known that when CSI is completely available at the destination, the optimal receiver for estimation of the transmitted signal $\mathbf{x}$ in (3) is the ML receiver, which is equivalent to solving the following optimization problem:

$$\arg\min_{\mathbf{x}} \left(\mathcal{A}\mathbf{x} + \mathcal{B}\mathbf{x}^* - \mathbf{y}\right)^H \mathbf{R}^{-1}\left(\mathcal{A}\mathbf{x} + \mathcal{B}\mathbf{x}^* - \mathbf{y}\right). \tag{5}$$

However, the computation complexity increases exponentially. Hence, in this paper, we seek for an orthogonal structure for the suboptimal SBSD. To do that, we notice that there exists a matrix family $\bar{\mathbf{A}}_k \in \mathbb{C}^{T\times T}$ and $\bar{\mathbf{B}}_k \in \mathbb{C}^{T\times T}, k = 1, 2, \ldots, K$ with the nth column of $\bar{\mathbf{A}}_k$ and $\bar{\mathbf{B}}_k$ being the kth column of $\mathbf{A}_n$ and $\mathbf{B}_n$, respectively, such that $\mathcal{A} = [\bar{\mathbf{A}}_1\mathbf{h}, \bar{\mathbf{A}}_2\mathbf{h}, \ldots, \bar{\mathbf{A}}_K\mathbf{h}]$ and $\mathcal{B} = [\bar{\mathbf{B}}_1\bar{\mathbf{h}}, \bar{\mathbf{B}}_2\bar{\mathbf{h}}, \ldots, \bar{\mathbf{B}}_K\bar{\mathbf{h}}]$.

3. ORTHOGONAL DESIGN CRITERIA

In this section, orthogonal design criteria are introduced. Similar to OSTBC, we assemble the original and conjugate version of (3) such that

$$\begin{bmatrix}\mathbf{y}\\ \mathbf{y}^*\end{bmatrix} = \underbrace{\begin{bmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{B}^* & \mathcal{A}^*\end{bmatrix}}_{\mathcal{H}}\begin{bmatrix}\mathbf{x}\\ \mathbf{x}^*\end{bmatrix} + \begin{bmatrix}\boldsymbol{\eta}\\ \boldsymbol{\eta}^*\end{bmatrix}. \tag{6}$$

Now, we are in the position to introduce the following definition:

Definition 1 *It is said that $\{\bar{\mathbf{A}}_k, \bar{\mathbf{B}}_k\}_{k=1}^K$ is orthogonally distributed if*

$$\mathcal{H}^H\mathcal{H} = \|\mathbf{h}\|^2\mathbf{I}. \tag{7}$$

for any complex numbers f_n and g_n, $n = 1, 2, \ldots, N$.

Theorem 1 $\{\bar{\mathbf{A}}_k, \bar{\mathbf{B}}_k\}_{k=1}^K$ *is orthogonally distributed if and only if the following three conditions are satisfied simultaneously:*

1) $\bar{\mathbf{A}}_k^H\bar{\mathbf{A}}_l + \bar{\mathbf{B}}_l^H\bar{\mathbf{B}}_k = \delta_{kl}\mathbf{I}$
2) $[\bar{\mathbf{A}}_k^H\bar{\mathbf{A}}_l]_{ij} = \mathbf{0}$, $[\bar{\mathbf{B}}_l^H\bar{\mathbf{B}}_k]_{ij} = \mathbf{0}$ *for* $i \neq j$
3) $\bar{\mathbf{A}}_k^H\bar{\mathbf{B}}_l + \bar{\mathbf{A}}_l^H\bar{\mathbf{B}}_k = \mathbf{0}$

By Theorem 1 and the definition of $\bar{\mathbf{A}}_n$ and $\bar{\mathbf{B}}_n$, we can attain the following corollary.

Corollary 1 *That $\{\bar{\mathbf{A}}_k, \bar{\mathbf{B}}_k\}_{k=1}^K$ is orthogonally distributed is equivalent to the fact that the following three conditions are satisfied simultaneously:*

1) $\mathbf{A}_n^H\mathbf{A}_n + \mathbf{B}_n^T\mathbf{B}_n^* = \mathbf{I}$
2) $\mathbf{A}_m^H\mathbf{A}_n = \mathbf{0}$, $\mathbf{B}_m^H\mathbf{B}_n = \mathbf{0}$ *for* $m \neq n$
3) $\mathbf{A}_m^H\mathbf{B}_n + \mathbf{B}_n^T\mathbf{A}_m^* = \mathbf{0}$

The proofs of Theorem 1 and its corollary are omitted because of space limitation. For ODSTBC, multiplying (6) with $\mathcal{H}^H$ and taking the first half of the signal vector yield

$$\tilde{\mathbf{y}} = \|\mathbf{h}\|^2\mathbf{x} + \tilde{\boldsymbol{\eta}}, \tag{8}$$

where $\tilde{\boldsymbol{\eta}} = \mathcal{A}^H\boldsymbol{\eta} + \mathcal{B}^T\boldsymbol{\eta}^*$. Hence, this leads us to propose the following suboptimal SBSD:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\|\tilde{\mathbf{y}} - \|\mathbf{h}\|^2\mathbf{x}\right\|. \tag{9}$$

Particularly, when the covariance matrix of $\tilde{\boldsymbol{\eta}}$, $\tilde{\mathbf{R}} = \mathrm{E}[\tilde{\boldsymbol{\eta}}\tilde{\boldsymbol{\eta}}^H]$, is an identity matrix up to a scale, the SBSD is equivalent to the ML detector. Here, it should be worth emphasizing that the channel model (9) is not equivalent to the channel model (3) from the viewpoint of detection theory. Now, let us analyze error performance of the SBSD (9). Note that since $\mathbf{A}_n^H\mathbf{A}_n + \mathbf{B}_n^T\mathbf{B}_n^* = \mathbf{I}$, and $T \geq K$, we have $\mathbf{0} \preceq \mathbf{A}_n\mathbf{A}_n^H + \mathbf{B}_n\mathbf{B}_n^H \preceq \mathbf{I}$ and thus, $\sigma^2\|\mathbf{h}\|^2\mathbf{I} \preceq \tilde{\mathbf{R}} \preceq \sigma^2(1 + \|\mathbf{g}\|^2)\|\mathbf{h}\|^2\mathbf{I}$. The SEP of SBSD for the square M-ary QAM constellation is upper and lower bounded by

$$\mathrm{P_{SEP}} \leq \mathrm{P}_1 = 4\left(1 - \frac{1}{\sqrt{M}}\right)\mathcal{Q}_1 - 4\left(1 - \frac{1}{\sqrt{M}}\right)^2\mathcal{Q}_1^2, \tag{10a}$$

$$\mathrm{P_{SEP}} \geq \mathrm{P}_2 = 4\left(1 - \frac{1}{\sqrt{M}}\right)\mathcal{Q}_2 - 4\left(1 - \frac{1}{\sqrt{M}}\right)^2\mathcal{Q}_2^2, \tag{10b}$$

where $\mathcal{Q}_1 = \mathrm{Q}\left(\frac{\|\mathbf{h}\|d}{\sigma\sqrt{2(1+\|\mathbf{g}\|^2)}}\right)$, $\mathcal{Q}_2 = \mathrm{Q}\left(\frac{\|\mathbf{h}\|d}{\sqrt{2}\sigma}\right)$, and $\mathrm{Q}(\cdot)$ is the *Q-function*, whose alternative expression is $\mathrm{Q}(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)\mathrm{d}\theta$, where $d = \sqrt{\frac{6E_s}{M-1}}$ is the minimum distance in QAM constellation and E_s is the average symbol energy. Based on the assumptions of the channel coefficients f_n and g_n, the probability density functions of $|f_n|^2$ and $|g_n|^2$ are all $\mathrm{p}(t) = \exp(-t)$. Since $\mathrm{E}_{\mathbf{f},\mathbf{g}}[\cdot] = \mathrm{E}_{\mathbf{g}}\left\{\mathrm{E}_{\mathbf{f}}[\cdot]\right\}$ and

$$\mathrm{E}_{\mathbf{f}}\left[\mathcal{Q}_1\right] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{n=1}^N \frac{1 + \|\mathbf{g}\|^2}{1 + \|\mathbf{g}\|^2 + \frac{|g_n|^2d^2}{4\sigma^2\sin^2\theta}}\mathrm{d}\theta \tag{11a}$$

$$\mathrm{E}_{\mathbf{f}}\left[\mathcal{Q}_2\right] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{n=1}^N \frac{1}{1 + \frac{|g_n|^2d^2}{4\sigma^2\sin^2\theta}}\mathrm{d}\theta, \tag{11b}$$

we can obtain

$$
\begin{aligned}
\mathrm{E}_{\mathbf{f},\mathbf{g}}\left[\mathcal{Q}_1\right] &\leq \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{E}_{\mathbf{g}}\left[\frac{(1+\sum_{n=1}^{N}|g_n|^2)^N}{\prod_{n=1}^{N}\left(1+\frac{|g_n|^2 d^2}{4\sigma^2\sin^2\theta}\right)}\right]\mathrm{d}\theta \\
\mathrm{E}_{\mathbf{f},\mathbf{g}}\left[\mathcal{Q}_2\right] &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{E}_{\mathbf{g}}\left[\frac{1}{\prod_{n=1}^{N}\left(1+\frac{|g_n|^2 d^2}{4\sigma^2\sin^2\theta}\right)}\right]\mathrm{d}\theta
\end{aligned}
\tag{12}
$$

After some calculus and algebraic manipulations, we can arrive at

$$
\begin{aligned}
&\mathrm{E}_{\mathbf{g}}\left[\frac{(1+\sum_{n=1}^{N}|g_n|^2)^N}{\prod_{n=1}^{N}\left(1+\frac{|g_n|^2 d^2}{4\sigma^2\sin^2\theta}\right)}\right] = \\
&\left(\frac{2(M-1)\sin^2\theta}{3}\right)^N\left(\frac{\ln\rho}{\rho}\right)^N + \mathcal{O}\left(\frac{\ln^{N-1}\rho}{\rho^N}\right)
\end{aligned}
\tag{13}
$$

where SNR is defined by $\rho = E_s/\sigma^2$. Therefore, the upper and lower bound of SER are

$$
\begin{aligned}
\mathrm{P}_1 \leq\ & \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\left(\frac{2(M-1)\sin^2\theta}{3}\right)^N\left(\frac{\ln\rho}{\rho}\right)^N\mathrm{d}\theta \\
&-\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}}\left(\frac{2(M-1)\sin^2\theta}{3}\right)^N\left(\frac{\ln\rho}{\rho}\right)^N\mathrm{d}\theta \\
&+\mathcal{O}\left(\frac{\ln^{N-1}\rho}{\rho^N}\right) \\
\mathrm{P}_2 =\ & \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\left(\frac{2(M-1)\sin^2\theta}{3}\right)^N\left(\frac{\ln\rho}{\rho}\right)^N\mathrm{d}\theta \\
&-\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}}\left(\frac{2(M-1)\sin^2\theta}{3}\right)^N\left(\frac{\ln\rho}{\rho}\right)^N\mathrm{d}\theta \\
&+\mathcal{O}\left(\frac{\ln^{N-1}\rho}{\rho^N}\right)
\end{aligned}
\tag{14}
$$

The above discussions can be summarized as the following theorem:

Theorem 2 *$\bar{\mathrm{P}}_{SEP}$, the average SEP of SBSD for the channel* (8) *can be represented by*

$$\bar{\mathrm{P}}_{SEP} = \phi(M,N)\left(\ln\rho/\rho\right)^N + \mathcal{O}((\ln\rho)^{N-1}/\rho^N),$$

when SNR *is sufficiently large, where* $\phi(M,N) = \frac{2^{N+2}(M-1)^N}{3^N\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin^{2N}\theta\mathrm{d}\theta + \frac{1}{\sqrt{M}}\int_0^{\frac{\pi}{4}}\sin^{2N}\theta\mathrm{d}\theta\right]$.

Theorem 2, whose detailed proof is omitted due to space limitation, reveals that using ODSTBC, the optimal diversity gain function $(\ln\rho/\rho)^N$ for the relay network is achieved with the SBSD.

4. ODSTBC DESIGNS

In this section, we provide two simple designs for ODSTBC. If we denote the code structure by $\mathcal{X}(\mathbf{s}) = [f_1\mathbf{A}_1\mathbf{x} + f_1^*\mathbf{B}_1\mathbf{x}^*, ..., f_N\mathbf{A}_N\mathbf{x} + f_N^*\mathbf{B}_N\mathbf{x}^*]$ and use $\mathcal{X}'(\mathbf{s}) = [\mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{x}^*, ..., \mathbf{A}_N\mathbf{x} + \mathbf{B}_N\mathbf{x}^*]$ to illustrate orthogonality for simplicity, one of simple designs based on the Alamouti coding scheme to construct Rate-$2/2^n$ ODSTBC is shown below.

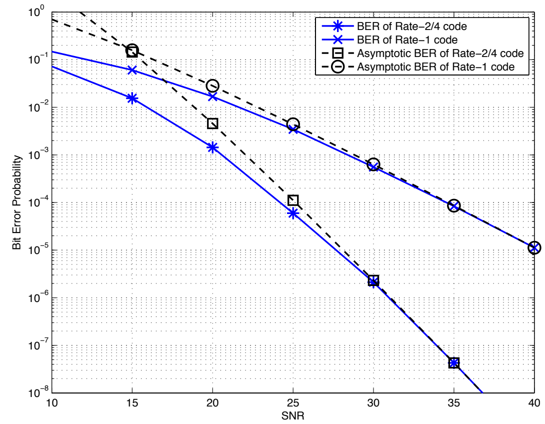

Fig. 2. BER performance comparison of Rate-1 and Rate-2/4 orthogonal design

Design 1 *For $2^n \times 2^n$ square orthogonal codes, one approach to construct Rate-$2/2^n$ orthogonal code is*

$$\mathcal{X}'(\mathbf{s}) = (\otimes^{n-1}\mathbf{I}) \otimes \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix},$$

where $\otimes^{n-1}\mathbf{I}$ means $n-1$ times Kronecker product of Identity matrix with itself.

The following is another simple design:

Design 2 *Any even number of symbols can be transmitted simultaneously in one process with the following structure.*

$$\mathcal{X}'(\mathbf{s}) = (\otimes^{n-1}\mathbf{I}) \otimes \left\{\mathbf{e}_1 \otimes \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \mathbf{e}_2 \otimes \begin{bmatrix} x_3 & -x_4^* \\ x_4 & x_3^* \end{bmatrix} + \cdots\right\},$$

where $\mathbf{e}_i = [0,\ldots,0,1,0,\ldots,0]^T$, the ith entry of which is 1.

The symbol rate for Designs 1 and 2 is $\mathcal{R} = 2^{1-n}$ per channel slot use. One of significant advantages of these two ODSTBC designs is that the noise covariance matrix $\tilde{\mathbf{R}}$ in (8) is the identity matrix up to a scale, thereby, resulting in the fact that SBSD is equivalent to the ML detector, which is the major difference between the OSTBC designs presented in this paper and the designs proposed in [18]. The following is a specific example to demonstrate this difference.

Example 1 *Consider a specific rate-2/4 ODSTBC for a cooperative network with four relays using Design 1, where*

$$\mathcal{X}'(\mathbf{s}) = \mathrm{diag}\left\{\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}, \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}\right\}.$$

In this case, the covariance matrix of the noise $\boldsymbol{\eta}$ in (3) *is $\mathbf{R} = \sigma^2\mathrm{diag}\{1+|g_1|^2+|g_2|^2, 1+|g_1|^2+|g_2|^2, 1+|g_3|^2+|g_4|^2, 1+|g_3|^2+|g_4|^2\}$, which is not an identity matrix. However, the covariance matrix of the noise $\tilde{\boldsymbol{\eta}}$ in* (9) *is an identity matrix up to a scale. In fact, $\tilde{\mathbf{R}} = \sigma^2(\|\mathbf{h}\|^2 + (|g_1|^2+|g_2|^2)(|h_1|^2+|h_2|^2) + (|g_3|^2+|g_4|^2)(|h_3|^2+|h_4|^2))\mathbf{I}$.*

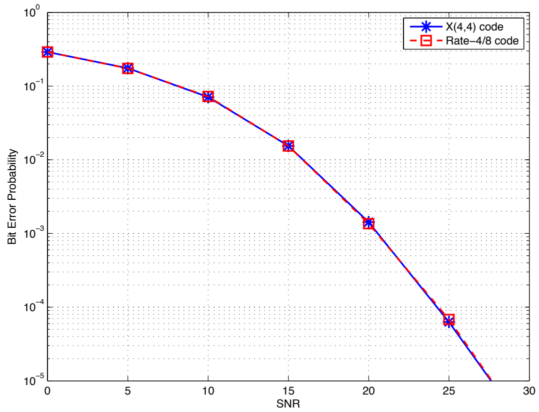

Fig. 3. BER performance comparison of Rate-2/4 orthogonal design and former X(4,4) code

5. SIMULATIONS

In this section, we carry out several numerical simulations to verify system performance of the codes designed in this paper. In all these simulations, the transmitted symbols are randomly, independently and equally likely chosen from the 16-QAM constellation.

Fig. 2 demonstrates the BER performance of Rate-1 code with two relays and Rate-2/4 code with four relays, respectively. As illustrated by Fig. 2, when SNR is sufficiently large, the simulated BER converges to the dominant theoretic curve. Fig. 3 shows the error performance comparison of the ODSTBC design (Example 1) proposed in this paper with $\mathbf{X}(4,4)$ in [18]. It can be seen that these two curves are matched very well. However, our design has much simpler decoding complexity, since it does not need to calculate the inverse matrix.

6. CONCLUSION

In this paper, we proposed a new orthogonal criterion for the design of orthogonally distributed space-time block codes. Some equivalent orthogonal conditions were discussed. It was shown by deriving asymptotic SEP formula that ODSTBC enables the sub-optimal SBSD to extract the optimal diversity gain function. In addition, two simple ODSTBC designs were presented, rendering SBSD was equivalent to the ML detector. Numerical results verified the diversity analysis and indicated better performance than direct implementation of standard OSTBC.

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