OPTIMAL DISTRIBUTED CONCATENATED SPACE-TIME BLOCK CODES FOR TWO-WAY RELAYING NETWORKS

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ABSTRACT

In this paper, we consider a half-duplex amplify-and-forward two-way relaying network consisting of two sources with each having a single antenna and N relays with each having two antennas. For such a system with a general distributed linear dispersion code, a tight lower bound of pairwise error probability (PEP) of the maximum likelihood (ML) detector is derived, showing that diversity gain function cannot decay faster than $\ln^N \mathtt{SNR}/\mathtt{SNR}^{2N}$, where SNR is signal to noise ratio. Particularly for $N = 2^b$, a new distributed concatenated space-time block code (STBC) is proposed and its asymptotic PEP formula is attained, showing that the code presented in this paper achieves the maximum diversity gain, i.e., meeting the lower bound of the diversity gain function, as well as the maximum coding gain.

1. INTRODUCTION

The pioneering study of memoryless two-way relaying networks from the standing point of information theory can date back to the early seminal works of Shannon [1] and Cover [2]. However, applications of the fundamental idea to wireless communication systems are more recent [3–10]. The major motivation for reviving this research interest is due to the promising spectrum efficiency gain of a two-way relaying network over a one-way relaying network. In this paper, we consider a transmitter design to enhance error performance for a two-way relaying network from the viewpoint of detection theory. In spite of the fact that quite a few of recent research results on the relaying networks clearly mimic those of the multiple input and multiple output (MIMO) systems, some major differences between the MIMO and relay systems are necessarily and explicitly pointed out here: (a) Unlike the MIMO system, the channel in the relay system is non-linear and multiplicative, and the covariance matrix of noise depends on the channel and the structure of the underlying distributed STBC. (b) Unlike STBC for the MIMO system, distributed STBC for the relay system is jointly performed through the source nodes and relay nodes with corrupted noisy signals, whereas the signals from different relaying nodes cannot be cooperatively processed for transmission. (c) Unlike the diversity gain for the MIMO system, the diversity gain function for the relay system involves the logarithm of SNR. Because of this, the optimal diversity gain function for a general distributed space-time block coded relay network is not as clear as for the MIMO system. To the best knowledge of the authors, only the upper bound of the diversity gain function for a general relay network was derived [11, 12], whereas asymptotic performance analysis is available only for some specific relaying protocols [6, 13–22]. Unfortunately, the upper bound cannot tells us what is either the best diversity gain or the optimal coding gain for a distributed space-time block code to possibly provide for the ML detector. (c) Power loading among source nodes and relay nodes significantly affects the overall performance of the whole relay system [11,23–25]. The principal goal of this paper is to design a new distributed concatenated STBC for a half-duplex amplify-and-forward two-way relaying network consisting of two sources with each having a single antenna and N relays with each having two antennas. This work is closely related to that in [23].

Notation: $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^H$ and $\det(\mathbf{A})$ denote the transpose, conjugate, and conjugate transpose, and the determinant of the matrix $\mathbf{A}$, respectively; $\mathbb{E}[\cdot]$ denotes the expected value of the expression in brackets; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. Notation $f(x) = O(g(x))$ with $g(x) \geq 0$ denotes that there exists a pair of constants, c_1 and c_2, independent of the variable x such that $c_1 g(x) \leq f(x) \leq c_2 g(x)$. A double factorial of n is denoted by $(n)!!$; Notation $\mathrm{diag}(a_1, a_2, \cdots, a_n)$ denotes a diagonal matrix whose diagonal entries are $a_1, a_2, \cdots, a_n$.

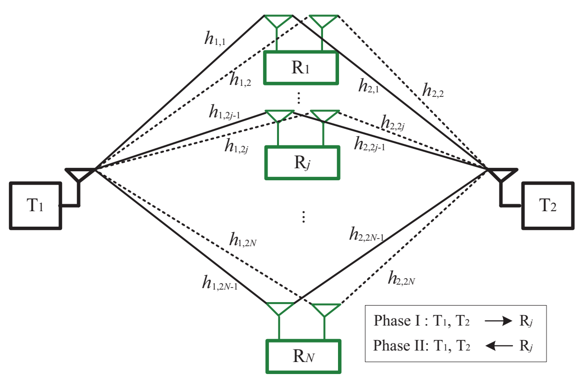

Fig. 1. Two-way relaying model with N dual-antenna relays.

2. TWO-WAY RELAY NETWORKS AND LOWER BOUND ON DIVERSITY GAIN

2.1. Two-Way Networks with Multiple Dual-Antenna Relays

In this section, we consider a half-duplex amplify-and-forward two-way relaying network consisting of two sources with each having a single antenna and N relays with each having two antennas, as shown in Fig. 1. The two-way relaying transmission can be described as follows.

There are in total $4N$ transmission time slots. In the first communication phase covering the first $2N$ consecutive time slots, source node T_k for $k = 1, 2$ transmits its messages $\mathbf{s}_k = \left[s_{k,1}, s_{k,2}, \cdots, s_{k,2N}\right]^T$ to all relay nodes, with the transmission power being P_k. At the l-th time slot, relay node R_j receives two by one signal vector $\mathbf{r}_{j,l} = [r_{j,l}^{(1)}, r_{j,l}^{(2)}]^T$, given by

$$\mathbf{r}_{j,l} = \sqrt{P_1}\mathbf{h}_{1,j}s_{1,l} + \sqrt{P_2}\mathbf{h}_{2,j}s_{2,l} + \mathbf{n}_{j,l}, l = 1, \cdots, 2N, \quad (1)$$

where $\mathbb{E}[\mathbf{s}_k\mathbf{s}_k^H] = \mathbf{I}_{2N}$ and $\mathbf{h}_{k,j} = [h_{k,2j-1}, h_{k,2j}]^T$ denotes the the channel vector between T_k and R_j, $j = 1, \cdots, N$. It is assumed that $h_{k,l}$ are independent complex Gaussian random variables with each having zero-mean and variance Ω_k, and remains constant in one transmission block. $\mathbf{n}_{j,l}$ denote 2×1 noise vectors $\mathbf{n}_{j,l} = [n_{j,l}^{(1)}, n_{j,l}^{(2)}]^T$ and are assumed that all entries $n_{j,l}^{(i)}$ for $i = 1, 2$ are independent complex Gaussian random variables with each having zero-mean and variance σ^2. If we let the $4N$ by one vector $\mathbf{r}_j = [\mathbf{r}_{j,1}^T, \mathbf{r}_{j,2}^T, \cdots, \mathbf{r}_{j,2N}^T]^T$, then, we have

$$\mathbf{r}_j = \sqrt{P_1}\mathbf{H}_{1,j}\mathbf{s}_1 + \sqrt{P_2}\mathbf{H}_{2,j}\mathbf{s}_2 + \mathbf{n}_j, \quad (2)$$

where $\mathbf{H}_{k,j} = \mathbf{I}_{2N} \otimes \mathbf{h}_{k,j}$ and $\mathbf{n}_j = [\mathbf{n}_{j,1}^T, \mathbf{n}_{j,2}^T, \cdots, \mathbf{n}_{j,2N}^T]^T$.

In the second communication phase capturing the next $2N$ consecutive time slots, each relaying node R_j first combines the $4N$ received signals into new symbols using linear dispersion coding such that

$$\mathbf{t}_{j,l} = \sqrt{\beta}(\mathbf{A}_{j,l}\mathbf{r}_j + \mathbf{B}_{j,l}\mathbf{r}_j^*), \quad (3)$$

where $\mathbf{A}_{j,l}$ and $\mathbf{B}_{j,l}$ are $2 \times 4N$ coding matrices for the j-th relay at the l-th time slot and β is an amplifier that is chosen in such a way that the average power per transmission per relay antenna is exactly P_r, and $\mathbf{t}_{j,l} = [t_{j,l}^{(1)}, t_{j,l}^{(2)}]^T$ with $t_{j,l}^{(i)}$ being the l-th transmitted signal from the i-th antenna of the R_j node. Without loss of generality, we assume that the total power per symbol transmission used in the whole network is fixed to be 1, i.e., $2NP_r + P_1 + P_2 = 1$. Then, these coded two by one signal vectors $\mathbf{t}_j^{(l)}$ are simultaneously transmitted to both the source nodes in the second $2N$ consecutive time slots. Hence, the signal received at source node T_k can be written as

$$\mathbf{y}_k = \sum_{j=1}^{N}\left(h_{k,2j-1}\mathbf{t}_j^{(1)} + h_{k,2j}\mathbf{t}_j^{(2)}\right) + \boldsymbol{\eta}_k, \quad (4)$$

where $\mathbf{t}_j^{(i)} = [t_{j,1}^{(i)}, t_{j,2}^{(i)}, \cdots, t_{j,2N}^{(i)}]^T$ and $\boldsymbol{\eta}_k = [\eta_{k,1}, \eta_{k,2}, \cdots, \eta_{k,2N}]^T$ is a $2N$ by one complex Gaussian noise vector received at T_k with zero mean and covariance matrix $\sigma^2\mathbf{I}_{2N}$.

2.2. Universal diversity gain bounds

The first major result of this paper is to establish a universal lower bound on the diversity gain function for any linear dispersion coded channel model (4), i.e.,

Theorem 1 *Let $P(\mathbf{s}_k \to \mathbf{s}_k')$ denote average pair wise error probability of the ML detector at source node T_k for $k = 1, 2$. Then, there exist two constants $C_{1,N}$ and $C_{2,N}$ independent of SNR, ρ such that*

$$P(\mathbf{s}_1 \to \mathbf{s}_1') \geq C_{1,N}\rho^{-2N}\ln^N\rho \quad (5a)$$

$$P(\mathbf{s}_2 \to \mathbf{s}_2') \geq C_{2,N}\rho^{-2N}\ln^N\rho \quad (5b)$$

Theorem 1, whose proof is omitted due to space limitation, tells us that the PEP of the ML detector for any linear dispersion coded channel model (4) cannot decay faster than $\ln^N\rho/\rho^{2N}$ as SNR tends to infinity, which is the best diversity gain function that is possibly enabled by a distributed linear dispersion code.

2.3. Optimal power loading

Power loading among source nodes and relay nodes significantly affects the overall performance of the whole relay system [11, 23, 24]. One solution of the optimal power allocation can be obtained by maximizing the received SNR of the worse link, which is given in the following theorem.

Theorem 2 *The optimal power loading to maximize the average received SNR of the worst link is determined as follows:*

$$P_1 = \frac{\sqrt{\Omega_2}}{2(\sqrt{\Omega_1} + \sqrt{\Omega_2})}, P_2 = \frac{\sqrt{\Omega_1}}{2(\sqrt{\Omega_1} + \sqrt{\Omega_2})}, P_r = \frac{1}{4N}.$$

Theorem 2, the proof for which is omitted because of space limitation, reveals that the optimal total power assigned to all relays is half of the total network power, regardless of the channel variances.

3. OPTIMAL DISTRIBUTED CONCATENATED STBC

The primary purpose of this section is to design an explicit distributed STBC for $N = 2^b$ to achieve the lower bound of the diversity gain provided by Theorem 1.

3.1. Optimal precoding

Let $\mathbf{x}_k = [x_{k,1}, x_{k,2}, \cdots, x_{k,2N}]^T$, where each entry $x_{k,l}$ for $l = 1, \cdots, 2N$ is the signal generated from a standard quadrature amplitude modulation (QAM) constellation $\mathcal{Q}$ with unit power. At source node T_k, $2N$ complex constellation symbols of $\mathbf{x}_k$ are divided into two groups, i.e., $\mathbf{x}_{k,\mathrm{o}} = [x_{k,1}, x_{k,3}, \cdots, x_{k,2N-1}]^T$ and $\mathbf{x}_{k,\mathrm{e}} = [x_{k,2}, x_{k,4}, \cdots, x_{k,2N}]^T$, as shown in Fig. 2. To achieve full diversity, both groups are first precoded individually by an angle rotation matrix $\mathbf{D}$ to be determined later, the inverse discrete Fourier transform matrix (IDFT) $\mathbf{W}_N^H$ and the Hadamard matrix, and then combined into the transmitted signal $\mathbf{s}_k$. More specifically, the whole described processing can be expressed by

$$\mathbf{s}_{k,\mathrm{o}} = \mathbf{P}\mathbf{x}_{k,\mathrm{o}}, \quad (6a)$$

$$\mathbf{s}_{k,\mathrm{e}}^* = \mathbf{P}^*\mathbf{x}_{k,\mathrm{e}}^*, \quad (6b)$$

where $\mathbf{P} = \frac{1}{\sqrt{N}}\mathrm{Hadamard}(N)\mathbf{W}_N^H\mathbf{D}$ with $\mathrm{Hadamard}(N)$ being an $N \times N$ Hadamard matrix and $\mathbf{W}_N$ being an $N \times N$ DFT matrix, i.e., $\mathbf{W}_N(p,q) = \frac{1}{\sqrt{N}}e^{-j2\pi pq/N}$, $p, q = 1, \cdots, N$. It will be shown that to ensure the maximum diversity gain and the maximum coding gain , the angle rotation matrix should be chosen to be $\mathbf{D} = \mathrm{diag}(1, e^{j\frac{\pi}{2N}}, \cdots, e^{j(N-1)\frac{\pi}{2N}})$. Now, combining (6a) with (6b) yields

$$\widetilde{\mathbf{s}}_k = \mathbf{E}\mathrm{diag}(\mathbf{P}, \mathbf{P}^*)\mathbf{E}^H\widetilde{\mathbf{x}}_k, \quad (7)$$

where $\widetilde{\mathbf{s}}_k = [s_{k,1}, s_{k,2}^*, \cdots, s_{k,2N-1}, s_{k,2N}^*]^T$, $\widetilde{\mathbf{x}}_k = [x_{k,1}, x_{k,2}^*, \cdots, x_{k,2N-1}, x_{k,2N}^*]^T$, and $\mathbf{E}$ is a $2N \times 2N$ elementary permutation matrix which permutes $[\mathbf{x}_{k,\mathrm{o}}^T, \mathbf{x}_{k,\mathrm{e}}^T]^T$ into $\mathbf{x}_k$, i.e.,

$$\mathbf{x}_k = \mathbf{E}\begin{pmatrix} \mathbf{x}_{k,\mathrm{o}} \\ \mathbf{x}_{k,\mathrm{e}} \end{pmatrix}.$$

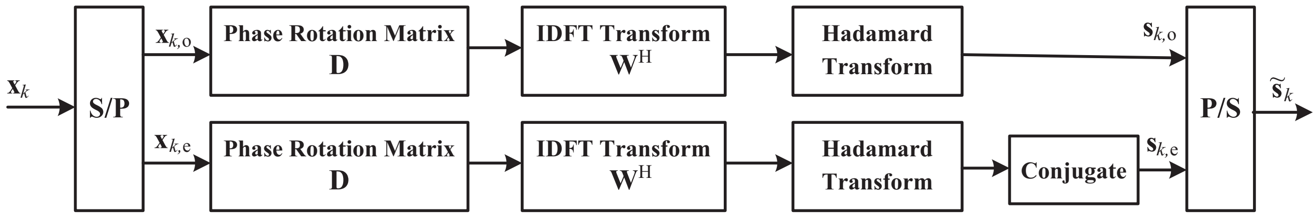

Fig. 2. Diagram illustration of the signal design at T_k for the proposed distributed concatenated RAC STBC

3.2. Distributed RAC STBC

In the first communication phase having $2N$ consecutive time slots, $\widetilde{\mathbf{s}}_1$ and $\widetilde{\mathbf{s}}_2$ are transmitted to the relays. Then, the relays generate the coded signals by using linear dispersion coding and *properly* combining the received noisy signals for transmission in the next consecutive $2N$ time slots. To clearly describe our coding scheme, we need to introduce the following two definitions.

Definition 1 *A family of $2^n \times 2^n$ matrices, each called recursive Alamouti circular matrix (RACM) and denoted by RACM_n, is recursively defined by*

$$\mathrm{RACM}_{\ell+1} = \left\{ \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_2 & \mathbf{M}_1 \end{bmatrix}, \mathbf{M}_1, \mathbf{M}_2 \in \mathrm{RACM}_\ell \right\},$$

for $\ell = 1, 2, \cdots, n-1$, where

$$\mathrm{RACM}_1 = \left\{ \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, a, b \in \mathbb{C} \right\},$$

Definition 2 *A $2^b \times 2^b$ index matrix $\mathbf{L}_b$ is recursively defined by*

$$\mathbf{L}_t = \begin{bmatrix} \mathbf{L}_{t-1} & 2^t \mathbf{1}_{t-1} + \mathbf{L}_{t-1} \\ 2^t \mathbf{1}_{t-1} + \mathbf{L}_{t-1} & \mathbf{L}_{t-1} \end{bmatrix}$$

for $t = 1, 2, \cdots, b$, where $\mathbf{L}_0 = 1$ and $\mathbf{1}_{t-1}$ is a $2^{t-1} \times 2^{t-1}$ matrix with each entry being one.

If we let $\mathbf{L}_b = (L_{j,i})_{N \times N}$, then, our coding scheme is described as follows: Based on (3), i.e., $\mathbf{t}_{j,l} = \sqrt{\beta}(\mathbf{A}_{j,l}\mathbf{r}_j + \mathbf{B}_{j,l}\mathbf{r}_j^*)$, where the coding matrices $\mathbf{A}_{j,l}$ and $\mathbf{B}_{j,l}$ are defined by

$$\begin{cases} \mathbf{A}_{j,2i-1}(1, 2L_{j,i}-1) & = 1 \\ \mathbf{A}_{j,2i-1}(2, 2L_{j,i}) & = 1 \\ \mathbf{A}_{j,2i}(1, 2L_{j,i}+1) & = 1 \\ \mathbf{A}_{j,2i}(2, 2L_{j,i}+2) & = 1 \end{cases}, \begin{cases} \mathbf{B}_{j,2i-1}(1, 2L_{j,i}+2) & = 1 \\ \mathbf{B}_{j,2i-1}(2, 2L_{j,i}+1) & = -1 \\ \mathbf{B}_{j,2i}(1, 2L_{j,i}) & = -1 \\ \mathbf{B}_{j,2i}(2, 2L_{j,i}-1) & = 1 \end{cases}$$

and all the remaining elements of $\mathbf{A}$ and $\mathbf{B}$ are zeros. Correspondingly, each codeword matrix at the relay nodes is given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{1,1} & \mathbf{R}_{1,2} & \cdots & \mathbf{R}_{1,N} \\ \mathbf{R}_{2,1} & \mathbf{R}_{2,2} & \cdots & \mathbf{R}_{2,N} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{R}_{N,1} & \mathbf{R}_{N,2} & \cdots & \mathbf{R}_{N,N} \end{bmatrix}, \quad (8)$$

where

$$\mathbf{R}_{j,i} = \begin{bmatrix} r^{(1)}_{j,L_{j,i}} + r^{(2)*}_{j,L_{j,i}+1} & r^{(1)}_{j,L_{j,i}+1} - r^{(2)*}_{j,L_{j,i}} \\ r^{(2)}_{j,L_{j,i}} - r^{(1)*}_{j,L_{j,i}+1} & r^{(1)*}_{j,L_{j,i}} + r^{(2)}_{j,L_{j,i}+1} \end{bmatrix}.$$

Then, the received signal at the source node T_k is represented by

$$\mathbf{y}_k = \sqrt{\beta}\mathbf{R}^T \mathbf{h}_k + \boldsymbol{\eta}_k, \quad (9)$$

where $\boldsymbol{\eta}_k = [\eta_{k,1}, \eta_{k,2}, \cdots, \eta_{k,2N}]^T$ still be a $2N$ by one complex Gaussian noise vector received at the source node T_k. Here, it should be pointed out that since the average transmission power per symbol used at each antenna of the relay nodes is normalized to be P_r, the amplifier β in (9) must be accordingly chosen in such a way that $\beta = \frac{P_r}{2(\Omega_1 P_1 + \Omega_2 P_2 + \sigma^2)} \approx \frac{P_r}{2(\Omega_1 P_1 + \Omega_2 P_2)}$. For detection convenience, equation (9) can be rewritten as

$$\widetilde{\mathbf{z}}_k = \sqrt{P_{3-k}\beta}\mathbf{G}_k \mathbf{s}_{3-k} + \boldsymbol{\epsilon}_k, , \quad (10)$$

where $\widetilde{\mathbf{z}}_k = [z_{k,1}, z^*_{k,2}, z_{k,3}, z^*_{k,4}, \cdots, z_{k,2N-1}, z^*_{k,2N}]^T$, with $\mathbf{z}_k$ denoting the received signal vector at $\mathrm{T_k}$ after its self-interference has been completely eliminated from $\mathbf{y}_k$, and $\mathbf{G}_k$ is exactly a $2N \times 2N$ RACM, the first two row of which is $[\mathcal{H}_{k,1}\mathcal{H}^T_{3-k,1}, \mathcal{H}_{k,2}\mathcal{H}^T_{3-k,2}, \cdots, \mathcal{H}_{k,N}\mathcal{H}^T_{3-k,N}]$. In addition,

$$\boldsymbol{\epsilon}_k = \sqrt{\beta} \begin{bmatrix} \sum_{j=1}^N \mathcal{H}_{k,j}\mathbf{w}_{j,1} \\ \sum_{j=1}^N \mathcal{H}_{k,j}\mathbf{w}_{j,2} \\ \cdots \\ \sum_{j=1}^N \mathcal{H}_{k,j}\mathbf{w}_{j,N} \end{bmatrix} + \begin{bmatrix} \widetilde{\boldsymbol{\eta}}_{k,1} \\ \widetilde{\boldsymbol{\eta}}_{k,2} \\ \cdots \\ \widetilde{\boldsymbol{\eta}}_{k,N} \end{bmatrix},$$

where $\widetilde{\boldsymbol{\eta}}_{k,j} = [\eta_{k,2j-1}, \eta^*_{k,2j}]^T$, $\mathbf{w}_{j,i} = [w^{(1)}_{j,i}, w^{(2)}_{j,i}]^T = [n^{(1)}_{j,L_{j,i}} + n^{(2)*}_{j,L_{j,i}+1}, n^{(1)*}_{j,L_{j,i}+1} - n^{(2)}_{j,L_{j,i}}]^T$ and $\mathcal{H}_{k,j} = \begin{bmatrix} h_{k,2j-1} & -h_{k,2j} \\ h^*_{k,2j} & h^*_{k,2j-1} \end{bmatrix}$. Substituting $\mathbf{s}_k = \mathbf{E}\mathrm{diag}(\mathbf{P}, \mathbf{P})\mathbf{E}^H \mathbf{x}_k$ into (10) yields

$$\widetilde{\mathbf{z}}_k = \sqrt{P_{3-k}\beta}\mathbf{G}_k \mathbf{E}\mathrm{diag}(\mathbf{P}, \mathbf{P})\mathbf{E}^H \mathbf{x}_{3-k} + \boldsymbol{\epsilon}_k, \quad (11)$$

Here, it is noted that the equivalent channel matrix $\mathbf{G}_k$ in (11) is a specific RACM, with each 2×2 block sub-matrix being the product of two Alamouti matrices. It is for this reason that we call our code as *distributed concatenated RAC STBC*. For visual understanding of our code design, we demonstrate three typical examples below.

Example 1 *Consider the two-way relaying network with one dual-antenna relay. In this case, we have $\mathbf{G}_k = \mathcal{H}_{k,1}\mathcal{H}^T_{3-k,1}$, $\mathbf{E} = \mathbf{I}_2$, and $\mathbf{P} = 1$, thereby, resulting in $\mathbf{s}_k = \mathbf{x}_k$.*

Example 2 *Consider the two-way relaying network with two dual-antenna relays. In this case, we have*

$$\mathbf{G}_k = \begin{bmatrix} \mathcal{H}_{k,1}\mathcal{H}^T_{3-k,1} & \mathcal{H}_{k,2}\mathcal{H}^T_{3-k,2} \\ \mathcal{H}_{k,2}\mathcal{H}^T_{3-k,2} & \mathcal{H}_{k,1}\mathcal{H}^T_{3-k,1} \end{bmatrix}.$$

In addition, $\mathbf{P} = \mathbf{D} = \mathrm{diag}(1, e^{j\pi/4})$ and $\mathbf{E}$ is obtained by exchanging the second row and the third row of $\mathbf{I}_4$. Hence, we have $\mathbf{s}_k = \mathrm{diag}(1, e^{j\pi/4}, 1, e^{j\pi/4})\mathbf{x}_k$.

3.3. PEP analysis

Theorem 3 *For any pair of distinct transmitted signal vectors* $\mathbf{x}_k$ *and* $\mathbf{x}'_k$*, the resulting error signal matrix* $\Delta\mathcal{S}_k$ *has full column rank and the following two asymptotic formulae of PEP hold:*

$$P\left(\mathbf{s}_2 \to \mathbf{s}'_2\right) = \frac{2^{2N-1}(4N-1)!!\rho^{-2N}\ln^N \rho}{(4N)!!N^{2N}P_2^{2N}\Omega_1^{2N}\Omega_2^{2N}\beta^{2N}\det(\Delta\mathcal{S}_2^H\Delta\mathcal{S}_2)} + O\left(\frac{\ln^{N-1}\rho}{\rho^{2N}}\right),$$

$$P\left(\mathbf{s}_1 \to \mathbf{s}'_1\right) = \frac{2^{2N-1}(4N-1)!!\rho^{-2N}\ln^N \rho}{(4N)!!N^{2N}P_1^{2N}\Omega_1^{2N}\Omega_2^{2N}\beta^{2N}\det(\Delta\mathcal{S}_1^H\Delta\mathcal{S}_1)} + O\left(\frac{\ln^{N-1}\rho}{\rho^{2N}}\right)$$

when SNR is sufficiently large.

The proof of Theorem 3 is omitted due to space limitation. We would like to make the following observations on Theorem 3.

1. Theorem 3 reveals that the code design presented in this paper enables the ML detector to achieve the diversity-gain lower bound given in Theorem 1.
2. It can be also observed that in addition to the optimal diversity gain function, the asymptotic PEP performance is dominated by two quantities, $\min\det(\Delta\mathcal{S}_1^H\Delta\mathcal{S}_1)$ and $\min\det(\Delta\mathcal{S}_2^H\Delta\mathcal{S}_2)$, each of which, following the concept from the MIMO system, is called *coding gain*. We can prove that the code design presented in this paper also enables the optimal coding gain.

4. SIMULATIONS

Throughout the simulations of this section, we assume that both the source nodes know perfect channel state information and that the relay nodes only know the first and second order statistics of the channels. All the bit error rate (BER) curves are shown as a function of SNR ρ. We carry out computer simulations and examine error performance by comparing the following half-duplex two-way relay networks:

(a) The two-way relaying network composed of two source nodes and $2N$ relay nodes with each employing a single antenna [12, 26–28].

(b) The two-way relaying network assisted by N dual-antenna relays using the code design proposed in Section 3.

Fig. 3 gives the BER comparison of network (a) and network (b) by using optimal power allocation (OPA) over the symmetric channels with $\Omega_1 = \Omega_2 = 1$, where the BER curves are obtained by averaging the BER values at two source nodes. It can be observed from Figs. 3 that the network (b) outperforms the network (a) in the whole SNR region, particularly when SNR becomes large, since the slopes for the network (b) are always better than those of the network (a). This observation is consistent with our asymptotic PEP analysis , i.e., the full diversity gain function for the network (a) is proportional to $\rho^{-2N}\ln^{2N}\rho$, whereas the full diversity gain function for the network (b) is proportional to $\rho^{-2N}\ln^{N}\rho$ when $2N$ relay antennas are used. Figs. 4 and 5 further illustrate the impact of power allocation on the BER performance of the proposed network (b) over asymmetric channels, i.e., $\Omega_1 = 1$, $\Omega_2 = 3$. Figs. 4 and 5 demonstrate the BER at T_2 and T_1, respectively. We

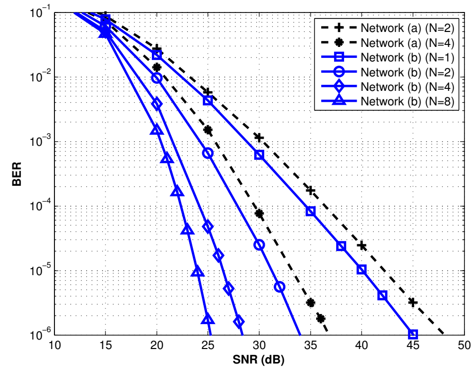

Fig. 3. Average BER performance comparison of the network (a) and the network (b) over symmetric channels $\Omega_1 = \Omega_2 = 1$ for 4-QAM with $P_1 = P_2 = \frac{1}{4}$ and $P_r = \frac{1}{4N}$.

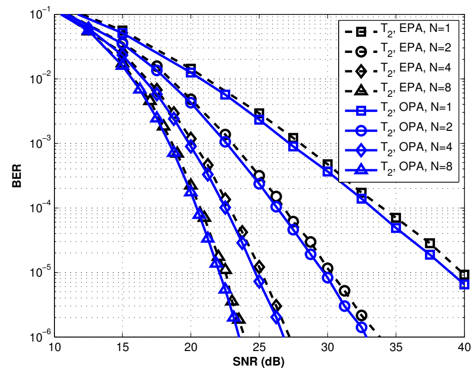

Fig. 4. BER of $\mathrm{T}_1 \to \mathrm{T}_2$ for $\Omega_1 = 1, \Omega_2 = 3$ and 4-QAM with OPA, $P_1 = \frac{3-\sqrt{3}}{4}, P_2 = \frac{\sqrt{3}-1}{4}, P_r = \frac{1}{4N}$ and EPA, $P_1 = P_2 = \frac{1}{3}, P_r = \frac{1}{6N}$

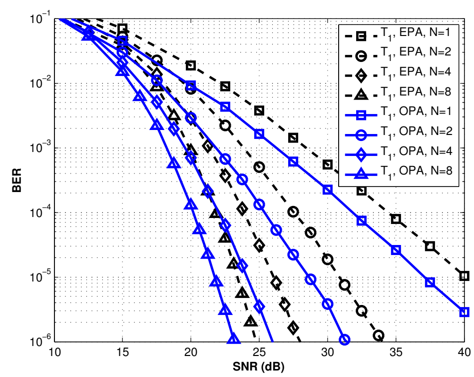

Fig. 5. BER of $\mathrm{T}_2 \to \mathrm{T}_1$, the same conditions as Fig. 4 are adopted

can observe from these two figures that the OPA given by Theorem 2 indeed enhances the error performance of the whole network, compared with conventional equal power allocation (EPA). Specifically, at the BER of 10^{-4}, the SNR gains of OPA over EPA are about 0.3 dB$-$0.8 dB for $\mathrm{T}_1 \to \mathrm{T}_2$ link, and about 1.5 dB$-$2.5 dB for the reverse $\mathrm{T}_2 \to \mathrm{T}_1$ link.

5. REFERENCES

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